Problems with the New Inflationary Universe

J. D. Breit, S. Gupta, and A. Zaks
The Institute for Advanced Study, Princeton, New Jersey 08540
(Received 25 April 1983)

It is demonstrated that in the new-inflationary-universe scenario based on a Coleman-Weinberg–type potential all the bubbles evolve through the SU(4) ⊗ U(1) local minimum. The transition from this local minimum to the SU(3) ⊗ SU(2) ⊗ U(1) global minimum is first order, and so the scenario does not solve all of the cosmological problems.

PACS numbers: 98.80.Bp, 12.10.En

The inflationary-universe scenario, first proposed by Guth,[1] resolves the horizon, flatness, and primordial monopole problems of standard hot "big bang" cosmology by abandoning the assumption of adiabatic expansion of the early universe. Instead it is assumed that there is a period in which the universe supercools in a "false" vacuum and the vacuum energy is dominated by a cosmological constant proportional to the value of the Higgs potential at the symmetric point $\varphi = 0$. This cosmological constant leads to a period of exponential expansion that explains the homogeneity and flatness of the universe and dilutes the density of primordial monopoles to acceptable levels.

Despite the elegance and intuitive appeal of the scenario, it suffers from a serious problem; in order to reach the stable asymmetric SU(3) ⊗ SU(2) ⊗ U(1) phase the universe must undergo a strongly first-order phase transition that occurs through the appearance of Coleman[2] bubbles of the asymmetric phase within the symmetric universe. The phase transition is completed when the universe is filled with those bubbles. Since all the energy gained from the phase transition is stored in the bubble walls, matter is generated by collisions of bubble walls, which leads to excessive inhomogeneity.[1,3]

To resolve this problem, Linde[4] and Albrecht and Steinhardt[5] developed the new-inflationary-universe scenario, in which the symmetry breaking is due to radiative corrections of the Coleman-Weinberg (C-W) type.[6] In this scenario the phase transition occurs in two distinct stages. As the temperature of the universe drops the Higgs field φ supercools in the false symmetric minimum. At $T \approx 10^{-7}\sigma$, where σ is the scale of the C-W potential, the potential barrier around the symmetric point at the origin is entirely due to finite-temperature corrections. This barrier is low enough that thermal fluctuations lead to the formation of a bubble of $\langle\varphi\rangle \approx 10^{-7}\sigma$ and the second stage of the transition begins. During this stage the $\langle\varphi\rangle$ field evolves semiclassically toward the minimum of the potential at $\langle\varphi\rangle = \sigma$. Because of the flatness of the C-W potential near the origin the $\langle\varphi\rangle$ field evolves very slowly, and through most of its evolution the vacuum energy is dominated by a cosmological constant that leads to exponential expansion. A detailed numerical study of the evolution equation[7] indicates that the expansion is sufficient to accommodate the observed universe within a single bubble. Matter is generated when the scalar field falls into the deep minimum in the C-W potential, heating the universe to $\sim 10^{14}$ GeV, and then oscillates about the minimum while cooling. This scenario is known to produce fluctuations in the scalar field with too large an amplitude.[8] In this paper we discuss another difficulty with this scenario that is independent of the fluctuation amplitudes.

We reinvestigate the new-inflationary-universe scenario, keeping in mind that the Higgs field has more than one dimension. Although we shall consider only Higgs fields belonging to the adjoint representation of SU(5), our results can be easily generalized to larger representations for the Higgs and to other groups. Previous studies of the new-inflationary-universe scenario have considered Higgs fields limited to one direction in the group space:

$$\varphi = \varphi_c \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1.5 & 0 \\ 0 & 0 & 0 & 0 & -1.5 \end{pmatrix},$$

for which the potential is

$$V(\varphi_c) = \frac{5625 g^4}{1024\pi^2}\,\varphi_c{}^4\left(\ln\frac{\varphi_c{}^2}{\sigma^2} - \frac{1}{2}\right). \quad (1)$$

We allow the field $\langle\varphi\rangle$ to evolve in the full 24-dimensional space of the adjoint of SU(5) in order to determine whether there are minima in which the field might become trapped in directions other than SU(3) ⊗ SU(2) ⊗ U(1).

We do not, in fact, have to consider the evolution of the full 24-dimensional adjoint of Higgs field. We can limit ourselves to diagonal 5×5 traceless matrices for the following reasons. At the instant the bubble of nonzero $\langle\varphi\rangle$ forms we assume that $\langle\dot{\varphi}\rangle = 0$ and $\langle\varphi\rangle$ is some arbitrary 5×5 Hermitian traceless matrix. We can then perform a global gauge transformation to diagonalize $\langle\varphi\rangle$; of course, this gauge transformation preserves $\langle\dot{\varphi}\rangle = 0$. As can be seen from the evolution equations for the full matrix, $\langle\varphi\rangle$ will then remain diagonal. We choose to write $\langle\varphi\rangle$ in the form

