Topological Theory and the Standard Electroweak Model

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Topological theory predicts four charged and four neutral electroweak vector bosons, together with one neutral scalar boson. There is a single coupling constant e allowing immediate prediction (up to radiative corrections), given the Fermi constant G, of a 75-GeV mass for left-handed charged vector bosons. The authors further predict vanishing of vector weak neutral-current coupling to charged leptons ($g_V = h_{VV} = h_{VA} = 0$). Dynamical assumptions motivated by meson spectra yield a vector boson spectrum whose lowest-lying four states correspond to the standard model with $\sin^2\theta_W = \frac{1}{4}$ ($M_{Z_0}{}^2 = \frac{4}{3}M_W{}^2$).

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This paper presents topological bootstrap-theory[1] predictions for the spectrum and couplings of electroweak bosons. We predict eight vector bosons, four charged and four neutral, but with a single independent coupling constant which can be fixed by knowledge of the elementary electric charge e. The ratio of e to the leptonic coupling of the left-handed charged bosons $W^\pm$ leads to the relation $M_W{}^2 = e^2/\sqrt{2}\,G$, where G is the Fermi constant; experimental values for e and G yield $M_W \cong 75$ GeV. We shall furthermore show below that the coupling of any neutral vector boson (except the photon) to charged leptons is purely axial vector. Both predictions follow also in the standard Weinberg-Salam model[2] for the special value of the Weinberg angle $\theta_W = 30°$—giving $\sin^2\theta_W = \frac{1}{4}$. In topological theory, however, there is no adjustable parameter analogous to θ_W.

Of the four neutral vector bosons, one is the photon and the other three we shall denote, in order of increasing mass, by Z_0, Z_0', and Z_0''. Topological theory in principle determines the masses and couplings of all three Z_0 bosons; in practice we presently lack sufficient understanding of topological dynamics to calculate all these quantities without further assumptions. We nevertheless present below some (hopefully plausible) dynamical assumptions which yield a Z_0 with the same leptonic couplings as the standard model if, again, in that model we set $\sin^2\theta_W = \frac{1}{4}$. We further, under these assumptions, obtain the Z_0' and Z_0'' couplings. The two predictions of the preceding paragraph (M_W and axial-vector Z_0, Z_0', Z_0'' coupling to charged leptons) do *not* depend on these extra assumptions.

We begin by describing the general structure of topological electroweak bosons; next come the additional assumptions needed for a complete set of leptonic couplings. We conclude with a brief discussion of hadronic couplings.

Topological bootstrap theory[1] represents electroweak bosons as each built from a pair of oppositely directed lines,[3] in a sense similar to that in which a meson is built in Harari-Rosner diagrams[4] from "quark" and "antiquark" lines. Because such lines carry a quantum number that can be taken as a definition of "fermion number," we shall call them "fermion" lines. Each fermion line effectively carries a pair of auxiliary orientations, one describable for reasons given below as "electrospin" and one associated with chirality; any fermion line is an electrospin doublet $\left\{ {c \atop n} \right\}$ (c for charged, n for neutral) and also a chiral doublet $\left\{ {o \atop p} \right\}$ (o for ortho, p for para, as defined in Refs. 1 and 5). At the elementary massless level there are then sixteen electroweak bosons—eight scalars and eight vectors as described below.[6]

Let us distinguish "fermion" from "antifermion," when denoting a pair of lines, by writing the former on the left and the latter on the right. Scalar bosons then have the topological structure oo and pp which, respectively, couple to leptons and to quarks as $1+\gamma_5$ and $1-\gamma_5$, while the vectors are $V_R \equiv op$ and $V_L \equiv po$. Because $V_{R,L}$ couple to leptons and to quarks as $\gamma_\mu(1\pm\gamma_5)$, we describe V_R and V_L, respectively, as "right handed" and "left handed."

Each of these scalars and vectors appears in four distinct electrospin forms—an electroquartet. The $\left\{ {c \atop n} \right\}$ degree of freedom associates with electric charge: A c fermion (antifermion) has charge +1 (−1), while an n fermion or antifermion is neutral. Thus the chirality and electric charge of the bosons usually designated by W^+ and W^- are carried, respectively, in topological theory by V_L^{cn} and V_L^{nc}. The photon is

$$\gamma = (1/\sqrt{2})\{V_R^{cc} + V_L^{cc}\}, \tag{1}$$

a neutral ortho-para symmetric vector coupling to electric charge, that is, to c directed lines.[7] Electric charge is the single conserved quantity carried by topological electroweak bosons.

