The confinement of the observed fluorescence to the $3p$-$5d$ transitions in C II and the absence of fluorescence with the Mg plasma were proof that three-body recombination pumping was not the cause of the observed fluorescence.

In summary, selective optical pumping of an upper state in one ion species using nearly coincident line radiation from another species has been experimentally demonstrated. While the fluorescence observed is in the ultraviolet, the concept verified here is the principal pumping scheme for a new generation of soft-x-ray laser experiments. With the modest pump powers available to us, experiments in the x-ray regime are not feasible. However, extreme ultraviolet fluorescence measurements at 62.2 and 48.4 nm based on the $2s2p^4$-$2s^22p^3$ Mg VI line at 29.135 nm pumping the $2s5p$-$2s^2$ C III transition at 29.133 nm are underway.

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Plasma Edge Turbulence

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Model mode-coupling equations for the resistive drift-wave instability are derived and numerically solved to study the properties of turbulence near a plasma edge. The wave-number spectrum of the turbulence is found to exhibit an inverse cascade to form an isotropic, two-dimensional Kolmogorov spectrum, k^{-3}, in the large-wave-number regime. The turbulence has a broad frequency spectrum with a large saturation level and produces Bohm-type particle diffusion.

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A number of experiments now clearly indicate that a tokamak-type plasma with a strong magnetic field exhibits a large level of density fluctuations which increase near the edge.[1,2] The observed frequency spectra are usually broader than the drift-wave frequency revealing their strongly turbulent nature.[1-3]

Recognizing that the classic weak-turbulence theory fails to explain these results, Fyfe and Montgomery[4] as well as Hasegawa, Kodama, and Maclennan[5] have presented theories of strongly turbulent drift waves based on the model equation derived by Hasegawa and Mima.[6] It was found that the wave-number spectrum rotates from that peaked in the azimuthal direction to that peaked in the radial direction[5] and that the spectrum obeys the two-dimensional (2D) Kolmogorov law.[4,7]

The importance of mode coupling in such strongly turbulent plasmas is now being recognized by many authors. In particular Waltz[8] as well as and Terry and Horton[9] have made extensive numerical studies of the spectrum evolution based on model mode-coupling equations and have successfully demonstrated that a broad frequency spectrum in fact originates from these models.

In this Letter we report a derivation of model equations suitable to explain drift-wave turbulence observed near the edge of tokamak plasmas, and the results of numerical solution of these equations. We present a dynamic evolution of the spectrum and the resultant particle diffusion across the magnetic field. The resulting wave-number spectrum at saturation is of the 2D Kolmogorov type ($\sim k^{-3}$) and the frequency spectrum is broad and peaked at $\omega \lesssim \omega_*$ (ω_* is the drift-wave frequency) in good agreement with observations.[1-3] In addition, a Bohm-type diffusion is found to result from the turbulence. The discovery of the Bohm-type diffusion is based on our theoretical recognition that the turbulence is found to be controlled by only two parameters, the normalized density gradient $\kappa\rho_s$ [where κ $(=|\partial \ln n_0/\partial x|)$ is a measure of the density gradient, and $\rho_s = (T_e/m_i)^{1/2}/\omega_{ci}$ is the ion Larmor radius at the electron temperature, T_e] and the normalized ion viscosity

$$c_2 = \mu/\rho_s^2\omega_{ci} = \tfrac{3}{10}(T_i/T_e)(\nu_{ii}/\omega_{ci})$$

(where T_i is the ion temperature and ν_{ii} is the ion-ion collision rate), and that the particle flux induced by the turbulence is found to depend almost entirely on $\kappa\rho_s$.

