Mean Free Path of a Nucleon in Nucleus-Nucleus Collision

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The mean free path of a nucleon for two colliding nuclei has been calculated in terms of the imaginary part of the optical potential and the k effective mass. The energy range 10–30 MeV/nucleon appears to be optimum for mean free paths in the range 3–4 fm.

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The long-standing discrepancy[1] between the theoretical prediction of the mean free path of a nucleon in nuclear matter and the experimental value, computed from the imaginary part of the optical potential, has been resolved recently.[2,3] The nonlocality of the optical potential, it appears, is the central cause—the equivalent local potential, when used properly, does indeed increase the theoretical prediction of a nucleon's mean free path to around 5–6 fm.

This is for a single nucleon in a nuclear medium. For two colliding nuclei, it is of great relevance to know the mean free path of a nucleon when the immediate medium is the overlap of the projectile and the target. At moderate energies (energy per nucleon of 10–100 MeV) the nucleon belonging to a nucleus differs from the free-nucleon case in two substantial ways: The nucleon for colliding nuclei is dressed with the internal Fermi motion, a fact necessarily implying that the relevant energy is scaled up to the sum of the relative energy and the internal Fermi energy; secondly, the Pauli blocking due to the occupied intrinsic states of the nuclei will tend to inhibit interactions between nucleons of the two nuclei unless the relative energy far exceeds the Fermi energy, when Pauli blocking gets switched off.

Pauli blocking in effect will increase the mean free path. Internal Fermi motion, on the other hand, will act to reduce the mean free path since the relevant energy is scaled up, thus decreasing the mean free path[2]; the last observation of course depends on the Fermi momentum, as shown later. For nonoverlapping Fermi spheres, however, the mean free path of a nucleon was recently computed by Müller.[4]

Several models invoked recently[5,6] to predict phenomena such as highly damped collisions (typically, for energies in the range of 2–3 MeV/nucleon above the Coulomb barrier) or the formation of hot spots rely implicitly on the assumption that the nucleons of the projectile have interacted with the target nucleons at least more than once, thus precipitating local equilibrium. The equilibration is possible only for nucleons with mean free paths less than the typical dimension of the nuclei. In this note I shall examine the validity of this assumption by computing the mean free path of a nucleon for colliding nuclei.

If we retain corrections up to first order in W, the imaginary part of the optical potential, the mean free path of a nucleon is given by[2,3]

$$\lambda = (2k_{\mathrm{Im}})^{-1} = -\hbar^2 k/2m_k W(E,k), \tag{1}$$

where $\hbar k$ is the momentum of the nucleon and m_k the momentum-dependent effective mass as defined by Jeukenne, Lejeune, and Mahaux.[7] It was noted that[2] since the value of m_k is less than unity at moderate energies, the mean free path turns out to be somewhat larger than the equivalent local expression $\lambda = -\hbar^2 k/2mW$, thus resolving the discrepancy between the experimental λ obtained from the phenomenological imaginary potential and that from the theoretical W indicated just now. It is interesting to note that using the so-called frivolous model which is exactly equivalent to the forward-scattering-amplitude approximation,[8] we get back the classical kinetic theory prediction of $\lambda = [\rho\langle\sigma\rangle]^{-1}$ since $W = \frac{1}{2}\hbar v\rho\langle\sigma\rangle$, $\langle\sigma\rangle$ being the Pauli-averaged two-body cross section in the nuclear medium.

For two colliding nuclei, the momentum of a single nucleon is given by

$$\vec{k} = \vec{K}/A_P + \vec{k}_1$$

where $K^2 = (2mA_P/\hbar^2)(E_C + U_C)$, with U_C the nucleus-nucleus potential, E_C the energy of relative motion, mA_P the projectiles mass, and k_1 the momentum for internal motion. The corresponding energy equation reads

$$E + U = \frac{(E_C + U_C)}{A_P} + \frac{\hbar^2}{m}\frac{\vec{K}\cdot\vec{k}_1}{A_P} + \frac{\hbar^2}{2m}k_1^2$$

for a nucleon with energy E and nucleon-nucleus potential U. Over a wide energy range U depends on E linearly,[8] so that the second term in the above equation is positive as often as it is negative and thus can be ignored. Furthermore, approximating $U \approx U_C/A_P$, one gets for the energy of a nucleon

$$E = E_C/A_P + (\hbar^2/2m)k_1^2.$$

In this work I use a simple Fermi-gas model to derive the average value of k_1^2, so that[2]

$$\langle E \rangle = E_C/A_P + 0.6(\hbar^2/2m)k_F^2, \tag{2a}$$

$$k(\rho) \equiv [k^2(\rho)]^{1/2} = \{(2m/\hbar^2)[\langle E\rangle + U(\rho)]\}^{1/2}, \tag{2b}$$

where k_F is the Fermi momentum and U is evaluated for a local density $\rho = k_F^3/(1.5\pi^2)$.

