Quenching of Stretched Magnetic Transitions

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Quenching of the $M6$ transition strengths in inelastic scattering to 6^- states in ^{28}Si is predicted when the $d_{5/2}$ shell-model space is enlarged to include the $s_{1/2}$ single-particle orbital. Nucleon transfer cross sections to the yrast $(I,T)=(6^-,0)$ and $(6^-,1)$ states and $B(M1)$ between them are also predicted to decrease when the extended space is used. The calculations suggest that the data may be understood in terms of only nucleon degrees of freedom once an adequate model space is used.

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In both the $(1d,2s)$ and $1f_{7/2}$ shells inelastic scattering experiments[1-6] on even-even nuclei populating the high-spin "stretched" magnetic states have cross sections very much smaller than predicted by the $(d_{5/2}{}^n)\to[d_{5/2}{}^{n-1}\otimes f_{7/2}]_{6^-;T}$ and $(f_{7/2}{}^n)\to[f_{7/2}{}^{n-1}\otimes g_{9/2}]_{8^-;T}$ models. This has led to the speculation that nonnucleonic degrees of freedom may have to be introduced to account for the observed quenching. In this note, we examine the effect of increasing the size of the model space and show that, for ^{28}Si, the inclusion of the $2s_{1/2}$ single-particle level substantially reduces the predicted cross sections. Other properties of these 6^- levels are examined and, in all cases, use of the larger space improves the agreement between theory and experiment. Although significant deviations from experimental results still exist, the calculations suggest that when a still larger model space is used the remaining discrepancies may become small.

The simplest model of ^{28}Si is one in which the 0^+ ground state is described by the configuration $(d_{5/2}{}^{12})_{I=0;T=0}$ and the $(I,T)=(6^-,0)$ and $(6^-,1)$ states have the form $[d_{5/2}{}^{11}\otimes f_{7/2}]_{I=6^-;T}$. With this model one predicts equal matrix elements for the transition operator $[a_{7/2;1/2}{}^\dagger\otimes\tilde{a}_{5/2;1/2}]_{6M;T0}$ which governs inelastic scattering between the ground state and these two 6^- levels,

$$|\langle\Psi_{I=6^-;T,0}\|[a_{7/2;1/2}{}^\dagger\otimes\tilde{a}_{5/2;1/2}]_{6;T,0}\|\Psi_{I=0;T=0,0}\rangle|^2=1. \tag{1}$$

In this expression $a_{jm;t\tau}{}^\dagger$ and $\tilde{a}_{jm;t\tau}$ are the spherical tensor creation and destruction operators for a nucleon in the state $(jm;t\tau)$; $[\ldots\otimes\ldots]_{IM;TT_z}$ implies vector coupling to angular momentum (IM) with isospin (TT_z); and the reduced matrix element $\langle\|\ldots\|\rangle$ times the Clebsch-Gordan coefficient equals the matrix element. Furthermore, the spectroscopic factor for populating these two 6^- states via the reaction ^{27}Al(^{3}He,d)^{28}Si is

$$C^2S=|\langle\Psi_{I=6^-;T,0}\|a_{7/2;1/2,-1/2}{}^\dagger\|\Psi_{I=5/2;1/2,1/2}\rangle|^2=\tfrac{1}{2}. \tag{2}$$

Experimentally, inelastic electron,[4] pion,[1] and proton[3,4] scattering to the yrast $(6^-,1)$ state all indicate that the square of the reduced matrix element in Eq. (1) between the physical $(0^+,0)$ and $(6^-,1)$ states should be approximately 0.3, while the pion and proton data to the yrast $(6^-,0)$ state yield an even smaller value, ~0.15, for this quantity. In stripping reaction Nann[7] finds $C^2S=0.24$ and 0.19 for these $T=0$ and $T=1$ states, respectively, whereas Kato and Okada[8] give 0.19 and 0.21 for the two values. Thus, the spectroscopic factors are approximately equal and substantially smaller than the $(d_{5/2},f_{7/2})$-model predictions. Finally, the value of $B(M1)$ between these two 6^- states has been measured[9] and is $(2.8\pm0.4)\mu_N{}^2$, where μ_N is the nuclear magneton. This is only about 20% of the $(d_{5/2},f_{7/2})$-model value of $14.439\mu_N{}^2$.

