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Time-Dependent Superposition of Spinors

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Inverting the spin state of one of the two coherent waves propagating within a neutron interferometer by means of a radio-frequency spin-flip device leads to a nonstationary interference pattern. By using stroboscopic neutron detection one can resolve this to demonstrate the nonclassical behavior of spinor superposition.

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Recently it has been shown that neutron interferometry can be used to demonstrate the quantum mechanical principle of linear spin-state superposition for fermions in an explicit way.[1-3] This was achieved by inverting the spin state of one of the two initially equally polarized coherent waves which propagate along separate paths inside an interferometer. After subsequent coherent superposition of these waves a final spin state results which is orthogonal to each of the interfering states, and hence has no classical analog. In these earlier experiments the spin inversion was accomplished by means of a static polarization-turn device, exploiting the fact that the neutron spin cannot follow a sudden, nonadiabatic spatial change of the direction of an applied static magnetic field. Since no explicitly time-dependent interaction is involved in such a flipping process the total energy of the neutron is a constant of the motion. This means that the associated change of the Zeeman potential energy is exactly compensated by a corresponding inverse change of the kinetic energy, i.e., of the neutron wavelength. The two interfering waves will therefore differ in wavelength by an amount $\Delta\lambda = 2m|\mu|B\lambda^3/h^2$, where m is the mass of the neutron, $\vec{\mu}$ is its moment, and B is the strength of the magnetic field.[2] Though in a field of about 5 mT the wavelength difference is as small as $\Delta\lambda \sim 10^{-8}$ Å, this should be sufficient according to the dynamical theory of diffraction[4] to cause an appreciable decrease of the observable interference contrast.

A completely different physical situation arises if a radio-frequency flipper is used instead of a static flip device to invert the spin state of one of the partial beams within the interferometer. There the total energy of the neutrons is no longer conserved because of an exchange of photons of energy $\hbar\omega_{\mathrm{rf}}$ between the neutrons and the rf field. For an rf field $\vec{B}_{\mathrm{rf}}(t)$ that rotates in a plane perpendicular to the static field component $\vec{B}_0$ this interaction has a resonant maximum if the photon energy equals exactly the Zeeman energy difference between the two spin eigenstates of the neutron within the static field, that is if $\hbar\omega_{\mathrm{rf}} = 2|\mu|B_0$. After passage through such a flipper, neutrons which were initially polarized parallel to the direction of $\vec{B}_0$ (z direction) and had an energy $E = \hbar^2 k_+^2/2m + |\mu|B_0$ will have been flipped into the opposite direction and lost an amount of energy $\Delta E = 2|\mu|B_0$, whereas they maintain their initial momentum $\vec{k}_+$. This inelasticity feature of the neutron-photon interaction has recently been measured[5] by means of the high-energy resolution of the neutron back-

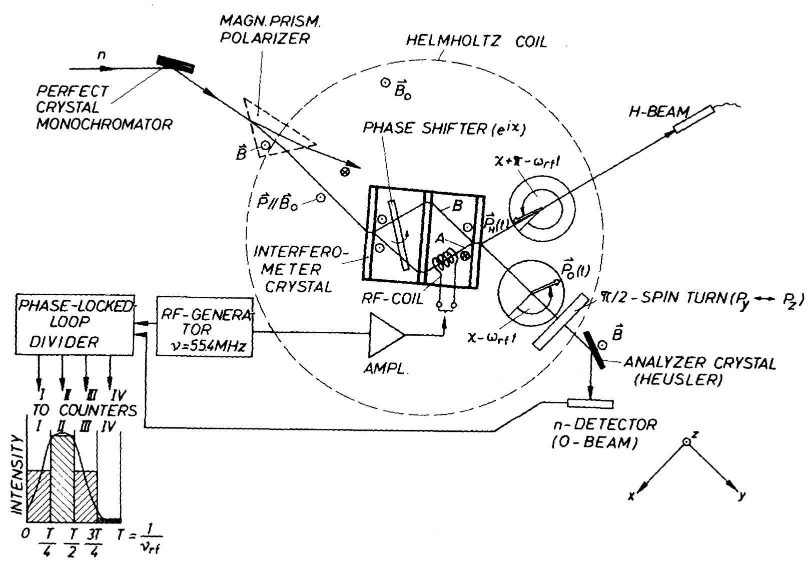

FIG. 1. Schematic of the spin-superposition experiment and of the stroboscopic neutron registration.

scattering technique.

