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High-Precision Spectroscopic Studies of Lyman α Lines of Hydrogenlike Iron: A Measurement of the $1s$ Lamb Shift

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The absolute energies of the Lyman- α lines of hydrogenlike iron have been measured with an accuracy of 90 ppm with use of a high-precision plane-crystal spectrometer calibrated directly with Co $K\alpha$ x rays. A value for the $1s$ Lamb shift of hydrogenlike iron has been deduced from this measurement.

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In this Letter an experiment is described in which absolute energy measurements of the Lyman- α lines of hydrogenlike iron ($Z=26$) were made. The principle of this experiment is to compare characteristic x-ray references with the energy of the x rays emitted in flight by a foil-excited hydrogenlike iron beam of high energy. In such experiments the main problems to be solved come from the large Doppler shifts associated with the high velocity of the ions, and the low rate of production of hydrogenlike ions. The most accurate way to make an absolute energy measurement with such fast beams is to use a flat-crystal spectrometer. Such a device provides, in contrast with those previously used (curved crystal, Soller slit, . . .), the two most important properties needed for an absolute energy measurement: (i) a very small angular acceptance (reduction of the Doppler broadening), (ii) a very accurate geometrical definition of the crystal in order to know precisely the relative angle between the line of flight of the ions and the detected x rays. A flat-crystal spectrometer,

whose characteristics are described below, has been specially designed with an improved transmission for measuring the x rays emitted by the foil-excited iron beam. With such a spectrometer, the transmission is, however, very poor and one needs a high flux of x rays. In this experiment, the new injector (ABEL) of the SuperHILAC of Berkeley which gave a very intense heavy-ion beam has been used. Moreover, in order to get a very large fraction of excited hydrogenlike ions, in a situation where the lines are free of contamination satellites (influence of outermost captured electrons), the experiment has been carried out at the maximum available energy, 8.5 MeV/u. All these improvements have made possible, for the first time, accurate absolute energy measurements for heavy ions. The precision is now sufficient to observe for the first time the $1s$ Lamb shift for a hydrogenlike atom heavier than neutral hydrogen. Prior to this measurement the $1s$ Lamb shift has been measured only in ordinary hydrogen¹ and in deuterium.²

The excited hydrogenlike iron ions in the $2p$

state were obtained by passing a Fe^{18+} ion beam through two carbon foils. The radiative decay of the excited ions in the $2p$ state, which is very fast and occurs within about $1 \mu\text{m}$ downstream of the target, was observed in the second, thin, carbon foil. The x rays emitted in the foil and downstream of the foil were observed by the spectrometer through a $100\text{-}\mu\text{m}$ entrance slit located nearby the target. The energy calibration was achieved by illuminating the crystal through the same fixed slit with the characteristic $\text{Co } K\alpha$ x rays from an x-ray tube working at 20 kV.

The x-ray spectrometer (Fig. 1) was located 3.6 m from the slit and consisted of a Si 220 flat crystal which reflected the x rays, at the Bragg angle, into a position-sensitive detector. The crystal was mounted on a goniometer having a precision of 3×10^4 deg. The position and the orientation of the target in front of the entrance slit were precisely adjusted in remote control before and during the experiment in order to optimize the transmission of the x rays into the spectrometer. We have thus reached the maximum transmission for such a spectrometer.

One of the main sources of error in this kind of experiment results from the large Doppler shifts associated with the high velocity of the ions. These errors result from two uncertainties:

(i) The uncertainty in the angles between the velocity of the ions and the trajectory of the detected x rays. To get rid of this uncertainty, we have used an alignment procedure which has enabled us to define the direction of the beam with a precision of 10^{-2} deg. At the same time the angular divergence of the beam was checked to be less than 10^{-2} deg. This highly parallel beam was achieved by means of a special achromatic tuner designed by the technical staff of the SuperHILAC. The $\text{Co } K\alpha_1$ rays were established as normal to the velocity of the ions by a precise

optical device which also gave a reference for the angular position of the crystal (90° Bragg angle). The crystal could be rotated from this reference to the theoretical Bragg angle of the calibration $\text{Co } K\alpha$ x rays with an accuracy of 3×10^{-4} deg. This procedure assured that the unknown Lyman- α line of hydrogenlike iron was compared with a Co x ray emitted at 90° with respect to the heavy-ion beam. The measurement was performed at two symmetric Bragg angles in order to get rid of remaining angular uncertainties.

