Consequences of a New Experimental Determination of the Quadrupole Moment of the Sun for Gravitation Theory

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A preliminary experimental determination by Hill, Bos, and Goode of the interior rotation of the sun leads to a nonzero value for the quadrupole moment coefficient J_2. This produces a deviation of 1.6% from Einstein's prediction of the precession of the perihelion of Mercury. A nonsymmetric gravitational theory can fit the measured precession with this J_2 and all other solar-system relativity experiments for one value of a post-Newtonian parameter in the theory. A prediction is made for the perihelion precession of Icarus.

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From observations of rotational splittings of the global oscillations detected in the limb-darkening function of the sun, Hill, Bos, and Goode[1-3] were able to make a preliminary determination of the interior rotation of the sun and obtained a value for the quadrupole-moment coefficient $J_2 = (5.5 \pm 1.3) \times 10^{-6}$. This should be compared with an earlier determination by Dicke and Goldenberg,[4] which yielded $J_2 = (24.7 \pm 2.3) \times 10^{-6}$, and the result obtained by Hill *et al.*,[5] $J_2 = (1.0 \pm 4.3) \times 10^{-6}$. The new value of J_2 leads to a small but significant departure from Einstein's prediction of the perihelion advance of Mercury. This is an important conclusion, since the perihelion precession of Mercury is considered to be a significant experimental check of general relativity (GR) both by virtue of its accuracy and since it is sensitive to the post-Newtonian parameter β that measures the nonlinearity in the superposition law for gravity.

In the following we shall be mainly concerned with the consequences of the result of Hill, Bos, and Goode for the nonsymmetric gravitational theory (NGT), based on a (real) nonsymmetric tensor $g_{\mu\nu}$ and a nonsymmetric connection $\Gamma_{\mu\nu}{}^{\lambda}$.[6,7] The field equations of NGT possess a spherically symmetric, static vacuum solution with the line element ($G = c = 1$)[6]

$$d\tau^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = (1 + l_4/r^4)(1 - 2m/r)dt^2 - (1 - 2m/r)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2). \tag{1}$$

When $l = 0$, Eq. (1) reduces to the Schwarzschild line element of GR.

I shall postulate that a planet, treated as a test particle, moves along a geodesic determined by $\delta\int d\tau = 0$. We get

$$\frac{d^2x^\mu}{d\tau^2} + \left\{ {\mu \atop \alpha\beta} \right\} \frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0, \tag{2}$$

where $\left\{ {\mu \atop \alpha\beta} \right\}$ are the Christoffel symbols formed from the symmetric $g_{(\mu\nu)}$ and its inverse $\gamma^{(\mu\nu)}$ ($\gamma^{(\mu\nu)} g_{(\nu\sigma)} = \delta_\sigma{}^\mu$). Equation (2) can be derived from the field equations and the generalized conservation laws in NGT,[8] if we require that locally at $x^\mu = x'^\mu$,

$$\left\{ {\lambda \atop \mu\nu} \right\}_{|x=x'} = 0, \tag{3a}$$

$$[(-g)^{1/2} T^{(\mu\nu)}]_{,\nu|x=x'} = 0. \tag{3b}$$

Here $T^{(\mu\nu)}$ is the symmetric part of the generalized nonsymmetric energy-momentum tensor $T^{\mu\nu}$ and $g = \det(g_{\mu\nu})$.

NGT has a well defined geometry and the local gauge group of the fiber bundle is $GL(4,R) \supset SO(3,1)$.[9] Moreover it leads to the following results: (1) The equivalence principle is satisfied and the theory contains GR and Newtonian theory in well-defined limits. A null result is predicted for the Nordtvedt effect[10,11] (i.e., the ratio of gravitational mass to inertial mass is unity up to the post-Newtonian order). (2) The real version of the theory has no ghost poles.[12] (3) There is consistency with all solar-system relativity experiments.[7,13] (4) The predictions[7] for the binary pulsar[14] PSR1913+16 are consistent with the data.[15] The gravitational radiation predicted in the linear approximation is quadrupole in form as in GR, since there is no dipole radiation.[16] (5) There exists a Cauchy initial-value solution to the theory.[17,18]