Pauli blocking, in effect, distorts the Fermi surface. For zero relative motion, the Fermi levels of both the nuclei overlap completely; with an increase in relative energy, the overlap becomes partial, and beyond a certain critical value of relative energy there is no overlap at all, switching off Pauli blocking completely. In this work I have used a simple geometrical model to incorporate Pauli blocking; the distorted Fermi momentum, in the model, is given by[9]

$$\tilde{k}_F^2 = F[0.5(k_{F_1}^2 + k_{F_2}^2)] + (1-F)[1.5\pi^2(\rho_1+\rho_2)]^{2/3}, \tag{3}$$

where k_{F_1} and k_{F_2} are the undistorted Fermi momenta and F is the fractional volume of one Fermi sphere which does not overlap with the other. F is given by[9]

$$F = 1.5(\mathcal{E}/4T_F)^{1/2} - 0.5(\mathcal{E}/4T_F)^{3/2}, \tag{4}$$

T_F being the Fermi energy at the Fermi surface for normal nuclear matter density given by $T_F = (\hbar^2/2m)(k_F{}^0)^2$, where $k_F{}^0 = 1.35\ \mathrm{fm}^{-1}$, and $\mathcal{E} = E_{\mathrm{c.m.}}(A_P + A_T)/A_P A_T$ is the reduced energy, with $E_{\mathrm{c.m.}}$ the relative energy in the center-of-mass system. For $\mathcal{E} \geq 4T_F$, $F=1$ and the Pauli blocking gets switched off; for $\mathcal{E} \to 0$, $F \to 0$ and the states are completely blocked. In the limiting case of a single free nucleon scattering off a nucleus, the only relevant Fermi momentum is that of the target nucleus and therefore the distortion of Fermi spheres is of no consequence. There is also the intrinsic temperature dependence arising as a result of the diffuseness of the Fermi surface, conveniently given by the simple expression[10]

$$k_F{}^0 \to k_F{}^0[1 - (\pi^2/24)(T/T_F)^2] \tag{5}$$

at a temperature T (MeV). An increase in temperature implies an effective decrease in $k_F{}^0$ and therefore further relaxation of Pauli blocking.

For a set of values of $k_{F(1,2)} = (1.5\pi^2\rho_{1,2})^{1/3}$ the mean free path is computed as a function of $\epsilon_c = E_c/A_P$ with use of Eqs. (1)–(5); the corresponding nuclear density is defined as $\rho_L = k_F{}^3/(1.5\pi^2)$. Further, for systems of realistic nuclei, the mean free path is computed with a local density approximation essentially in the spirit of Ref. 7 as shown below. To compute m_k, W, and U the polynomial expansion of Ref. 7 is used.

For colliding nuclei the geometry of collision is rather crucial; I therefore wish to devise a geometrical averaging procedure for λ motivated by the observation that for nucleus-nucleus collisions, the classical attenuation of flux is given by $\exp[-S(b)/\rho_L\lambda]$, where $S(b)$ is the thickness function given by

$$S(b) = \int_{-\infty}^{\infty} dz \int \rho_2(\vec{R} - \vec{r}_1)\rho_1(r_1)d^3r_1$$

for an impact parameter b defined by $b^2 + z^2 = R^2$ with r_1 being the internal coordinate of a nucleus and ρ_L the local density. Thus an average $\lambda(b)$ can be defined by

$$\langle\lambda(b)\rangle^{-1} \equiv \frac{[S(b)/\rho_L\lambda]}{[S(b)/\rho_L]}. \tag{6}$$

The brackets indicate the geometrical averaging procedure as shown in the following:

$$\langle\lambda(b)\rangle^{-1} = \frac{\int_{-\infty}^{\infty} dz \int\{\rho_1(r_1)\rho_2(\vec{R}-\vec{r}_1)/\lambda(R,r_1)\rho_L(R,r_1)\}d^3r_1}{\int_{-\infty}^{\infty} dz \int\{\rho_1(r_1)\rho_2(\vec{R}-\vec{r}_1)/\rho_L(R,r_1)\}d^3r_1}, \tag{7}$$

where $\rho_L(R,r_1)$ in the local density approximation is given by[7] $\rho_L(R,r_1) = \tilde{k}_F{}^3(R,r_1)/(1.5\pi^2)$, and the nuclear density functions are evaluated at $\rho_1(r_1)$ and $\rho_2(\vec{R}-\vec{r}_1)$, respectively.