Elementary topological couplings among fermions and electroweak bosons are all $c \longleftrightarrow n$ and $o \longleftrightarrow p$ symmetric, a circumstance implying SU(2) electrospin symmetry (through the Paton-Chan argument[8]) and parity symmetry so long as *junction lines* are neglected. Topological theory contains junction lines[1] as well as fermion lines and, because junction lines are exclusively $o \times c$ in auxiliary orientation, their electroweak couplings break both SU(2) and parity symmetries.[5] All electroweak couplings are nevertheless determined by the single parameter e.

Closed-loop topologies can give masses to physical electroweak bosons and we assume that, because electric charge is the only conserved quantum number, Ward identities preserve masslessness only for the photon. There will then be seven massive physical vector bosons, each of which must have "eaten" a scalar of matching charge in acquiring longitudinal helicity. One physical neutral scalar boson will survive; this paper will not consider its properties.

Topological couplings among electroquartets of elementary (massless) vector bosons are of the Yang-Mills type [equivalent to SU(2) ⊗ U(1) when the quartet is decomposed into triplet plus singlet] with left-handed vectors not coupled to right-handed ones. We assume that massive *physical* vector bosons are to a good approximation either right or left handed, apart from states which mix with the photon. The four charged vectors, $V_{L,R}^{cn}$ and $V_{L,R}^{nc}$, do not mix. Motivated by experimental weak-interaction facts, we assume the junction-line parity asymmetry to generate substantially higher masses for right-handed vector bosons than for left-handed ones. To the extent that the Fermi constant G is dominated by left-handed vectors, we then immediately relate G to the W mass:

$$G/\sqrt{2} = e^2/2M_W^2. \tag{2}$$

A feature of the weak neutral current follows immediately from the photon structure (1) even though the four neutral vectors have different masses and couplings. Any cc superposition *orthogonal* to the photon is proportional to

$$V_R^{cc} - V_L^{cc},$$

which couples like $\gamma_\mu\gamma_5$ (axial vector) to charged leptons. Because any nn vector boson fails to have charged-lepton coupling, topological theory is characterized by the *absence* of weak *vector* neutral-current coupling to charged leptons.

Although with less generality than the foregoing, it is possible to contact the standard electroweak model further through dynamical assumptions suggested by observed meson spectra. These assumptions, to be explained below, lead to certain easily stated results.

If we define

$$W^0 \equiv (1/\sqrt{2})\{V_L^{cc} - V_L^{nn}\}, \tag{3}$$

which transforms under SU(2) as the neutral member of a left-handed W triplet, then the least massive neutral vector boson after the photon is a mixture of W^0 and γ. We shall find

$$\begin{aligned} Z_0 &= (1/\sqrt{3})\gamma - (2/\sqrt{3})W^0 \\ &= (1/\sqrt{6})\{2V_L^{nn} - V_L^{cc} + V_R^{cc}\}. \end{aligned} \tag{4}$$

Comparison with the standard model again reveals the equivalent of a 30° Weinberg angle. The value $\sin^2\theta_W = \frac{1}{4}$ may be understood by recognizing that the topological photon is half left handed and half right handed and, simultaneously, half electrotriplet and half electrosinglet; the photon is 25% left-handed electrotriplet.

The next neutral vector boson is

$$Z_0' = (1/\sqrt{3})\{V_L^{nn} + V_L^{cc} - V_R^{cc}\}. \tag{5}$$

This is an $SU(2)_L$ singlet. Up to radiative corrections, Eq. (5) determines all Z_0' couplings to leptons in terms of e, just as Eq. (4) gives Z_0 couplings. (Notice that both Z_0 and Z_0' couplings to charged leptons are axial vector, as anticipated.) The most massive neutral vector is $Z_0'' = V_R^{nn}$, an $SU(2)_L$ singlet which couples only to neutral leptons.

The Z_0' contribution to weak neutral currents depends inversely on $M_{Z_0'}^2$. The standard-model success—without Z_0'—indicates a substantial Z_0'-Z_0 mass gap; the current "best value" for $\sin^2\theta_W$, in differing from $\frac{1}{4}$ by about 10%, may be signaling a Z_0' mass roughly 3 times larger than M_{Z_0}.

Our guess at a dynamics which yields the above is motivated by the observed meson spectrum.