As long as no rf field is acting on the two coherent beams A and B their wave functions belong to the same polarization state but may differ by an arbitrary phase factor $\exp(i\chi)$, where $\chi = k(1-n)\Delta D$ is given by the wave number k of the neutron, the index of refraction n, and the path difference ΔD within the phase shifter (Fig. 1). Choosing for simplicity a single-plane-wave representation for the neutron wave function, we describe the two beams by the spinors

$$|A\rangle = \exp(i\vec{k}_+\cdot\vec{r}_A)\exp[-(i/\hbar)Et]\,|\uparrow_z\rangle, \quad |B\rangle = \exp(i\chi)\exp(i\vec{k}_+\cdot\vec{r}_B)\exp[-(i/\hbar)Et]\,|\uparrow_z\rangle. \tag{1}$$

Here the unit spinor $|\uparrow_z\rangle$ is the "up" eigenstate of the Pauli spin matrix σ_z and it was implicitly assumed that the incident neutrons are fully polarized in the $+z$ direction. With neglect of the propagation-direction difference of the two interfering beams and omission of all common phase factors, the coherent superposition of states $|A\rangle$ and $|B\rangle$ yields a final state

$$|0\rangle = \tfrac{1}{2}|A\rangle + \tfrac{1}{2}|B\rangle = \tfrac{1}{2}(1+e^{i\chi})\,|\uparrow_z\rangle, \tag{2}$$

which is polarized in the $+z$ direction as well and exhibits the well-known oscillatory behavior of the intensity as a function of the phase shift χ, since

$$I(\chi) \propto \langle 0|0\rangle = \tfrac{1}{2}(1+\cos\chi). \tag{3}$$

If, however, the rf flipping field acts on one of the partial beams, say on beam A, the corresponding spinor is transformed into

$$|A\rangle' = \exp(i\vec{k}_+\cdot\vec{r}_A)\exp[-(i/\hbar)(E-\Delta E)t]\,|\downarrow_z\rangle. \tag{4}$$

Now the two interfering beams differ in energy; that means that they can never lead to a stationary interference pattern. In fact, written as a formula, the coherent superposition of state $|A\rangle'$ and state $|B\rangle$ gives

$$|0\rangle = \tfrac{1}{2}|A\rangle' + \tfrac{1}{2}|B\rangle = \tfrac{1}{2}\begin{pmatrix}\exp[(i/\hbar)\Delta E t]\\ \exp(i\chi)\end{pmatrix} = \frac{1}{\sqrt{2}}\cos\left[\frac{1}{2}\left(\chi - \frac{\Delta E}{\hbar}t\right)\right]|\uparrow_x\rangle - \frac{1}{\sqrt{2}}\,i\sin\left[\frac{1}{2}\left(\chi - \frac{\Delta E}{\hbar}t\right)\right]|\downarrow_x\rangle, \tag{5}$$

where again all common phase factors have been omitted. One can see that the outgoing beam is in a spin state which is orthogonal to the spin states of both interfering beams. But unlike the static spin-superposition experiment[1-3] this state is not stationary in time. The polarization vector beyond the interferometer will rotate in a plane perpendicular to the incident polarization direction with the frequency $\omega_{rf} = \Delta E/\hbar$ of the rf field, as can be seen immediately from

$$\vec{P}_0(t) = \frac{\langle 0|\vec{\sigma}|0\rangle}{\langle 0|0\rangle} = \begin{pmatrix}\cos(\chi - \omega_{rf}t)\\ \sin(\chi - \omega_{rf}t)\\ 0\end{pmatrix}. \tag{6}$$

This time-dependent rotation of the polarization can be detected if a stroboscopic registration of the neutrons is applied synchronously with the phase of the rf field. Figure 1 shows the scheme of the experiment that we have performed. Aside from the rf flipper and the phase-sensitive neutron detection system the setup is the same as that described in detail in Ref. 3. The incident monochromatic beam of mean wavelength $\lambda_0 = 1.835$ Å ($\Delta\lambda/\lambda_0 \sim 0.015$) is polarized by magnetic-prism refraction parallel to a static guide field B_0 which is produced by a large (diam ~ 60 cm) Helmholtz coil pair. Its absolute value at the position of the rf flip coil (length ~ 1 cm) is 1.90 mT, corresponding to a Larmor frequency $\omega_0/2\pi = 55.4$ kHz. Since the rf field is of oscillatory nature there is a Bloch-Siegert shift[6,7] between the resonance frequency $\omega_0 = 2|\mu|B_0$ that follows from the energy difference between the Zeeman levels and the effective resonance frequency $\omega_{eff} = \omega_0(1 + B_{rf}^2/4B_0^2)$ at which the flipping efficiency achieves its maximum. In our experiment $\omega_{rf} = \omega_0$ was chosen at the cost of flip efficiency in order to avoid any wavelength change of the beam passing through the rf coil. To transform the time-dependent interference pattern into an observable stationary one and to discriminate the polarization rotation given by Eq. (6) against a beam modulation caused by an unflipped contribution [Eq. (3)] the following procedure was applied. By means of an electronic phase-locked loop circuitry one period $T = 1/\nu_{rf}$ of the rf frequency was divided synchronously into four equal counting subintervals I–IV. Any time-independent contribution to the intensity that is reflected from the analyzer can be averaged out by subtracting the counts accumulated in different subintervals from each other. On the other hand, an intensity component which varies in time synchronously with the rf field leads to a pronounced mutual deviation of the count rates of the four subintervals as indicated also in Fig. 1. The optimal contrast is achieved by forming the difference count rate of intervals I and III or II and IV, respectively, since these have a mutual phase shift of just half a period.