The theory has a conserved vector current density $s^\mu\sqrt{-g}$ corresponding to the fermion-number current density, and $F \equiv l^2 = \int S^4(-g)^{1/2} d^3x$ is the conserved fermion number. Moreover $F = \sum_i f_i N_i$ where N_i is the number of fermions and f_i is a universal coupling constant.[7,17] This physical interpretation differs significantly from the Einstein-Straus theory[19] in which $g_{[\mu\nu]}$ is identified with the electromagnetic field. It has been shown in the weak-field approximation[16] that $g_{[\mu\nu]}$ is a spin-0 field, which precludes the identification of $g_{[\mu\nu]}$ with the electromagnetic field.[12]

A parametrized post-Newtonian (PPN) analysis of NGT has been carried out.[20,21] If we adopt the Lagrangian $L = 1 - d\tau/dt$, and use

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v^i}\right) = \frac{\partial L}{\partial x^i}, \tag{4}$$

where v^i is the particle velocity, then for the motion of a freely falling test particle in the gravitational field of the sun, we obtain in the post-Newtonian order of approximation ($\sim v^4$)

$$d\vec{v}/dt = -M_\odot \vec{x}/r^3 + \vec{\eta}, \tag{5}$$

where

$$\vec{\eta} = -[\nabla(\epsilon + 2\varphi^2)] + 2l_\odot{}^4\vec{x}/r^6 + 4\vec{v}(\vec{v}\cdot\nabla)\varphi - v^2\nabla\varphi. \tag{6}$$

Here $\varphi = -M_\odot/r$; ϵ is a term that includes the potentials of the other planets, a Newtonian quadrupole-moment term, possible preferred-frame effects, and a contribution due to the angular momentum of the sun.

The precession can be calculated by determining the rate of change of the Runge-Lenz vector $\vec{L}$. I shall use

$$\vec{L} = -M_\odot \vec{x}/r + (\vec{v}\times\vec{w}), \tag{7a}$$

$$\dot{\omega} = (\vec{w}\times\vec{L})\cdot(d\vec{L}/dt)(wL^2)^{-1}, \tag{7b}$$

where ω is the angle of the perihelia (the orbit is taken to lie in the plane $\theta = \pi/2$), $\dot{\omega} = d\omega/dt$, $\vec{w} = \vec{x}\times\vec{v}$, and $w = |\vec{w}|$. The precession is found from the integral

$$\Delta\omega = \int_0^{2\pi}(d\omega/dt)(dt/d\omega)d\omega. \tag{8}$$

The result obtained is

$$\Delta\omega = [6\pi GM_\odot/c^2a(1-e^2)]\lambda_p, \tag{9}$$

where a is the semimajor axis of the orbit and

$$\lambda_p = \Gamma - \frac{l_\odot{}^4c^4(1+\frac{1}{4}e^2)}{G^2M_\odot{}^2a^2(1-e^2)^2} + \frac{J_2R_\odot{}^2c^2}{2GM_\odot a(1-e^2)} - \frac{4}{3}\,\frac{J_\odot wc}{GM_\odot{}^2a(1-e^2)}. \tag{10}$$

Here $\Gamma = \frac{1}{3}(2+2\gamma-\beta)$ where γ and β are PPN parameters, $J_\odot$ is the angular momentum of the sun, and $l_\odot$ is the value of l for the sun. An estimation of the last term in (10) using $w = [a(1-e^2)MG/c^2]^{1/2} = 9.04 \times 10^3$ km and $J_\odot \lesssim 3.9\times10^{49}$ g cm^2 sec^{-1} gives $\Delta\omega \lesssim -0''.04$ per century, too small to be detected. For GR and NGT the PPN parameters are $\alpha = \beta = \gamma = 1$, so that $\Gamma = \frac{1}{3}(2+2\gamma-\beta) = 1$. When $J_\odot = J_2 = l_\odot = 0$ we obtain the GR prediction $\lambda_p = 1$ for Mercury's perihelion precession. From (10) we can derive an equation for $l_\odot$ (we neglect the contribution of $J_\odot$):

$$l_\odot = \left[\frac{G^2M_\odot{}^2a^2(1-e^2)^2}{c^4(1+\frac{1}{4}e^2)}\left(1 + \frac{J_2R_\odot{}^2c^2}{2GM_\odot a(1-e^2)} - \lambda_p\right)\right]^{1/4}. \tag{11}$$