We suggest an analogy between *left-handed* electroweak vector bosons and the lower-mass (up-down) *first* generation of mesons, while *right-handed* vectors are seen as analogous to the higher-mass (charmed-strange) *second* generation. SU(2) electrospin symmetry, while badly broken in general, is relatively good for the up-down hadrons, allowing recognition of "strong isospin" triplets and singlets. For example, $\rho^{\pm}$, ρ^0 is recognizable as a triplet and ω as a singlet. Thus

$$\rho^0 \approx (1/\sqrt{2})\{cc - nn\}_{1\text{st gen.}}, \quad \omega \approx (1/\sqrt{2})\{cc + nn\}_{1\text{st gen.}}, \tag{6}$$

whereas

$$\varphi \approx \{nn\}_{2\text{nd gen.}}, \quad \psi \approx \{cc\}_{2\text{nd gen.}}. \tag{7}$$

Here $c_{1\text{st gen.}}$ represents the $\mathcal{P}$ quark, $n_{2\text{nd gen.}}$ the λ quark, etc.

Cylindrical closed-loop topologies, even when SU(2) symmetric, mix cc with nn, in contrast to planar loops which do not.[9] The examples (6) and (7) reflect a general rule that in the first generation the SU(2)-symmetric cylindrical components are larger than any planar components that break $c \leftrightarrow n$ symmetry. In the second generation a large planar breaking of $c \leftrightarrow n$ symmetry overwhelms the cylinder.

We propose a similar rule for electroweak bosons: One of the two right-handed neutral vectors (V_R^{cc} or V_R^{nn}) shifts *upward* to a mass so far above the left-handed neutrals that (analogously to the ψ) it undergoes negligible mixing therewith; the other right-handed neutral vector shifts *down* into the vicinity of the left-handed neutral vectors (and so is analogous to the φ). Knowing the photon wave function [Eq. (1)] that must eventually emerge, we conclude that it is V_R^{cc} which shifts down. (V_R^{cc} couples to junction lines; V_R^{nn} does not.)

The second step also follows the meson pattern —where the second-generation state which has shifted downward mixes with the isosinglet first-generation state. For the mesons this mixing gives a (small) $\lambda\bar{\lambda}$ component to the ω and a (substantial) $\lambda\bar{\lambda}$ component to the η. Analogously we assume that V_R^{cc} and the isosinglet left-handed vector—orthogonal to the W^0 of Eq. (3)—will mix. For reasons to become evident below we denote by B the lower-mass state resulting from this mixing and by Z_0' the higher-mass state. We do not know, at this stage of the argument, the wave functions of B and Z_0'.

The final step involves SU(2)-breaking for the left-handed vectors, allowing the isotriplet W^0 [Eq. (3)] to mix with isosinglets. We assume that the Z_0' has been pushed to so high a mass that the W^0 mixes significantly only with the B. Since the photon [Eq. (1)] must result from this last mixing, the orthogonality between γ and Z_0' determines uniquely the Z_0' wave function to be that given by Eq. (5). The B, furthermore, is uniquely determined to be

$$B = (1/\sqrt{6})\{2V_R^{cc} + V_L^{cc} + V_L^{nn}\}. \tag{8}$$

According to this last equation the leptonic couplings of our B are exactly the same as those of the standard-model B: The coupling is to "weak hypercharge." The photon [Eq. (1)] is a linear combination of the B [Eq. (8)] and the W^0 [Eq. (3)], with the Z_0 being the orthogonal combination, which turns out to be Eq. (4). If we neglect $SU(2)_L$ symmetry breaking except through the off-diagonal terms of the mass-squared matrix that mix B and W^0, then that matrix is completely determined and one finds

$$M_{Z_0}{}^2 = \tfrac{4}{3} M_W{}^2. \tag{9}$$

This condition also is obtained in the standard model with $\sin^2\theta_W = \frac{1}{4}$.

The foregoing sequential dynamics will be unrecognizable in a calculation where Ward identities are satisfied step by step; the photon mass then never moves from a zero value. On the other hand, in a calculation where junction-line couplings are treated on a different basis from purely fermion-line couplings, and where planar topologies are computed before cylindrical, the foregoing sequence is conceivable.[10]

Electroweak *hadron* couplings seem at first sight to provide a basis for experimental discrimination between topological theory and the standard model, because topological quarks are integrally charged (like leptons) and hadron junction lines not only carry electric charge but break parity together with isospin symmetry.[5] At low momentum transfer, however, the combined electroweak couplings to topological quarks and to junction lines tend to be controlled by the total values of conserved hadron quantum numbers and thereby to be difficult to distinguish from couplings to fractionally charged quarks in the standard model. At large momentum transfers to hadrons it is not yet known how to evaluate the experimental predictions of topological theory. In the near future, therefore, experimental evidence for or against our Z_0' may prove the most significant test for topological theory. The simpler but higher-mass Z_0'', which couples neither to junction lines nor to charged leptons, remains for the more distant future, together with the right-handed charged vectors.

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