Supersymmetry Breaking by Instantons

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It is shown that instantons generate a superpotential in supersymmetric QCD with N colors and $N-1$ flavors.

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Supersymmetry offers the hope of understanding the gauge hierarchy.[1,2] As Witten has emphasized, such a hierarchy could come about if a term in the effective Lagrangian of a theory, which was zero to all orders in perturbation theory (because of nonrenormalization theorems[3]), received an exponentially small contribution from nonperturbative effects. This might lead to small, spontaneous supersymmetric breaking. Witten in fact exhibited examples of this phenomenon in one dimension, but results to date in higher dimensions have been discouraging.[4-6]

In this Letter, we will show that instantons in fact do generate a superpotential in supersymmetric QCD, for certain values of the number of flavors. This superpotential may spontaneously break supersymmetry in the massless limit. Supersymmetric QCD is a theory with an SU(N) gauge interaction, with N_f fields, Q_{ia} ($i=1,\ldots,N$; $a=1,\ldots,N_f$), in the N representation and N_f in the $\overline{N}$ representation, $\overline{Q}_a{}^i$.[7] For zero quark mass, the classical potential of this theory has a large vacuum degeneracy.[4,8] This degeneracy persists to all orders in perturbation theory as a result of the nonrenormalization theorems.

However, it is possible that nonperturbative effects may lift this degeneracy. Remarkably, there is a unique superpotential which respects all the nonanomalous flavor and color symmetries[8] for $N_f < N$:

$$W = b\int d^2\theta[\det_{aa'}\overline{Q}_a Q_{a'}]^{-1/(N-N_f)}. \qquad (1)$$

If this term is generated, and if a small (flavor-invariant) mass term is also present (at tree level), the theory has N supersymmetric vacuum states, in agreement with Witten's index theorem.[8] As $m\to 0$, these states move off to infinity.

The dynamics of this theory depend strongly on N_f. It is asymptotically free for $N_f < 3N$, and index-theorem arguments indicate unbroken supersymmetry in the massless limit for N_f an integer multiple of N.[9] The existence of this F term, and its origin, might thus be expected to depend on N_f. (For $N_f \geq N$, there is no invariant F term!)

Let us consider more explicitly how such a term might be generated. It is simplest to study the massless theory, and to focus on the flat directions in the potential.[8] We expect that the nonperturbative term in the effective potential of the massless theory (at large field strength) survives when a small mass term is added. To calculate the effective Lagrangian at nonzero $\langle Q\rangle$ and $\langle\overline{Q}\rangle$ in the massless limit, we can study the theory with boundary conditions on the scalars which break SU(N) but leave unbroken subgroups of the color and flavor symmetries including an anomalous chiral R symmetry.[8]

For $N_f \geq N-1$ the color group is completely broken. The spectrum of the theory, at tree level consists of massive vector and chiral multiplets and some massless, weakly interacting chiral fields. Thus when $\langle Q\rangle$ is large enough that $g^2(\langle Q\rangle) \ll 1$, all properties of the theory can be studied by a weak-coupling analysis. All nonperturbative effects are then in principle calculable with use of instanton methods. If the superpotential of Eq. (1) is in fact generated by instantons, we would expect the Yukawa term, W'', to be generated by a single instanton and to be $O(\exp(-8\pi^2/g^2))$. Then the scalar potential, $|W'|^2$, would be $O(\exp(-16\pi^2/g^2))$, and would arise from two-instanton effects.[5]

In the theory with SU(N) broken by the boundary conditions, the Yukawa coupling would produce, for the massless fermions, a mass term, $m\overline{\psi}\psi$, with

$$m \sim \langle Q\rangle^{-2N/(N-N_f)}.$$

However, this theory has an unbroken chiral R

symmetry under which the gluinos and quarks transform with opposite phase. This chiral symmetry is anomalous and the associated index theorem requires that there be at least $2|N-N_f|\nu$ zero modes in the sector of topological charge ν. Thus this mass term can only be generated in the one-instanton sector and then only if $N_f = N-1$. It is amazing that for $N_f = N-1$ a simple renormalization-group argument gives the power in Eq. (1). If a mass term is generated by an instanton effect, it must obey

$$m \sim \langle Q\rangle \exp[-8\pi^2/g^2(Q)] \sim \langle Q\rangle^{1-3N+N_f}.$$