By inserting the experimental value for Mercury[22-24] $\lambda_p = 1.003 \pm 0.005$ and the preliminary experimental value[1] $J_2 = (5.5\pm1.3)\times10^{-6}$ into (11) we get

$$l_\odot = (3.1\pm0.4)\times10^3 \text{ km}. \tag{12}$$

This result is close to the upper bound $l_\odot \leq (2.92 \pm 0.10)\times10^3$ km obtained in earlier work using an average value for the world's Mercury data.[13] The GR prediction for the perihelion precession of Mercury is $\Delta\omega^{GR} = 42''.98$ per century. If we neglect for the moment l, the predicted value of $\Delta\omega$ including J_2 is $\Delta\omega = 43''.68 \pm 0.16$ per century which is 1.6% larger than the GR prediction and 1.3% larger than the observed value $\Delta\omega^{obs} = 43''.11 \pm 0.21$ per century. By using $l_\odot = (3.1\pm0.4)\times10^3$ km, we can get agreement with the observed value of $\Delta\omega$ when the new experimental value of J_2 is included. If we assume that the solar system preferred-frame velocity is just that given by the galactic rotation, a preferred-frame effect would yield[25] $\Delta\omega = 28''.8\alpha_1$, where α_1 is one of the PPN preferred-frame parameters. We find that in GR and NGT all three PPN parameters α_1, α_2, and α_3 must vanish which requires that $\alpha_1 \lesssim 10^{-4}$.

The large eccentricity ($e = 0.8266$) and the periodic close approaches to Earth make the minor planet Icarus very suitable for testing the predictions of gravitational theories.[26,27] The result of an analysis of radar data for Icarus performed by Shapiro *et al.* yielded $\lambda_p = 0.95 \pm 0.08$.[28] This result is consistent with an earlier analysis of Lieske and Null.[29] The GR prediction for the perihelion precession of Icarus, including the J_2

of Hill *et al.*, is $\Delta\omega^{GR} = 10''.18 \pm 0.04$ per century. In NGT we predict $\Delta\omega^{NGT} = 10''.0 \pm 0.04$ per century which is in better agreement with the observed value $\Delta\omega^{obs} = 9''.5 \pm 0.80$ per century. However, the discrepancy with GR is only about 1 standard deviation. The discrepancy of GR with the combined observed precessions of Mercury and Icarus is $\approx 2\frac{1}{4}$ standard deviations. A more accurate measurement of Icarus's orbit could decide which theory of gravitation is correct.

The deflection of light grazing the limb of the sun is predicted in NGT to be[13]

$$\Delta = \frac{4GM_\odot}{c^2R_\odot}\left(1 + J_2 - \frac{3}{16}\,\frac{\pi l_\odot{}^4c^2}{GM_\odot R_\odot{}^3} - \frac{9}{16}\,\frac{l_\odot{}^4}{R_\odot{}^4}\right). \tag{13}$$

From this result we obtain the upper bound $l_\odot \lesssim 1.9\times10^4$ km. Using the value $J_2 = 5.5\times10^{-6}$ yields an additional solar deflection of 10 arc μsec,[30] while for $l_\odot = 3.1\times10^3$ km we obtain the second-order contribution -1.9×10^2 arc μsec. In GR the second-order deflection is 4 arc μsec.[30] The red shift of spectral lines emitted by the sun is[6,13]

$$\frac{\Delta\lambda}{\lambda} = -\frac{\Delta\nu}{\nu} \simeq \frac{GM_\odot}{c^2R_\odot} - \frac{l_\odot{}^4}{2R_\odot{}^4}\,. \tag{14}$$