We consider an edge plasma where the temperature is sufficiently low such that the electron mean free path is shorter than qR (q is the safety factor and R is the major radius). There the Landau damping is less important than the collisional damping, yet the parallel heat conductivity is sufficiently large that the electrons may be treated as an isothermal fluid. These conditions are met if $\omega\nu_e^{-1} \ll v_{Te}^2(\nu_e qR)^{-2} \lesssim O(1)$, where ω is the typical frequency of the turbulence, ν_e is the electron collision rate, and v_{Te} is the thermal speed. The resulting mode-coupling equations significantly simplify the earlier attempt for collisional drift-wave turbulence.[10]

We treat ions as a 2D warm fluid with ordering similar to that of the Hasegawa-Mima equation.[6] The equation for ion vorticity $\nabla\times v = (\nabla^2\varphi/B_0)\hat{z}$ is then given by[6]

$$\frac{d}{dt}\left(\frac{\nabla^2\varphi}{B_0\omega_{ci}} - \frac{n_1}{n_0} - \ln n_0\right) - \mu\frac{\nabla^4\varphi}{B_0\omega_{ci}} = 0, \tag{1}$$

where n_1 is the density perturbation, φ is the electrostatic potential, $n_0(x)$ is the background density which varies in the x direction, and μ $[=3T_i\nu_{ii}/(10\, m_i\omega_{ci}^2)]$ is the kinematic ion-viscosity coefficient. The convective derivative d/dt is given by

$$\frac{d}{dt} = \frac{\partial}{\partial t} - \frac{\nabla\varphi\times\hat{z}}{B_0}\cdot\nabla, \tag{2}$$

where $\hat{z}$ is the unit vector in the direction of the magnetic field.

The quasineutrality condition relates the ion number density to the electron number density through the continuity equation:

$$\frac{d}{dt}\left(\frac{n_1}{n_0} + \ln n_0\right) = \frac{1}{en_0}\frac{\partial J_z}{\partial z}, \tag{3}$$

where d/dt is the same as Eq. (2) and J_z is the perturbed current density in the $\hat{z}$ direction. The assumption of the isothermal electron fluid relates J_z to n and φ through the equation of motion in the $\hat{z}$ direction by

$$J_z = \frac{T_e}{e\eta}\frac{\partial}{\partial z}\left(\frac{n_1}{n_0} - \frac{e\varphi}{T_e}\right). \tag{4}$$

Here T_e ($\simeq$const) is the electron temperature and η is the resistivity.

If we eliminate J_z, we can construct coupled nonlinear equations for φ and n_1. If we use the normalization $e\varphi/T_e \equiv \varphi$, $n_1/n_0 \equiv n$, $\omega_{ci}t \equiv t$, and $x/\rho_s \equiv x$, the coupled equations become

$$\left(\frac{\partial}{\partial t} - \nabla\varphi\times\hat{z}\cdot\nabla\right)\nabla^2\varphi = c_1(\varphi - n) + c_2\nabla^4\varphi \tag{5}$$

and

$$\left(\frac{\partial}{\partial t} - \nabla\varphi\times\hat{z}\cdot\nabla\right)(n + \ln n_0) = c_1(\varphi - n), \tag{6}$$

where

$$c_1 = -\frac{T_e}{e^2 n_0\eta\omega_{ci}}\frac{\partial^2}{\partial z^2}, \quad c_2 = \frac{\mu}{\rho_s^2\omega_{ci}}. \tag{7}$$

Similar equations were recently derived also by Bekki *et al.*[11] The conservation laws for the energy E and potential enstrophy U result from Eqs. (5) and (6):

$$\frac{1}{2}\frac{\partial}{\partial t}\int[n^2 + (\nabla\varphi)^2]\,dV \equiv \frac{\partial}{\partial t}E(t) = -c_1\int(n-\varphi)^2\,dV - c_2\int(\nabla^2\varphi)^2\,dV - \int n(\hat{z}\times\vec{\kappa})\cdot\nabla\varphi\,dV, \tag{8}$$

$$\frac{1}{2}\frac{\partial}{\partial t}\int(\nabla^2\varphi - n)^2\,dV \equiv \frac{\partial}{\partial t}U(t) = -c_2\int(n - \nabla^2\varphi)\nabla^4\varphi\,dV - \int n(\hat{z}\times\vec{\kappa})\cdot\nabla\varphi\,dV, \tag{9}$$

where $\bar{\kappa} = -\rho_s \nabla \ln n_0 = \kappa \rho_s$.