In the remainder of this note we consider the effect upon the foregoing predictions of including any number of particles in the $s_{1/2}$ orbit while still restricting the number of $f_{7/2}$ particles to be one. In this case the ground state of ^{28}Si becomes

$$\begin{aligned}\Psi_{00;00}=\alpha_1(d_{5/2}{}^{12})_{00;00}&+\alpha_2[(d_{5/2}{}^{10})_{1;0}\otimes(s_{1/2}{}^2)_{1;0}]_{00;00}+\alpha_3[(d_{5/2}{}^{10})_{0;1}\otimes(s_{1/2}{}^2)_{0;1}]_{00;00}\\&+\alpha_4[(d_{5/2}{}^9)_{1/2;1/2}\otimes(s_{1/2}{}^3)_{1/2;1/2}]_{00;00}+\alpha_5[(d_{5/2}{}^8)_{0;0}\otimes(s_{1/2}{}^4)_{0;0}]_{00;00}\end{aligned} \tag{3}$$

with the expansion coefficients, α_i, determined by diagonalizing the nuclear Hamiltonian. The dimensionality of the 6^- energy matrices is much larger than this and, when one includes all states of the configuration $[(d_{5/2}, s_{1/2})^{11} f_{7/2}]$, is 95 for $T=0$ and 144 for $T=1$. The nuclear Hamiltonian has two parts: (1) The single-particle and two-body interaction energies to be used in the $(d_{5/2}, s_{1/2})$ space. We examine the consequences of taking two different sets of matrix elements for these parameters. We consider those values determined by Wildenthal, McGrory, Halbert, and Glaudemans[10] (WMHG) from a least-squares fit to spectra and also those obtained by use of the modified surface delta potential with parameters given by Van Hienen, Glaudemans, and Van Lidth de Jeude.[11] (2) The two-body interactions involving the $(d_{5/2}, f_{7/2})$ and $(s_{1/2}, f_{7/2})$ configurations. These were calculated using the best-fit central spin-dependent potential of Schiffer and True[12] with $r_1 = 1.45$ fm and $r_2 = 2.0$ fm. Oscillator wave functions with $\nu = 0.293$ fm^{-2} $[\psi \sim \exp(-\frac{1}{2}\nu r^2)]$ were used in the evaluation. The only other parameter, the $f_{7/2}$ single-particle energy, was chosen to fit the excitation energy of the yrast $(6^-, 0)$ state. (When one allows only one $f_{7/2}$ particle this does not affect other properties of the negative-parity states.)

Even with this extended model space, the nuclear structure factor governing inelastic scattering to the stretched 6^- states is still given by the matrix element of $[a_{7/2;1/2}{}^{\dagger} \otimes \tilde{a}_{5/2;1/2}]_{6M;\,TT_z}$. Instead of the strength being concentrated in a single level for each isospin, as it is in the $d_{5/2}$ model, it is now spread over many states. If the strength to all possible states is summed, we obtain for either isospin a total strength

$$\Sigma = \tfrac{1}{12}\{12\alpha_1^2 + 10(\alpha_2^2 + \alpha_3^2) + 9\alpha_4^2 + 8\alpha_5^2\} \tag{4}$$

as opposed to the value unity given by Eq. (1). Therefore, even if all the strength is in one $(6^-, 0)$ and one $(6^-, 1)$ the value is less than unity because $\sum_i \alpha_i^2 = 1$. For the WMHG interaction $\Sigma = 0.854$ and for the modified surface delta potential $\Sigma = 0.872$. Any fractionation of the strength will yield a value less than Σ for the square of the matrix element to the yrast $T=0$ and $T=1$ states.

In Table I we list all 6^- states for which the quantities

$$R = \left| \frac{\langle \Psi_{6^-;\,T,0} \| [a_{7/2;1/2}{}^{\dagger} \otimes \tilde{a}_{5/2;1/2}]_{6;\,T,0} \| \Psi_{0;0,0} \rangle}{\langle [(d_{5/2}{}^{11})_{5/2;1/2} \otimes f_{7/2;1/2}]_{6^-;\,T,0} \| [a_{7/2;1/2}{}^{\dagger} \otimes \tilde{a}_{5/2;1/2}]_{6;\,T,0} \| (d_{5/2}{}^{12})_{0;0,0} \rangle} \right|^2, \tag{5}$$

$$Q = \left| \frac{\langle \Psi_{6^-;\,T0} \| a_{7/2;1/2,-1/2} \| \Psi_{5/2;1/2,1/2} \rangle}{\langle [(d_{5/2}{}^{11})_{5/2;1/2} \otimes f_{7/2;1/2}]_{6^-;T,0} \| a_{7/2;1/2,-1/2}{}^{\dagger} \| (d_{5/2}{}^{11})_{5/2;1/2,1/2} \rangle} \right|^2 \tag{6}$$