For smaller values of N_f, the broken theory has a strongly interacting gauge sector and neither of these arguments applies. One may be able to argue that supersymmetry is broken in the massless limit of such theories because of strong interaction effects. However, that is beyond the scope of this paper.[10]

We now turn to the simplest theory in which instantons may generate an F term: that with two colors and one flavor. Before describing the calculation, we must say a few words about instantons in spontaneously broken gauge theories.[11-13] With $\langle Q\rangle = 0$, the classical solution is just the pure Yang-Mills instanton with all the scalar fields set equal to zero. With $\langle Q\rangle \neq 0$, a simple scaling argument shows that no nontrivial, finite-action classical solution exists. However, when the coupling constant is small, one expects that these are "almost" solutions. One can make this notion precise by introducing a constraint into the functional integral, and looking for classical solutions of the resulting effective theory, "constrained instantons." It is usually convenient to constrain the space-time integral of a local operator.[13] Upon Fourier transformation of the δ function which implements the constraint, one is led to work with a new Lagrangian (the "constraint Lagrangian") which includes the constrained operator. The constraint is chosen so as not to modify the long-range behavior of the new classical equations. Thus the massive scalar fields decay exponentially, as do the gauge fields (up to a gauge transformation). The value of the constraint introduces a length scale, ρ, into the classical solution. The constraint can be chosen so that, when $\rho\langle Q\rangle \ll 1$, the classical equations can be approximated by (for $r\langle Q\rangle \ll 1$)[11,13]

$$D_\mu F_{\mu\nu} = 0, \quad D^2 Q = D^2\overline{Q} = 0. \tag{2}$$

These equations possess well-known, exact solutions[11,14]:

$$\begin{aligned} A_\mu{}^a &= 2\eta_{a\mu\nu}x^\nu/(x^2+\rho^2), \\ Q_{\rm cl} &= \sigma\hat{x}[x^2/(x^2+\rho^2)]^{1/2}\langle Q\rangle, \\ \overline{Q}_{\rm cl} &= -\overline{\sigma}\hat{x}[x^2/(x^2+\rho^2)]^{1/2}\langle\overline{Q}\rangle. \end{aligned} \tag{3}$$

Here $\eta_{a\mu\nu}$ is 't Hooft's η symbol. These represent the first terms in an expansion in powers of $\langle Q\rangle$ of the solutions. (This expansion only gives the correct long-range behavior after it is summed.) Substituting the approximate constrained instanton into the action gives

$$S = g^{-2}[8\pi^2 + 2\pi^2\rho^2\langle Q\rangle^2].$$

Upon integrating over the constraint, ρ, we find that $\rho\langle Q\rangle$ becomes $O(g)$, so that the expansion in $\langle Q\rangle$ ultimately becomes an expansion in g.

It is simplest to constrain the value of a bosonic operator, i.e., an operator consisting only of bosonic fields. In this case the constraint Lagrangian, for fixed ρ, is not supersymmetric. The ρ integration, however, restores supersymmetry. Alternatively, one may constrain a supersymmetric operator. This introduces Yukawa (and multifermion) operators into the constraint Lagrangian. This complicates the analysis, but has the virtue of preserving supersymmetry at intermediate stages of the computation. We will work here with bosonic constraints, but we will discuss some features of supersymmetric constraints at the end of this Letter.

For $\langle Q\rangle = 0$, index theorems require six zero modes. Four of the zero modes are pure gluino. Two are obtained from supersymmetry transformations of the instanton,

$$\lambda_\alpha{}^{SS\,a(n)} \sim \sigma_\alpha{}^{\mu\nu n} F_{\rm inst}{}^{a\mu\nu} \sim \sigma_{\alpha n}{}^a (x^2+\rho^2)^{-2}, \tag{4}$$

where α is a spinor index and n labels the zero mode. Two are obtained from superconformal transformations,

$$\lambda_\alpha{}^{SC\,a(n)} \sim \sigma_\alpha{}^{\mu\nu}{}_\beta \not{x}_{\beta n} F_{\rm inst}{}^{a\mu\nu} \sim \sigma_{\alpha\beta}{}^a \not{x}_{\beta n} (x^2+\rho^2)^{-2}. \tag{5}$$