$$\varphi = \begin{bmatrix} \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \alpha_3 & 0 & 0 \\ 0 & 0 & 0 & \alpha_4 & 0 \\ 0 & 0 & 0 & 0 & \alpha_5 \end{bmatrix},$$

where $\sum_{i=1}^{5}\alpha_i = 0$. The general C-W potential for an adjoint of Higgs field with one-loop corrections due only to the gauge fields included is then

$$V = \frac{3g^4}{256\pi^2}\left\{b\left[\sum_{i=1}^{5}\alpha_i{}^4 - \frac{7}{30}\left(\sum_{i=1}^{5}\alpha_i{}^2\right)^2\right] + \sum_{i,j=1}^{5}(\alpha_i-\alpha_j)^4\left[\ln\left(\frac{\alpha_i-\alpha_j}{\mu}\right)^2 - \frac{1}{2}\right]\right\} + V_0, \tag{2}$$

where V_0 is chosen to make the value of the potential 0 in the $SU(3)\otimes SU(2)\otimes U(1)$ minimum, b is a dimensionless parameter, g is the gauge-field coupling constant, and $\mu = \frac{5}{2}\sigma$. We have arranged the potential in this way to reproduce Eq. (1) in the $SU(3)\otimes SU(2)\otimes U(1)$ direction. We discuss limits on the allowed values for b later.

The classical equations of motion for the α_i's are

$$\ddot{\alpha}_i + (3\dot{R}/R + ag^2|\varphi|)\dot{\alpha}_i + \partial V/\partial\alpha_i + \lambda = 0, \tag{3}$$

where λ is a Lagrange multiplier chosen to enforce $\sum\alpha_i = 0$, $|\varphi| = (\sum\alpha_i{}^2)^{1/2}$,

$$\dot{\rho}_T = -4(\dot{R}/R)\rho_T + ag^2|\varphi|\sum\dot{\alpha}_i{}^2, \tag{4}$$

$$\rho = \tfrac{1}{2}\sum\dot{\alpha}_i{}^2 + \rho_T + V, \tag{5}$$

and

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3m_p{}^2}\rho - \frac{\kappa}{R^2}, \tag{6}$$

where $\kappa = 0, \pm 1$. Following Albrecht *et al.*[7] we have included a friction term $ag^2|\varphi|\sum\dot{\alpha}_i{}^2$ to account for the loss of energy to radiation.

We can gain considerable insight into the evolution of $\langle\varphi\rangle$ by looking more closely at $V(\varphi)$ around the $SU(3)\otimes SU(2)\otimes U(1)$ direction. In the tangent directions the first derivatives of the potential are zero and some of the second derivatives are negative for $\langle\varphi\rangle < (1.2)^{1/2}\mu\exp(-\frac{1}{3} - b/120)$ for $b > 0$ and $\langle\varphi\rangle < (1.2)^{1/2}\mu\exp(-\frac{1}{3} - b/45)$ for $b < 0$, where we have normalized $\mathrm{tr}\varphi^2$ to 1. So for $\langle\varphi\rangle$ in that range the bubble falls away from the $SU(3)\otimes SU(2)\otimes U(1)$ direction. Furthermore, for $b < 15$, there is a local minimum in the $SU(4)\otimes U(1)$ direction into which the bubble can fall, and for $b < -15\ln(1.5)$ this minimum becomes the global minimum. Furthermore, the $SU(4)\otimes U(1)$ minimum always lies at a smaller $\langle\varphi\rangle$ than the $SU(3)\otimes SU(2)\otimes U(1)$ minimum and is therefore more accessible to a $\langle\varphi\rangle$ starting from the neighborhood of the origin. So for $b < 15$, it seems likely that the evolution of the φ field will depart markedly from the $SU(3)\otimes SU(2)\otimes U(1)$ direction.

To place an upper bound on the allowed range of b, recall that the general quartic Higgs coupling

$$\lambda_1\mathrm{tr}\varphi^4 + \lambda_2(\mathrm{tr}\varphi^2)^2$$

was written as

$$\frac{3g^4}{256\pi^2}b\left[\mathrm{tr}\varphi^4 - \frac{7}{30}(\mathrm{tr}\varphi^2)^2\right] + \frac{3g^4}{256\pi^2}b'[10\,\mathrm{tr}\varphi^4 + 6(\mathrm{tr}\varphi^2)^2], \tag{7}$$