are greater than 0.05, i.e., all 6^- states that are predicted to have more than 5% of the pure $(d_{5/2}, f_{7/2})$ strength in either inelastic scattering, R, or stripping, Q. In contrast to the open-shell random-phase approximation,[13] which predicts that the entire strength for the stretched magnetic transitions should be concentrated in one state, we obtain a very substantial fractionation of Σ. The matrix element governing inelastic scattering to the yrast $(6^-, 1)$ state is reduced by almost a factor of 2 whereas that to the yrast $T=0$ is down by a factor of between 3 and 4. Thus, in agreement with experiment the extended model leads to the prediction that the $T=0$ strength is quenched more than the $T=1$. In agreement with observation, the spectroscopic factor for stripping to the yrast $(6^-, 0)$ level calculated with the exact $(d_{5/2}, s_{1/2})^{11}$ wave function for the ^{27}Al ground state is predicted to be between 40% and 50% of the $(d_{5/2}, f_{7/2})$ value. Stripping to the yrast $(6^-, 1)$ level and $B(M1)$ between the 6^- states are both substantially smaller than the $(d_{5/2}, f_{7/2})$ predictions, although they are both still larger than the experimental values.

It is important to ascertain the amount of spurious component in the wave functions describing the 6^- states. We have calculated this percentage and listed it in the columns labeled SP in Table I. Clearly the yrast $(6^-, 1)$ and $(6^-, 0)$ states are almost free of this so that their properties will not be affected when the spurious component is projected out. On the other hand, states with SP $\gtrsim 5\%$ are likely to have their properties appreciably changed when this deficiency is removed. Therefore, one should view with skepticism the predicted properties of states with SP greater than this amount.

Thus, the extended $(d_{5/2}, s_{1/2}, f_{7/2})$-model space greatly improves the agreement between theory and experiment. The main features that still require improvement are as follows:

(1) The energy splitting between the yrast $(6^-, 0)$ and $(6^-, 1)$ states is observed to be 2.78 MeV whereas both interactions give less than 1.5 MeV. Note, however, that the pure $(d_{5/2}, f_{7/2})$ model gives a splitting of only 704 keV. Extending the model space has increased this value by almost

TABLE I. All 6^- states in ^{28}Si predicted to have either inelastic scattering strength, R of Eq. (5), or spectroscopic strength, Q of Eq. (6), greater than 5% of the pure $(d_{5/2}, f_{7/2})$ sum-rule value. E is the predicted excitation energy and those states with $T=1$ have their energies underlined. The percent probability that the state is spurious is given in the columns labeled SP. The WMHG matrix elements are given in Ref. 10 and the parameters of the modified surface delta potential in Ref. 11. The value of $B(M1)$ connecting the yrast $(6^-, 1)$ and $(6^-, 0)$ states is given.

WMHG Interaction				Modified Surface Delta			
E	R	Q	SP	E	R	Q	SP
11.576	0.258	0.412	1.4	11.576	0.389	0.495	2.2
12.818	0.135	0.190	2.5	11.918	0.065	0.078	44.5
<u>13.020</u>	0.522	0.737	0.7	<u>12.765</u>	0.585	0.689	1.1
14.108	0.101	0.162	24.9	13.952	0.084	0.044	1.4
<u>14.845</u>	0.058	0.138	15.2	14.050	0.131	0.067	13.4
15.143	0.136	0.120	15.4	<u>14.571</u>	0.037	0.050	40.8
15.409	0.037	0.078	9.3	14.721	0.023	0.068	26.3
<u>16.327</u>	0.053	0.019	3.6	15.153	0.043	0.064	20.2
<u>18.474</u>	0.058	0.008	1.6	<u>15.764</u>	0.157	0.133	4.8
				<u>16.888</u>	0.039	0.056	6.7
$B(M1;(6^-,1) \to (6^-,0))$		$6.47\ \mu_N^2$			$8.32\ \mu_N^2$		

a factor of 2.

(2) The predicted inelastic cross sections and spectroscopic factors, particularly for the yrast $(6^-, 1)$ state, together with $B(M1; (6^-, 1) \to (6^-, 0))$ are still too large. As far as $B(M1)$ is concerned it is well known that when one does not include both spin-orbit partners in a calculation, theory and experiment are usually at variance.