The other two are the usual isospin-$\frac{1}{2}$ zero modes for the quark and antiquark, ψ and $\overline{\psi}$:

$$\psi_{i\alpha} = \frac{\epsilon_{i\alpha}}{(x^2+\rho^2)^{3/2}}, \quad \overline{\psi}_\alpha{}^i = \frac{\delta_\alpha{}^i}{(x^2+\rho^2)^{3/2}}. \tag{6}$$

Note that there are only two zero modes associated with supersymmetry (and conformal supersymmetry) as a result of the self-duality of the instanton.

Let us consider how this picture is altered by a nonzero $\langle Q\rangle$, with a bosonic constraint. We must now study the spectrum of the generalized Weyl operator which includes the Yukawa coupling to the classical scalar field:

$$\mathcal{L}_Y = i\sqrt{2}(Q_{\rm cl}{}^{*i}\lambda_i{}^{\alpha j}\psi_{j\alpha} - \bar{\psi}^{i\alpha}\lambda_{i\alpha}{}^j\bar{Q}_{{\rm cl}\,j}{}^*).$$

The index theorem now requires only two zero modes. We may calculate the eigenvalues of the Weyl operator perturbatively in $\langle Q\rangle\rho$. To lowest order, we need consider only the mixing of the λ, ψ, and $\bar{\psi}$ zero modes due to $\mathcal{L}_Y$. If we examine the behavior of the classical solution, Eq. (3), and the zero modes, Eqs. (4)–(6), under $x_\mu \to -x_\mu$ it is easy to see that to lowest order the ψ and $\bar{\psi}$ modes can mix with the superconformal zero modes, $\lambda^{\rm SC}$, while the supersymmetry modes $\lambda^{\rm SS}$ must remain at zero. A simple computation verifies that in fact four of the zero modes are raised above zero by an amount of order $\rho\langle Q\rangle$, while two remain at zero, as required by the index theorem. As a result, a certain $\lambda\lambda$ matrix element is generated, of no particular interest. However, the Yukawa coupling also perturbs the two remaining zero-mode eigenfunctions, adding quark terms. This is most easily seen by noting that, in lowest order, the equation for $\psi^\dagger$ reads (suppressing indices)

$$\not{D}^\dagger\psi^\dagger = \sqrt{2}\,\lambda^{\rm SS}Q^{\rm cl\,\dagger}. \qquad (7)$$

This equation has the solution, noting $\lambda^{\rm SS} \sim \sigma^{\mu\nu}F^{\mu\nu}$,

$$\psi^\dagger = \sqrt{2}\not{D}Q^{\rm cl\,\dagger}. \qquad (8)$$

A similar expression holds for $\bar{\psi}^\dagger$. Since the quark fields have massless components, the zero-mode eigenfunctions fall off as $1/r^3$ [one can easily check that the massless modes are excited in Eq. (8)]. Thus they produce a $\bar{\psi}^\dagger\psi^\dagger$ mass term just as do the zero modes in ordinary QCD with one flavor. It is easy to show that this is precisely of the form expected from the superpotential of Eq. (1); the detailed calculation will be presented elsewhere.[10] One can also show that they produce a nonzero expectation value,

$$\langle 0|\{\bar{Q}^{\dot\alpha}, \psi_{\dot\alpha}{}^\dagger(x)\bar{Q}^\dagger(x)\}|0\rangle \sim \langle 0|\bar{\psi}^\dagger(x)\psi^\dagger(x)|0\rangle$$

(the $\bar{Q}^{\dot\alpha}$ are supersymmetry charges), indicating that the "vacuum energy" is nonzero in the theory with a boundary condition. (Strictly speaking, once a potential is generated, a source term must be added to keep $\langle Q\rangle$ fixed.) It is straightforward to show that higher-order corrections to the effective action are down by powers of g^2,[10] and cannot cancel the contribution that we have found here.