Near the plasma edge, we assume that the density gradient has a constant value which is equivalent to an exponential density profile. Then Eqs. (5) and (6) become uniform in space. Equations (5) and (6) show that the nonlinear behavior of resistive drift waves is completely characterized only by three parameters, $\kappa\rho_s$, c_1, and c_2.

We postulate that in a toroidal geometry the plasma will choose the parallel wave number such that the growth rate of the instability is maximized for a given value of the perpendicular wave number of a perturbation. Then c_1 is given by[12] $c_1 = 4k^2 k_y \bar{\kappa}/(1+k^2)^2$, where $k = (k_x^{\,2} + k_y^{\,2})^{1/2}$ is the perpendicular wave number which is normalized by ρ_s^{-1}. Under this assumption, only two free parameters are left: $\kappa\rho_s$ and c_2.

We have solved Eqs. (5) and (6) numerically to study the nature of the turbulence described by these model equations for various values of $\kappa\rho_s$ and c_2. The number of modes used in the calculation is 24×24. The integration time step is taken to be $\frac{1}{20} - \frac{1}{40}$. The numerical error is checked with the energy conservation law, Eq. (8), and is kept within 5%. When we start from an initial condition of $|\varphi_k|^2 = 0.005/(1+k^2)$, saturation in the total energy $E(t)$ is found to appear at $t \simeq 150$. $E(t)$ is then found to oscillate around the saturation level. The saturation level is found to increase linearly with $\bar{\kappa}$ indicating that the source of the instability is the density gradient.

Figures 1(a) and 1(b) show the temporal variations of the k_x and k_y spectra obtained by integrating the energy spectrum $\frac{1}{2}(|n_k|^2 + k^2|\varphi_k|^2)$ with respect to k_y and k_x, respectively. Near saturation, the wave-number spectrum of the energy is seen to form an inverse cascade in the y direction from a peaked spectrum and to cascade to a larger wave number in the x direction, a tendency qualitatively similar to that found earlier[5,9] for collisionless drift-wave turbulence. The wave-number spectrum at large k is very close to the 2D Kolmogorov spectrum,[7] $E_k \simeq k^{-3}$. This spectrum corresponds to the inertial range of enstrophy,[7] and thus the energy does not cascade in this range.

The frequency spectrum of the number-density fluctuation for a fixed value of $k = k_0$, $|n_{k_0\omega}|$, is shown in Fig. 2 for $k_0 = (0.59, 0.59)$. It is obtained by averaging over a time interval after the saturation. Here ω_* is the drift-wave frequency $k_y\bar{\kappa}/(1+k^2)$ for $k = k_0$. The frequency spectrum for a larger value of k was also studied and found to be significantly broader. The qualitative features