From the experimental data we obtain the upper bound $l_\odot \lesssim 2\times10^4$ km. The time delay in the round-trip travel time for radar signals in NGT is[13,21]

$$(\Delta\tau)_{\max} \simeq \frac{(1+\gamma)}{2}\,\frac{4GM_\odot}{c^3}\,\ln\left(\frac{4r_\oplus r_p}{R_\odot{}^2}\right) - \frac{l_\odot{}^4}{2cR_\odot{}^2}\left[\frac{1}{r_p} + \frac{1}{r_\oplus} + \frac{1}{R_\odot}\tan^{-1}\left(\frac{r_p}{R_\odot}\right) + \frac{1}{R_\odot}\tan^{-1}\left(\frac{r_\oplus}{R_\odot}\right)\right], \tag{15}$$

where $r_\oplus$ and r_p are the distances of the sun from Earth and the planet, respectively, and γ is equal to unity in GR and NGT. The latest accurate data obtained from radar ranging to the Viking spacecraft[31] yield the upper bound $l_\odot \lesssim 1.13\times10^4$ km. A calculation of the Nordtvedt effect in NGT[11] shows that the two PPN parameters η and ζ that occur in the calculations are zero, leading to a result consistent with a null variation in the Earth-Moon distance.[23]

From these results we see that with $l_\odot = (3.1 \pm 0.4)\times10^3$ km a fit can be obtained in NGT to all the solar system data, including the precessions of the perihelia of Mercury, Icarus, Venus, Earth, and Mars, taking into account the new value of J_2. It is clear that further data for the radar tracking of Mercury and Icarus must be analyzed and that new experimental studies of the quadrupole moment of the sun should be performed.

An important check of these results would be guaranteed by a successful solar-probe mission.[21] For a solar-probe satellite orbit with $e = 0.993\,31$, a perihelion $R_p = 4R_\odot = 2.7839\times10^6$ km, $J_2 = 5.5\times10^{-6}$, we find that $\Delta\omega^{GR} = 1''.20$ per revolution. By using NGT we predict $\Delta\omega^{NGT} = -0''.57$ per revolution. GR predicts for $J_2 = 0$ the result $\Delta\omega^{GR} = 1''.03$ per revolution. The predictions are significantly different and would decide conclusively which theory is valid.

In the Brans-Dicke theory[32] the predicted λ_p including J_2 is

$$\lambda_p = \frac{3\omega+4}{3\omega+6} + \frac{J_2R_\odot{}^2c^2}{2GM_\odot a(1-e^2)}\,, \tag{16}$$

where ω is the adjustable dimensionless coupling constant in the Brans-Dicke theory. If we choose $\omega = 48.24$, then we can fit the data using the Hill, Bos, and Goode value for J_2. However, from the Viking spacecraft delay-time data[31] we have $\omega \geq 550$, so that the Brans-Dicke theory cannot account for the new experimental determination of J_2. In the bimetric theory of gravitation of Rosen[33] the predictions for the solar system are the same as GR up to the post-Newtonian order.

Since predictions in NGT for early-universe cosmology,[7,34] black holes, and other phenomena involving strong gravitational fields are expected to differ radically from GR, it is important to make further experimental checks involving the sun and the orbits of Mercury and Icarus to decide which theory of gravitation is correct.

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$$\Gamma = \tfrac{1}{3}\left[1 + 2\left[\frac{1+\omega}{2+\omega}\right] - \frac{\omega'}{(3+2\omega)^2(4+2\omega)}\right]$$

and with $J_2 = (5.5 \pm 1.3)\times 10^{-6}$ and $\lambda_p = 1.003 \pm 0.005$ a fit to the observed value of $\Delta\omega$ for Mercury and to the Viking spacecraft data can be achieved when $\omega = 550$ and $\omega' = 4.87\times 10^7$.

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