Thus, we have generated the desired Yukawa coupling. Nothing in the calculation suggests that the complete effective Lagrangian is not supersymmetric. The $\lambda^4\bar{\psi}\psi$ and $\lambda\lambda$ operators generated by the instanton are presumably parts of D terms in the effective Lagrangian, examples being $\mathfrak{D}^\alpha W^2 Q\mathfrak{D}_\alpha W^2\bar{Q}|_D$ and $\bar{D}_{\dot\alpha}(Q^\dagger e^V Q)\bar{D}^{\dot\alpha}(\bar{Q}^\dagger e^V Q)$, where $\mathfrak{D}_\alpha$ denotes the supersymmetric covariant derivative.

Finally, let us discuss briefly how the calculation would look with a supersymmetric constraint. We assume that the constraint preserves the chiral symmetry. The classical solution in the presence of the constraint cannot be self-dual. Thus, we must have four supersymmetric zero modes (the theory does not have superconformal symmetry). Two of these are chirality minus and two are chirality plus, and so there must be two additional zero modes to satisfy the index theorem. We may again proceed perturbatively in $\langle Q\rangle\rho$. To lowest order there are six zero modes as before. However, they must remain at zero in this case, to all orders in $\langle Q\rangle\rho$. This is entirely possible since the constraint introduces additional Yukawa couplings which modify the perturbation of the zero modes. Clearly, the two initial supersymmetry modes, $\lambda^{\rm SS}$, retain their interpretation. The two quark zero modes, ψ and $\bar{\psi}$, become the other two supersymmetry zero modes. This can be seen by noting that

$$\{\bar{Q}_{\dot\alpha}, \psi_\alpha\} = \not{D}_{\alpha\dot\alpha}{}^\dagger Q, \quad \{\bar{Q}_{\dot\alpha}, \lambda_\alpha\} = 0,$$

and that $\not{D}^\dagger Q_{\rm cl}$ and $\not{D}^\dagger\bar{Q}_{\rm cl}$ are, in fact, the lowest-order quark zero modes since

$$\not{D}\not{D}^\dagger Q = D^2 Q + \sigma^{\mu\nu}(F^{\mu\nu} - \tilde{F}^{\mu\nu})Q \cong 0$$

[by Eq. (2)]. The mixing of the $\lambda^{\rm SS}$ modes with $\psi^\dagger$, $\bar{\psi}^\dagger$ is also predicted by supersymmetry since

$$\{Q^\alpha, \psi^{\dagger\dot\alpha}\} = \not{D}^{\dot\alpha\alpha}Q^\dagger.$$

This is precisely the perturbation obtained previously. Since the six zero modes remain precisely at zero, in this case it must be the quantum fluctuations of the scalar fields which, through their Yukawa couplings, are responsible for the $\psi^\dagger\bar{\psi}^\dagger$ matrix element. This calculation seems considerably more complicated than the previous one, since the constraint must be considered explicitly. We have not performed it. There is no reason, however, to think it should give a different result.

The last statement requires some discussion. In general, when it is possible to introduce four supersymmetry collective coordinates, one cannot generate F terms.[1,5,15] However, one cannot introduce such coordinates for supersymmetry charges which annihilate the classical solution (the perturbative sector being an obvious example). In our case, the charges $\bar{Q}$ *nearly* annihilate the classical solution. As a result, as we have shown, one can only admit two collective coordinates, and thus hope to generate F terms.[10]

In the general case of N colors and $N-1$ flavors, we have also shown that the superpotential of Eq. (1) is generated. For most orientations of the instanton, all but two of the zero modes are again lifted, and integration over the gauge group yields a nonzero result.[10]

We cannot, unfortunately, say how the theory behaves at small field strength, where the effective coupling is large. It may well be that the theory has no vacuum state at all, and can be given, at best, a cosmological interpretation. On the other hand, we cannot rule out the possibility that the theory has some number of strongly interacting, supersymmetric vacuum states. (However, we *have* saturated the index for nonzero mass.) There is also the possibility that the theory possesses some number of local minima. These may be separated by an infinite barrier from the large $\langle Q \rangle$ region.[16]

In any case, we have exhibited a nonperturbative breakdown of the nonrenormalization theorems.[5] Clearly this raises the hope that some theories (e.g., chiral theories) yield a more useful breaking of supersymmetry.

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