

^(a)Present address: CMB 5, MS 730, Los Alamos National Laboratory, Los Alamos, New Mexico 87545.

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Chaotic Noise Observed in a Resistively Shunted Self-Resonant Josephson Tunnel Junction

Robert F. Miracky and John Clarke

Department of Physics, University of California, Berkeley, California 94720, and Materials and Molecular Research Division, Lawrence Berkeley Laboratory, Berkeley, California 94720

and

Roger H. Koch

IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

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The current-voltage characteristics of Josephson tunnel junctions shunted by a resistance with a substantial self-inductance exhibit stable negative-resistance regions. Very large increases in the low-frequency voltage noise with temperatures of 10^6 K or more observed near these regions arise from switching between subharmonic modes. Moderate increases in the noise, with temperatures of about 10^3 K, arise from chaotic behavior. These results are substantiated by analog and digital simulations.

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Huberman, Crutchfield, and Packard¹ were the first to point out that a resistively shunted Josephson junction with appropriately chosen parameters should exhibit chaotic behavior when driven by an external rf field. Subsequently, other authors²⁻⁶ have developed and extended this work using simulations. Apart from the inherent interest in chaotic behavior, these ideas may be highly relevant

to the design of devices based on the Josephson effect; for example, it is likely that the large levels of excess noise observed in parametric amplifiers⁷⁻⁹ were due to chaos. We have chosen to study possible chaotic effects in a different system, consisting of a Josephson tunnel junction with capacitance C shunted by a resistance R with a self-inductance L . In experiments on many

junctions, we have observed large increases in the low-frequency noise which are associated with negative resistance regions on the current-voltage (I - V) characteristics. This noise arises from at least two different mechanisms: chaos, and switching between relaxation oscillations at subharmonics of the Josephson frequency. We have simulated the I - V curves and the noise behavior on both analog and digital computers. With appropriate parameters, the simulations show Feigenbaum¹⁰ period-doubling transitions to chaos, regions of Pomeau-Manneville¹¹ intermittency, and subharmonic windows in the chaotic regions, as well as switching between subharmonic modes.

The model of the junction is shown in the inset in Fig. 3. The coupled equations of motion of the system can be written in the form

$$I = I_0 \sin \delta + \hbar C \ddot{\delta} / 2e + I_s + I_{qp}(V), \quad (1)$$

and

$$\hbar \dot{\delta} / 2e = I_s R + \dot{I}_s L + V_N(t). \quad (2)$$

Here, I_0 is the critical current, $I_s(t)$ is the current flowing through the inductor and resistor, $I_{qp}(V)$ is the quasiparticle current, δ is the phase difference across the junction, $V_N(t)$ is the Johnson noise voltage generated across the resistor, and we have set $\dot{\delta} = 2eV/\hbar$, where the dot denotes time differentiation. The junction is characterized by the parameters $\beta_c = 2\pi I_0 R^2 C / \Phi_0$ and $\beta_L = 2\pi L I_0 / \Phi_0$ ($\Phi_0 = \hbar/2e$), with the control parameter $i = I/I_0$. The magnitude of the Johnson noise is set by $\Gamma = 2\pi k_B T / I_0 \Phi_0$.

We fabricated 350×400 - μm^2 Nb-NbO_x-PbIn Josephson junctions shunted with a narrow bridge of Cu in a PbIn strip (inset, Fig. 1). The junction and shunt were covered with an insulating layer of SiO and a PbIn ground plane to reduce the inductance of the shunt. Typical values of the parameters were $I_0 = 1$ mA, $C = 6$ nF, $R = 2$ m Ω , and $L = 3$ pH. Figure 1 shows the noise temperature, T_N , and dynamic resistance, dV/dI , as a function of I for a typical junction at 3.18 K. The voltage noise was measured by connecting a cooled LC-resonant circuit across the junction, and mixing down the noise across the capacitor with a local oscillator tuned to the tank circuit frequency, 117 kHz. The Q of the tank circuit was about 480 for $I > I_0$. We define the noise temperature as $T_N = \langle V_N^2 \rangle / 4k_B R B$, where $\langle V_N^2 \rangle$ is the mean square voltage noise across the junction in the bandwidth B of the tank circuit.