For realistic values of $k_{F(1,2)}$, the mean free path is computed as a function of energy per nucleon. For zero temperature and low values of one or both Fermi momenta the mean free path decreases as a function of energy; beyond (approximately) the Fermi energy, however, the mean free path tends to increase again. The Pauli blocking effect in this regime of Fermi momenta essentially determines λ; around the Fermi energy the blocking is substantially relaxed, yielding small values of λ. For energies beyond the Fermi energy, W decreases with energy, pushing up the values of λ; it appears that m_k remains ap-

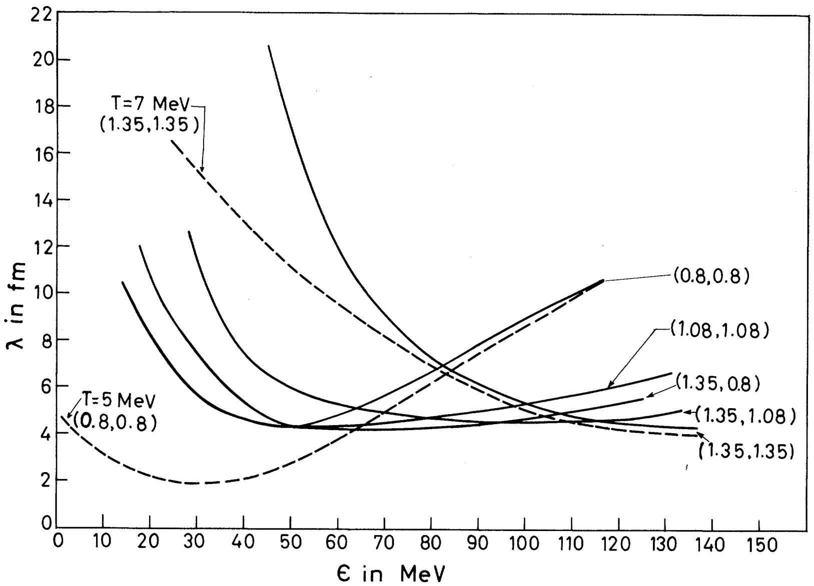

FIG. 1. The mean free path of a nucleon as a function of energy per nucleon for various values of Fermi-momentum combinations (k_{F1}, k_{F2}); T refers to the "temperature" of the Fermi distribution.

proximately constant. For normal values of $k_{F(1,2)} \sim 1.35\ \mathrm{fm}^{-1}$ the situation changes; both W and m_k tend to increase with energy, thus turning λ into a decreasing function of ϵ_c.[2,3] For nonzero temperature, Pauli blocking is further relaxed, thus decreasing λ (Fig. 1); the general trend of λ as a function of energy is, however, sustained as in the case of zero temperature. Evidently, for low values of one or both Fermi momenta and energies around $\epsilon_c \sim 30$ MeV the mean free path of a nucleon has the lowest value and is less than, or at least comparable with, the typical dimension of a nucleus. For finite nuclei, this observation is validated more directly by what follows.

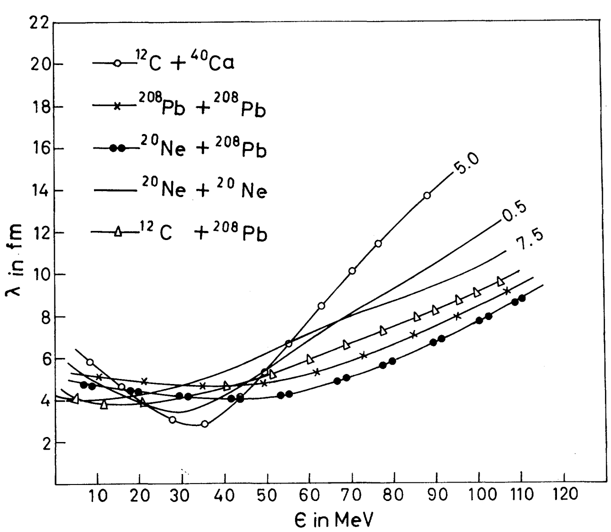

FIG. 2. The mean free path for collision between finite nuclei; the numbers on the lines denote impact parameters.