Figure 2 shows the result of an interference experiment according to such a measuring procedure. As long as the $\pi/2$-spin-turn device in front of the analyzer is not in action, which means that the z component of the polarization is analyzed, no counting difference is observed if the geometrical path difference ΔD, and hence the phase shift χ between the interfering waves, is varied. This demonstrates that beyond the interferometer there is no z component of the polarization [Eq. (6)]. And in fact, after the z and y components of the polarization vector have been mutually interchanged by means of the static $\pi/2$-spin-turn device, the coherent oscillations of the detected intensity as a function of χ, which

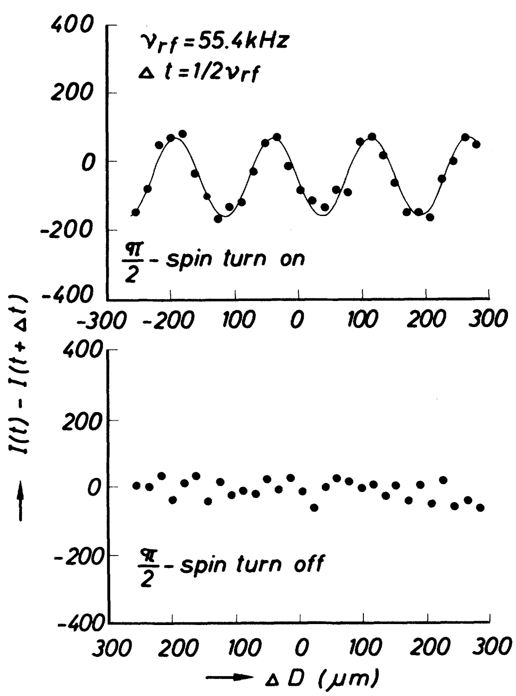

FIG. 2. Stroboscopic picture of the interference pattern. Observed intensity difference between two phase-locked subintervals separated in time by half a period of the rf field vs the path difference ΔD of the interfering beams. Coherent intensity oscillations beyond the analyzer appear only if the polarization component orthogonal to the initial spin directions is measured ($\pi/2$-spin-turn "on").

are an immediate consequence of Eq. (6), are clearly observed.

Having thus explicitly demonstrated the scheme of nonstationary superposition of spinors by this neutron interferometric experiment some clarifying remarks have to be made. Intuitively one might argue that, at least in principle, it should be possible to detect the passage of the neutron through the rf coil by detecting the change of the damping of the electronic resonance circuit which is caused by the emission or absorption of a photon during the flipping process. Although this would allow one to find out over which of the two possible paths within the interferometer the neutrons have propagated, at first sight it looks as if the interference pattern could nevertheless be observed in that case. However, to convert the time-dependent interference pattern into a stationary one by means of stroboscopic neutron registration the phase of the rf field has to be known with an accuracy $\Delta\Phi < 2\pi$. Because of the particle-number–phase uncertainty relation[8–10]

$$\Delta N \, \Delta\Phi \geqslant 2\pi, \tag{7}$$

the mean number of photons would be known only to $\Delta N > 1$. Therefore it is in principle impossible to detect single-photon transitions simultaneously with the interference pattern.

A similar argument holds if one were to try to find out which path the neutrons have taken by measuring the energy change $\Delta E = \hbar\omega_{\mathrm{rf}}$ that they undergo on their passage through the rf coil. In that case the spectral width of the incident beam has to be smaller than ΔE which outside of the region of the static guide field $\vec{B}_0$ results in a measurable velocity difference[5,11] $\Delta v = \mu B_0/mv$ between neutrons belonging to different propagation paths inside the interferometer. The observation of the interference pattern affords that the widths of the stroboscopic time channels fulfill the condition $\Delta t < 1/\omega_{\mathrm{rf}}$. But on the other hand, neutrons with a defined polarization phase can be accumulated within the correct time channel only if the condition $\Delta t > \Delta l/\Delta v$ is fulfilled, which runs into conflict with the momentum-position uncertainty relation concerning the possible distance Δl of the detector beyond the field region.

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