(ii) The uncertainty in the velocity of the ions. This uncertainty provided the largest source of error in this measurement. The energy of the ions was determined with 0.4% precision by two independent measurements of the beam energy. This precision gave rise to an error of less than 0.25 eV in the energy of the x rays. In the first measurement, a surface-barrier detector was calibrated against a 400-MeV Fe beam delivered by the Orsay cyclotron (CEVIL) which was analyzed in two different bending magnets whose magnetic maps were accurately known.³ This procedure yielded two absolute values of the energy which agreed within 0.4%. An iron ion with this energy produces a charge in the surface-barrier detector which was compared to the charge on a capacitor from a dc voltage standard, and to the charge produced by the 6.04-MeV α particle of a ^{212}Bi source. The charge collected with this surface-barrier detector under the same conditions with the iron beam from the Berkeley SuperHILAC gave us a relative value for the energy of the beam. This value was found to agree within 0.3% with crystal and beam-phase measurements carried out independently by the SuperHILAC operators. The overall error for these two uncertainties was thus set at 0.3 eV.

Figures 2 and 3 display the $\text{Co } K\alpha$ reference spectrum and the Fe Lyman- α line spectrum, respectively. We take, following Bearden and Shaw,⁴ as a reference point in the complex $\text{Co } K\alpha$ spectrum the peaks of the observed lines (Fig. 2). The energies of these reference points were set equal to 6930.41 eV for the $K\alpha_1$ line and 6915.39 eV for the $K\alpha_2$ line which are the values given in Bearden and Shaw's table⁴ corrected for recent values of the fundamental constants.^{5,6} The error in our determination of the peaks of the lines was estimated to be 0.15 eV for the $\text{Co } K\alpha$ lines and 0.05 eV for the Lyman- α peaks. In this last case, the natural width is negligible compared to the experimental one and the line fits very well with

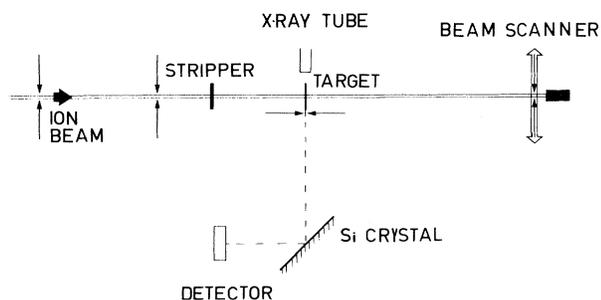


FIG. 1. Schematic of the experimental setup.

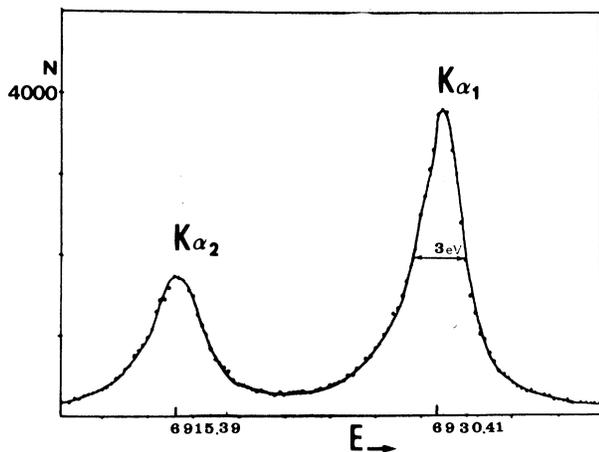


FIG. 2. Co $K\alpha$ spectrum (energy scale in electronvolts).

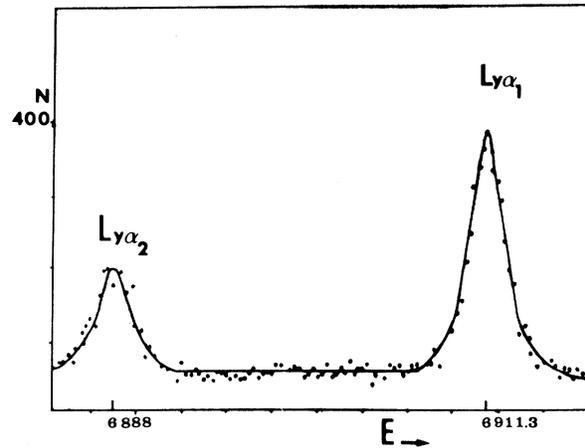


FIG. 3. Hydrogenlike iron Lyman- α spectrum (energy scale not corrected for Doppler effect).

a Gaussian distribution.