For finite nuclei also (Fig. 2) the mean free path is lowest at or around the Fermi energy; for symmetric systems of nuclei the sensitivity of λ with respect to the impact parameter is not as pronounced as for asymmetric systems such as light nuclei incident on medium or heavy nuclei. For the latter system, the large-impact-parameter interaction in effect gives rise to smaller mean free paths—this is due to the relaxation of the Pauli blocking for peripheral interaction of nuclei. The optimum regime for lowest mean free path for finite-nuclei interactions is therefore light nuclei incident on medium or heavy nuclei at or around the Fermi energy. The mean free path for finite nuclei is not terribly sensitive to temperature; for temperature typically 5–8 MeV, about the order of the diffuseness of the Fermi surface, λ is reduced by approximately 20%.

The discussion presented here is valid for energies below the threshold of particle production —the relevant physics for mean-free-path computation beyond the threshold of particle production happens to be different and is not the subject of the present work.

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Left-Right Symmetry in Nuclear Beta Decay under Investigation with a New Bhabha Polarimeter

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Precise observation of the ratio P_F/P_{GT} of beta polarizations for Fermi and Gamow-Teller decays can provide stringent constraints on electroweak models. The result of a first measurement, with a Bhabha polarimeter of novel design, is $P_F/P_{GT}=0.986\pm 0.038$. Resulting bounds, being comparable to those from existing muon polarization experiments, are limited by statistics. Precision of order 10^{-3} or better may be anticipated with an extended system.

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The degree of longitudinal polarization of beta rays is an important pseudoscalar quantity for studies of the electroweak interaction.[1-3] Stringent bounds on a possible left-right symmetry are set by the outcomes of two accurate experiments: the degree of longitudinal polarization P_{GT} in allowed Gamow-Teller (GT) beta decay, known[3] to 1% accuracy, and the Michel ρ parameter for muon decay, known[1] to 0.4% accuracy. In this communication, we stress the importance of the longitudinal polarization P_F in superallowed Fermi (F) beta decay and we report a first result obtained with a new positron polarimeter. To illustrate the potential of this arrangement we relate the accuracy for the parameter P_F with the accuracies reached in previous experiments: 20% for ^{14}O (Hopkins *et al.*[4]), 14% for ^{3}H (Koks and van Klinken[5]), and 4% with ^{26m}Al (present result). We shall promulgate the feasibility of arriving at progressively narrower bounds on electroweak parameters via the ratio $R=P_F/P_{GT}$.

In the standard $SU(2)_L\otimes U(1)$ model with pure $V-A$ charged currents, parity violation is maximal: $P_F=P_{GT}=1$ (in units of v/c). Any $V+A$ admixture to the weak charged current, as predicted[6] by left-right-symmetric $SU(2)_R\otimes SU(2)_L \otimes U(1)$ models with predominantly left- and right-handed W bosons of finite mass (M_1 and M_2), is reflected in a nonmaximal degree of beta polarization.[7] In terms of the ratio $\delta=(M_1/M_2)^2$ and $\epsilon=(1+\tan\zeta)/(1-\tan\zeta)$, where ζ is the mixing angle, one finds

$$P_F=2\epsilon(1+\epsilon^2)^{-1}(1-\delta)(1+\epsilon^2\delta)(1+\epsilon^2\delta^2)^{-1}, \quad (1)$$

$$P_{GT}=2\epsilon(1+\epsilon^2)^{-1}(1-\delta)(\epsilon^2+\delta)(\epsilon^2+\delta^2)^{-1}. \quad (2)$$

Clearly, a $V+A$ contribution, and thus a nonzero value of δ and ζ, has a different effect on F and GT decays, so that the ratio $R=P_F/P_{GT}$ in general differs from unity. It is this ratio that we aim to measure with high precision.

From the experimental point of view a comparative measurement is advantageous. This approach reduces the effects of various systematic errors, as noted long ago by Heintze *et al.*[8] Recently, Skalsey *et al.*[9] reported on progress with an interesting β^+ comparator based on positronium formation in a magnetic field, and made a detailed proposal for a high-precision comparison to within the 10^{-4} region. This instrument,