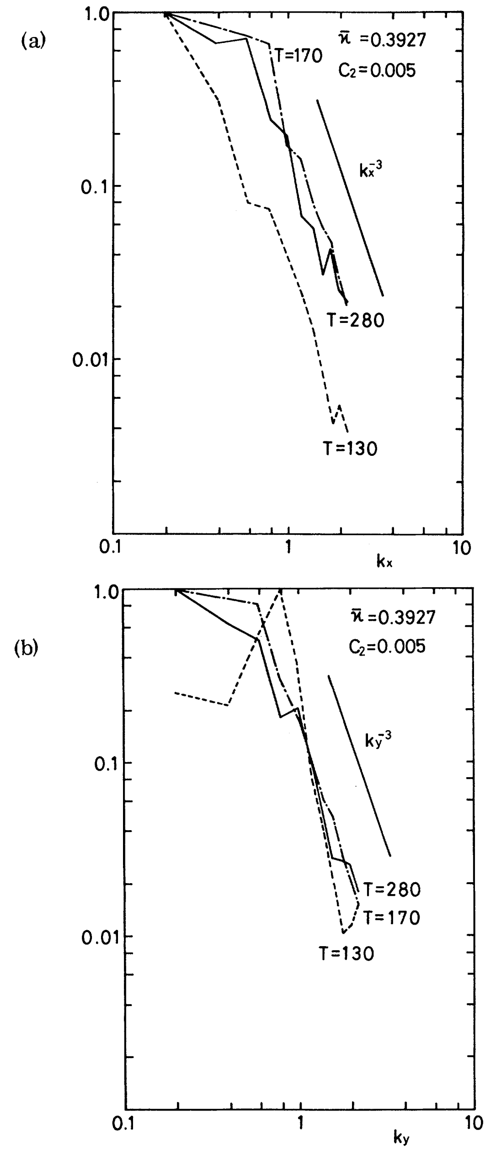

FIG. 1. (a) k_x and (b) k_y dependencies of the energy spectrum $\Sigma_{k_y} E_k$ and $\Sigma_{k_x} E_k$, where $E_k = \frac{1}{2}(|n_k|^2 + k^2|\varphi_k|^2)$.

of the broad spectrum peaked near $\omega \lesssim \omega_*$ agree well with the experimental observations.[1-3]

The total energy $E(t) = \frac{1}{2}\sum_k(|n_k|^2 + k^2|\varphi_k|^2)$ at saturation, E_{sat}, is found to be proportional to $\bar{\kappa}$ and c_2 for the range of parameters, $0.2 \lesssim \kappa \lesssim 0.8$ and $5\times10^{-3} \lesssim c_2 \lesssim 3\times10^{-2}$. (This range of c_2 is chosen for numerical reasons.) The best fit to the several runs yields

$$E_{sat} \simeq 6\bar{\kappa}(1 + 140c_2), \tag{10}$$

while the saturated squared density fluctuation $\frac{1}{2}\sum_k |n_k|^2$ was approximately 75% of E_{sat}. The reason that E_{sat} increases with c_2 is probably the inverse cascade nature of the turbulence. We

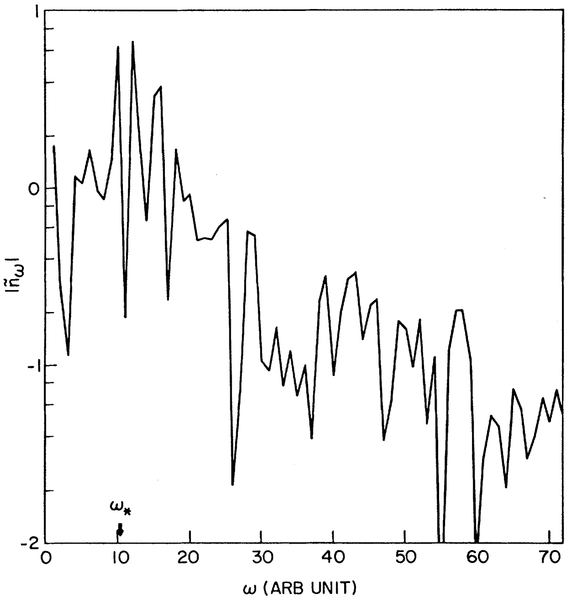

FIG. 2. Frequency spectrum of the density perturbation at a fixed value of the wave number $k_0 = (0.59, 0.59)$.

note that the saturation can occur by excitation of a mode with k_y in the direction of the ions' diamagnetic drift.

Since $\bar{\kappa}$ is taken to be constant, the numerical results give a spatially uniform particle flux in the x direction, $\overline{\Gamma}(t) = \langle n v_x \rangle$. $\overline{\Gamma}(t)$ grows in time and approaches a stationary level as the instability saturates. The saturated level of the flux $\overline{\Gamma}$ then also depends only on the two parameters $\bar{\kappa}$ and c_2. From several numerical runs, $\overline{\Gamma}$ is also found to increase linearly with $\bar{\kappa}$ and c_2 for the range of the parameters as shown in Fig. 3. From the results shown in this figure, we find $\overline{\Gamma} \simeq 1.0\bar{\kappa}(1 + 60\, c_2)$.