The dynamic resistance shows a great deal of

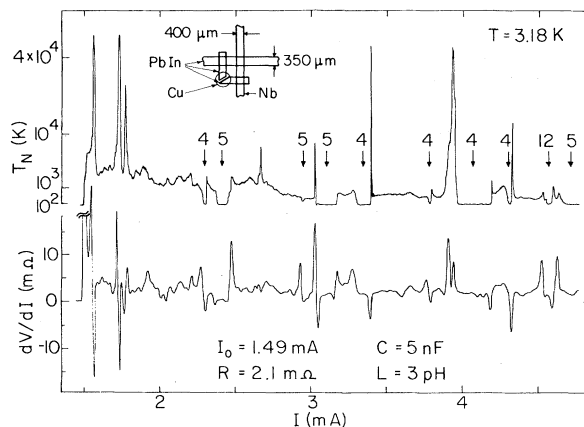


FIG. 1. T_N (upper) and dV/dI (lower) vs I (for increasing I) for a typical junction with $\beta_c \approx 0.105$, $\beta_L \approx 12.0$, and $\Gamma = 8.9 \times 10^{-5}$. Numbers indicate order of subharmonics. Noise measured in a bandwidth of about 244 Hz about 117 kHz. Inset shows junction configuration.

structure, including regions of negative resistance. The noise also shows considerable structure, and one can distinguish three types of features. First, there are regions of bias current for which the junction has a noise temperature below the system noise temperature, 70 K. Second, there are a number of fairly broad regions for which the noise temperature varies from about 300 K at high bias currents to about 2300 K at currents just above the critical current. Third, there are a number of narrow but exceedingly noisy peaks for which the noise temperature is greater than 5×10^4 K, the saturation value for the electronics; in other measurements we have determined that these peaks may have noise temperatures as high as 10^6 to 10^8 K. These noise spikes are usually in the vicinity of negative-resistance regions, but do not appear consistently at any specific feature: For example, the spike at 1.56 mA in Fig. 1 occurs near $dV/dI = 0$, while that at 4.32 mA occurs near a minimum in dV/dI .

To investigate further the origin of this structure we connected the junctions directly to a low-noise preamplifier with a bandwidth of about 1 GHz and examined the output on a spectrum analyzer. Because of the extreme impedance mismatch, the preamplifier noise temperature was very high, about 7×10^5 K. In the regions of very low noise, we were able to observe stable subharmonic oscillations of the Josephson frequency; the subharmonic number is indicated in Fig. 1. These relaxation modes have been studied by several other authors.¹²⁻¹⁴ In regions of bias current

where the junction noise temperature was of the order of 10^3 K, the subharmonic spectral components vanished, but the noise temperature of the high-frequency amplifier was far too high for us to make any observations of the nature of the noise. The very large noise spikes generally arise from switching between two different regimes. As an example, Fig. 2 shows two well-defined peaks at 335 and 377 MHz observed when a different junction was biased on a large noise spike. As one sweeps the bias current through the region where the noise spike occurs, one observes first one peak, then the growth of the second peak as the first one shrinks, and finally the disappearance of the first peak. Thus, the junction switches between two subharmonic relaxation modes, giving rise to copious levels of noise at frequencies below the characteristic switching frequencies. In fact, switching between subharmonic and chaotic modes can also occur, as our simulations have demonstrated (see below). A manifestation of this behavior is the appearance of noise spikes at the boundaries between noise-free and moderately noisy regions ($\sim 10^3$ K), as illustrated in Fig. 1.

We have simulated the junction using an electronic analog of a Josephson junction involving a phase-locked loop (Philipp Gillette and Associates, model JA-100) and an active inductance. Figure 3 shows an I - V characteristic with values of the parameters chosen to approximate those of the junction illustrated in Fig. 1. The measured low-frequency noise is also plotted in a bandwidth of 10 to 50 Hz corresponding to frequencies between 24 and 120 MHz for the experimental junction; 117 kHz for the real junction corre-

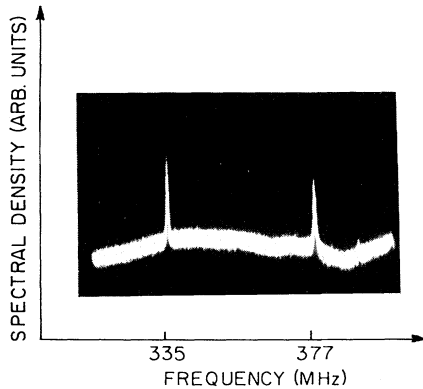


FIG. 2. Spectral density of voltage across junction at 4.2 K with $I_0 = 2.35$ mA, $I = 6.68$ mA, $C \approx 5$ nF, $R = 1.7$ m Ω , and $L \approx 3$ pH.

sponds to 0.049 Hz for the analog.

Although there is certainly not a one-to-one correspondence between the simulations and the real junction, the simulations enable us to understand the general features. For example, as the current is lowered we see period-doubling bifurcations to chaos, and even- and odd-period windows within chaotic regimes; these regions are labeled in Fig. 3. Furthermore, return maps demonstrate that many of the transitions from chaotic regions to limit cycles as the current is lowered occur via Pomeau-Manneville intermittency. The low-frequency noise in the chaotic regions has a noise temperature of typically 600 K at the higher bias currents, increasing somewhat as the current is lowered. In addition, we see large levels of noise, sometimes with noise temperatures as high as 10^6 K, over relatively narrow ranges of current. To illustrate the chaotic and switching behavior, in Fig. 4 we plot the spectral densities of the voltage at the points A and B of Fig. 3. In Fig. 4(a), the noise due to chaos is plotted for no injected noise and for an injected noise equivalent to 3.18 K. (The intrinsic noise temperature of the analog was ≈ 7 mK.) The broadened peak at high frequencies is from a residual subharmonic mode. The noise at low frequencies is white with a noise temperature of about 700 K, and is relatively unaffected by the presence or absence of thermal noise. In Fig.