(3) In pion scattering near the (3, 3) resonance, the cross section for excitation of a $T=0$ state is about a factor of 4 greater[1] than for a $T=1$ level with the same value of R. Therefore those $T=0$ states in Table I with $R \gtrsim 0.1$ and a small spurious component should have been observed. Only one state, the second $(6^-, 0)$ predicted by the WMHG interaction, satisfies these criteria and no evidence for this transition is seen.

There is abundant evidence, both experimental and theoretical, that the $d_{3/2}$ single-particle level must be included when one discusses nuclei near the middle of the (ds) shell. For example, a recent analysis of stripping and pickup experiments[14] leads to the conclusion that the ground state of ^{28}Si has 9.0 $d_{5/2}$, 1.5 $d_{3/2}$, and 1.5 $s_{1/2}$ nucleons outside the $A=16$ closed core. This experimental result is in excellent agreement with the theoretical expectations of Singhal *et al.*[15] who predict 8.95 $d_{5/2}$, 1.41 $d_{3/2}$, and 1.64 $s_{1/2}$ particles. Even when the $d_{3/2}$ orbit is included, inelastic scattering is still mediated by the matrix element of Eq. (1) and, according to the experimental occupation probabilities, Σ of Eq. (4) becomes $\frac{9}{12}=0.75$. Thus including $d_{3/2}$ decreases Σ and if additional fragmentation occurs, agreement between theory and experiment for the transition strengths appears possible. The model, nevertheless, must not predict strength where none is seen. Once the $d_{3/2}$ orbit is included, however, the size of the calculation, even in the simplest case, is more than an order of magnitude larger.[16]

In these calculations we have not attempted to fit experimental data, but instead have merely looked at the effect upon the properties of the 6^- levels in ^{28}Si of including not only the $d_{5/2}$ single-particle state but also the $s_{1/2}$. We find that with this inclusion one substantially reduces the scattering and stripping cross sections to the yrast (6^-, 0) and (6^-, 1) states and decreases the value of $B(M1)$ between the two. The improvement brought about by this extension suggests that once some aspects of the $d_{3/2}$ orbit are included, one may be able to bring theory and experiment into reasonable agreement. Although we have performed calculations only for ^{28}Si, one would expect the same sort of results for other (sd) nuclei and nuclei in the $f_{7/2}$ shell. Thus the massive quenching observed for the high-spin stretched magnetic states may be merely a manifestation of overtruncation of the shell-model configuration space.

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[1]C. Olmer *et al.*, Phys. Rev. Lett. 43, 612 (1979).

[2]S. Yen *et al.*, Phys. Lett. 93B, 250 (1980).

[3]G. S. Adams *et al.*, Phys. Rev. Lett. 38, 1387 (1977).

[4]R. A. Lindgren *et al.*, Phys. Rev. Lett. 42, 1524 (1979), and references therein.

[5]B. Zeidman and D. F. Geesaman, Nucl. Phys. A396, 419 (1983).

[6]R. A. Lindgren *et al.*, Phys. Rev. Lett. 46, 706 (1981).

[7]H. Nann, Nucl. Phys. A376, 61 (1982).

[8]S. Kato and K. Okada, J. Phys. Soc. Jpn. 50, 1440 (1981).

[9]K. A. Snover *et al.*, Phys. Rev. C 27, 493 (1983).

[10]B. H. Wildenthal, J. B. McGrory, E. C. Halbert, and P. W. M. Glaudemans, Phys. Lett. 26B, 692 (1968).

[11]J. F. A. Van Hienen, P. W. M. Glaudemans, and J. Van Lidth de Jeude, Nucl. Phys. A225, 119 (1974).

[12]J. P. Schiffer and W. W. True, Rev. Mod. Phys. 48, 191 (1976).

[13]D. J. Rowe and S. S. M. Wong, Nucl. Phys. A153, 561 (1970); D. J. Rowe, S. S. M. Wong, H. Chow, and J. B. McGrory, Nucl. Phys. A298, 31 (1978).

[14]B. S. Ishkhanov, I. M. Kapitonov, and A. V. Shumakov, Nulc. Phys. A394, 131 (1983).

[15]R. P. Singhal *et al.*, Nucl. Phys. A323, 91 (1979).

[16]If one restricts the calculation to at most one nucleon in $d_{3/2}$ and one in $f_{7/2}$ there are 1056 ways to make the (6^-, 0) and 1670 ways to make the (6^-, 1). For the complete configuration space $[(d_{5/2}, s_{1/2}, d_{3/2})^{11} f_{7/2}]$ (i.e., all possible distributions of eleven nucleons in $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ and at most one in $f_{7/2}$) these sizes become 28 908 and 53 637.