where the second term in Eq. (7) was included in the one-loop correction due to the vector bosons. In order that the C-W potential be a consistent approximation we need $b' \sim O(1)$,[5] and so a natural choice for λ_1 and λ_2 will be $\lambda_1, \lambda_2 \sim O(\alpha^2)$, which in turn will make $b \sim O(1)$. A more concrete bound can be imposed if we demand that the one-loop correction due to scalars be no larger than 10% of the usual C-W terms. Renormalizing the potential at the $SU(3)\otimes SU(2)\otimes U(1)$ minimum fixes b' to be $\frac{11}{3}$. A comparison of the vector and scalar one-loop contributions for $\varphi \sim 10^9$ GeV ($\alpha \sim \frac{1}{15}$) yields $b_{\max} < 10$. In this calculation we use the running coupling for the terms that multiply b' and the vector loop, but for the terms multiplying b we hold α fixed at $\frac{1}{45}$ because to leading order bg^4 is invariant under a change in the renormalization point μ. We note that this stringent bound on b is due to the fact that the terms proportional to b' already come very close to making up 10% of the usual C-W potential. As a result our bound is very sensitive to the value of α used and the fact that we have demanded that the scalar loop be less than 10% of the vector. For example, changing the limit to 20% raises the bound on b by more than an order of magnitude. A more conservative bound would be obtained by neglecting the terms proportional to b' and demanding that the terms proportional to b alone be less than 10% of the vector loop, in which case we find that $b < 500$.

We have numerically solved the evolution equations for a number of initial conditions and values for the parameters $b < 10$ and α of $O(1)$. We have used the zero-temperature potential since after the bubble has formed the temperature is always at least two orders of magnitude less than $\langle\varphi\rangle$. Our initial conditions are $\langle\varphi\rangle \approx 10^{-6}\mu$, and we start in a direction close to (within 10% of) SU(3) $\otimes$SU(2)$\otimes$U(1). Clearly the probability to tunnel exactly in the direction SU(3)$\otimes$SU(2)$\otimes$U(1) is zero and need not be considered. In addition, even if we start exactly in this direction, since we are on a ridge quantum fluctuations will break up the bubble and move the pieces away from the ridge. We do not let the gauge coupling run since that would affect only the time scales in the problem, not the directions in the group space into which the bubble evolves. We also hold the Higgs coupling b fixed.

Our results demonstrate that the evolution of $\langle\varphi\rangle$ is initially away from the SU(3)$\otimes$SU(2)$\otimes$U(1) direction and towards the SU(4)$\otimes$U(1) minimum. For all values of g, a and $b < 1000$. If $b < 14.5$ then the bubble gets trapped in the SU(4)$\otimes$U(1) minimum for all values of g and a. In the range $14.5 < b < 15$, the bubble classically always passes through the SU(4)$\otimes$U(1) minimum, but depending on the value of g and a either gets stuck in the SU(4)$\otimes$U(1) minimum, or continues its evolution towards the SU(3)$\otimes$SU(2)$\otimes$U(1) minimum. In the case when it gets stuck in the SU(4)$\otimes$U(1) minimum, it always goes to the SU(3)$\otimes$SU(2)$\otimes$U(1) minimum by a strongly first-order transition, which leads to the usual problems.[9]

To summarize our results, we have found that the new-inflationary-universe scenario based on a Coleman-Weinberg type potential does not cure the defects of the original Guth scenario. This failure is due to the presence of an additional local minimum in the SU(4)$\otimes$U(1) direction in the C-W potential and slopes in the potential that lead away from the global SU(3)$\otimes$SU(2)$\otimes$U(1) minimum and towards this local minimum. The only way out that we can see is to choose an unnaturally large Higgs coupling b. In that case one should for consistency include the Higgs loop in the potential, which could radically change the nature of the potential and destroy the features that made it attractive in the first place. Moreover, since the Higgs loop is formally of $O(g^8)$ we must include all other radiative corrections of $O(g^6)$ and $O(g^8)$. If corrections of such high order are important, the consistency of perturbation theory is doubtful. It would still be worthwhile, however, to explore this possibility in greater detail.

We thank Erick Weinberg for a critical reading of the manuscript and valuable comments and Alan Guth for bringing to our attention a mistake in the original draft of this paper and suggesting improvements. S. G. would like to acknowledge valuable conversations with H. R. Quinn. This work is supported by the Department of Energy under Grant No. DE-AC02-76ER022220.

[1]A. H. Guth, Phys. Rev. D 23, 850 (1979).

[2]S. Coleman, Phys. Rev. D 15, 2929 (1977).

[3]S. W. Hawking, I. G. Moss, and J. M. Stewart, Phys. Rev. D 26, 2681 (1982); A. H. Guth and E. J. Weinberg, Nucl. Phys. B212, 321 (1983).

[4]A. D. Linde, Phys. Lett. 108B, 389 (1982), and 114B, 431 (1982).

[5]A. Albrecht and P. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).

[6]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[7]A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, Phys. Rev. Lett. 48, 1437 (1982).

[8]A. A. Starobinsky, Phys. Lett. 117B, 175 (1982); A. H. Guth and S.-Y. Pi, Phys. Rev. Lett. 49, 1110 (1982); S. W. Hawking, Phys. Lett. 115B, 295 (1982); J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. 28, 679 (1983); S. W. Hawking and I. G. Moss, to be published.

[9]A. H. Guth and E. J. Weinberg, Phys. Rev. D 23, 876 (1981).