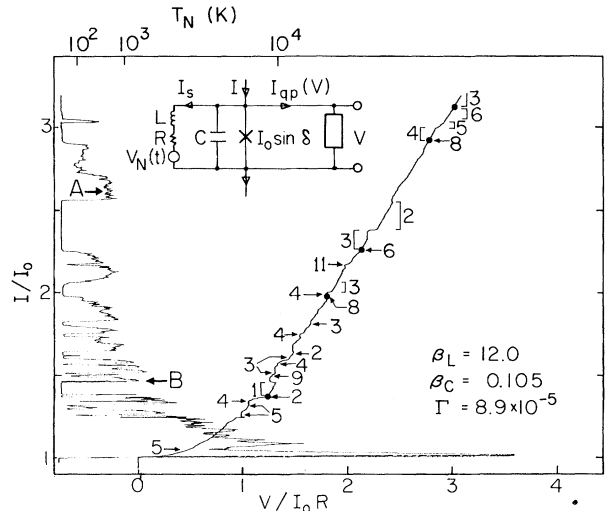


FIG. 3. T_N and $V/I_0 R$ vs I/I_0 (for increasing I) for analog junction with $\beta_L = 12.0$, $\beta_C = 0.105$, and $\Gamma = 8.9 \times 10^{-5}$. Noise measured at frequencies between 10 and 50 Hz. Solid circles indicate bifurcation points. Inset shows model of junction.

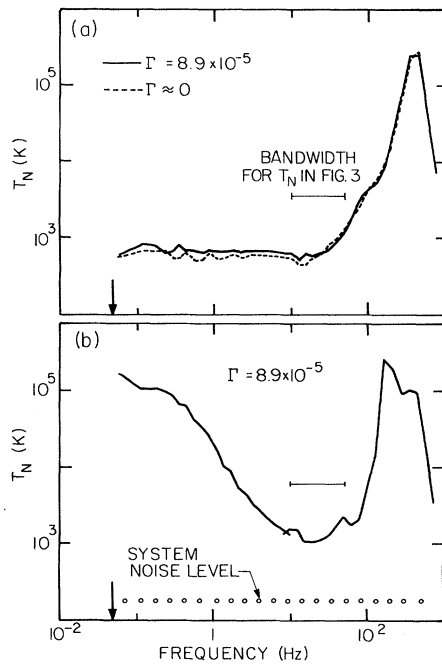


FIG. 4. Spectral density of voltage across analog at (a) point A and (b) point B of Fig. 3, with and without injected thermal noise. In (b), the junction noise was below the system noise in the absence of thermal noise. Arrows indicate frequency equivalent to 117 kHz in real junction. Bandwidth over which noise measurements in Fig. 3 were made is also shown.

4(b), the noise temperature of the junction in the absence of thermal noise was below the noise temperature of the measurement system. The addition of thermal noise greatly enhanced the noise temperature at low frequencies, where T_N increases approximately as $1/f$ to a value of about 10^5 K at 0.1 Hz. This high noise temperature was not observed in Fig. 3 because the bandpass of the measurement (indicated in Fig. 4) was in a higher-frequency region where the noise was relatively low. In this particular instance, the bias point is at a metastable subharmonic mode, but is sufficiently close to a chaotic regime that transitions between the subharmonic mode and the chaotic regime can be induced by the added thermal noise. Results from digital simulations are in excellent agreement with those from the analog.

To conclude, we note that while we can under-

stand much of the observed behavior of the real junctions using analog and digital simulations, for some regions of bias current these junctions, and also those of smaller area ($2 \times 2 \mu\text{m}^2$), often displayed much higher noise temperatures than those that can be explained using simulations. The inclusion of several second-order effects in the simulations has not yet resolved these discrepancies.

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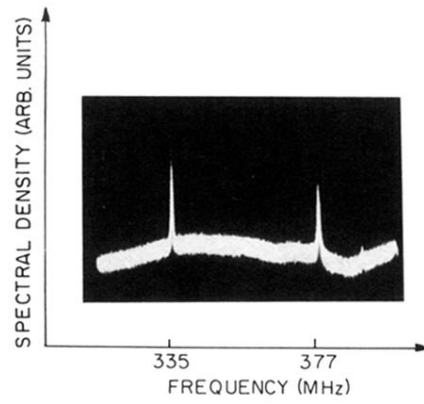


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