An additional contribution of 0.1 eV to the total error arose from uncertainties on energy loss of the particles in the foil before x-ray emission. The static screening induced by the electron gas of the target on the energies of the Lyman- α lines has been studied and is found to be negligible.⁷

Table I compares the values obtained in this experiment with theoretical predictions. A value of the 1s Lamb shift of hydrogenlike iron has been deduced by subtracting the experimental values of the Lyman- α_1 and Lyman- α_2 lines from those calculated⁷ from the Dirac equation corrected for the finite size of the nucleus. The small corrections which arise from the $2p_{1/2}$ and $2p_{3/2}$ Lamb shifts¹⁰ were neglected. The values thus obtained are 3.4(0.6) eV from Lyman α_1 and 4.1(0.7) eV from Lyman α_2 . These values can be compared to the sum of the 1s self-energy (4.263

TABLE I. Comparison between our experimental values and theoretical calculations. All fundamental constants used in the tables are those quoted in Ref. 8. All energies in electronvolts.

Line	$Ly\alpha_1$	$Ly\alpha_2$
Erikson total energy ^a	6 973.11	6 951.90
Point nucleus (Dirac)	6 977.243	6 956.073
Finite nucleus ^b	6 977.195	6 956.025
Mohr total energy ^c	6 973.292	6 952.077
Experiment	6 973.8 ± 0.6	6 951.9 ± 0.7

^aRef. 9.

^bRef. 7.

^cRefs. 10 and 11.

eV)¹¹ and the 1s vacuum polarization⁷ (-0.33 eV) obtained by direct numerical integration of the expectation value of the Uehling potential with Dirac wave functions, following Wichmann and Kroll.¹² This leads to a 1s Lamb shift of 3.93 eV in good agreement with our results (Table II).

It must be emphasized that, while the energy of the $Ly\alpha_2$ line seems to be in good agreement with the theory, that of the $Ly\alpha_1$ is larger, but within the error bar. The experimental energy of the fine-structure separation is 21.9 ± 0.4 eV and is not in agreement with theory (21.2 eV), a point we cannot explain for the moment. This discrepancy may be due to some polarization effects of the $2p_{3/2}$ level (effects that cannot occur for the $2p_{1/2}$) and some interactions in the foil. Further theoretical and experimental studies on this sub-

TABLE II. Contributions of the different QED corrections to the energy of the iron Ly α lines.

State	Contribution to	$Ly\alpha_1$	$Ly\alpha_2$
1s	Self-energy		
	$1s_{1/2}$ ^a	4.263	4.263
	Vacuum polarization		
	$1s_{1/2}$ ^b	-0.33	-0.33
	1s Lamb shift	3.933	3.933
2p	Self-energy $2p_{1/2}$ ^c		+0.015
	Self-energy $2p_{3/2}$ ^d	-0.03	
	Total radiative correction to the transition	3.903	3.948
	Experiment	3.4 ± 0.6	4.13 ± 0.7

^aRef. 11.

^bRef. 7.

^cRef. 10.

^dRef. 13.

ject are now in progress.

A problem which plagues precision measurements of energies in beam-foil experiments is the possible presence of additional electrons in high-Rydberg states. In varying the thickness and the nature of the target we were able to control the number of outermost shell spectator electrons and then to choose the conditions where the lines were free of satellites.¹⁴

In conclusion we would like to point out how such precise absolute energy measurements for hydrogenlike ions of high Z can be useful to check the validity of the fundamental theories. This may be illustrated in the case of hydrogenlike iron in which the precision in the measurement of the Lyman- α line allows the observation, for the first time, of the higher-order relativistic correction, a correction which cannot be appreciated even in measuring the Rydberg constant in hydrogen. In the case of hydrogenlike iron, the energy difference between the Dirac eigenvalue and the Schrödinger equation corrected at the first order for relativistic effects (change of mass, spin-orbit, and Darwin), neglecting in both cases QED and nuclear motion corrections, is $9281.87 - 9280.30 = 1.57$ eV which is much larger than our error bar in the energy measurement of the Lyman- α lines. This is in sharp contrast with the one we can get in the case of hydrogen where the two eigenvalues only differ by the tenth significant figure (i.e., one order of magnitude less than the accuracy with which the Rydberg constant has been measured by use of the characteristic lines of hydrogen).

The emittance and the precision of the beam-energy measurements in the new high-energy heavy-ion accelerators is being improved by at least one order of magnitude. Then the possibility to control by the previously described techniques the Doppler aberrations in beam-foil spectroscopy will certainly allow more precise measurements of Lyman- α absolute energies and $1s$ Lamb shifts of heavier elements. Since QED corrections scale as Z^4 while the Lyman- α energy scales only as Z^2 , the relative precision of $1s$ Lamb-shift measurements will be improved to a value of 1% which is the typical precision of the $n=2$ experiment for $10 \leq Z \leq 18$. Moreover, it has already been pointed out¹⁵ that all these meth-

ods fail for $Z \geq 20$ and that $1s$ Lamb-shift measurements are for the moment the only practical means of testing QED at very high field.

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