Now the unnormalized particle flux Γ_0 is given by

$$\Gamma_0 = -\langle (\partial\varphi/\partial y) n_1 \rangle = \overline{\Gamma} n_0 c_s .$$

For most plasmas $c_2 \simeq 0$, thus $\overline{\Gamma} \simeq 1.0\,\bar{\kappa} = 1.0\kappa\rho_s$, and hence

$$\Gamma_0 \simeq \kappa\rho_s n_0 c_s . \tag{11}$$

This flux corresponds to a diffusion coefficient D given by

$$D = \Gamma_0/\kappa n_0 = T_e/eB_0 , \tag{12}$$

which is Bohm-type diffusion (but with the coefficient 16 times that given by Bohm). The particle diffusion near the edge of tokamaks is in fact found to be Bohm type in several experiments.[13,14]

If we take the example of the Caltech Research Tokamak ($R = 45$ cm, $a = 15$ cm, $B_T = 4$ kG, T_{edge}

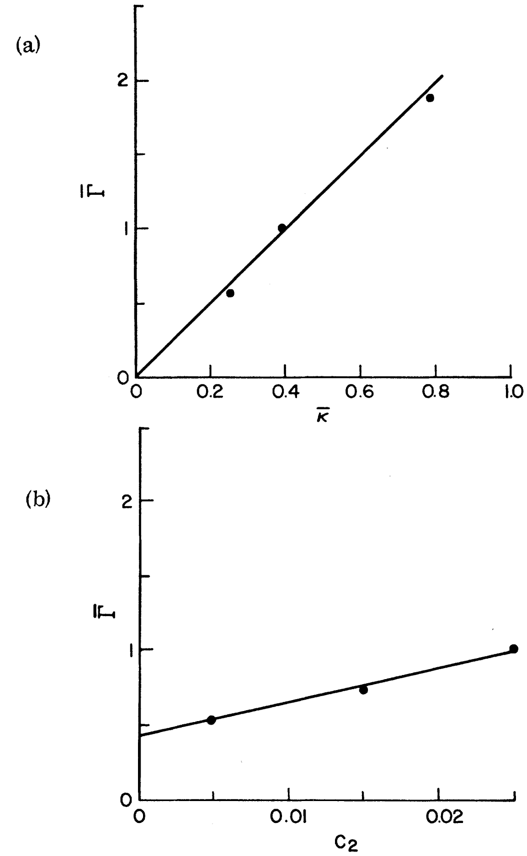

FIG. 3. Dependencies of the saturated particle flux $\overline{\Gamma}$ on parameters (a) $\bar{\kappa}$ and (b) c_2. Examples shown are for $c_2 = 0.025$ and $\bar{\kappa} = 0.4$, respectively.

$\simeq 25$ eV, $n_{\text{edge}} \simeq 10^{12}$ cm^{-3}, the plasma is marginally collisional), then $c_s = 5\times10^6$ cm/sec, $\rho_s = 0.13$ cm, and thus if we take $\kappa \simeq 6^{-1}$ cm^{-1} and $\bar{\kappa} = \frac{1}{48}$, Eq. (11) gives $\Gamma_0 = 1.0\times10^{17}$ cm^{-2}sec^{-1}. In addition, Eq. (10) gives $E_{\text{sat}} \simeq 0.13$. Since $\frac{1}{2}\sum|n_k|^2$ is 75% of E_{sat}, this gives $|(n_1/n_0)| \simeq 0.31$. Both of these numbers are in good agreement with the observation.[14]

In summary, we find that the plasma turbulence excited by a resistive drift-wave instability is characterized primarily by only one parameter, $\kappa\rho_s$. The resulting particle diffusion is of the Bohm type, $D \simeq T_e/eB_0$, and the wave-number spectrum is of the Kolmogorov type, k^{-3}. The results compare well with the experimental observations qualitatively as well as quantitatively.

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Kinetics of Formation of Randomly Branched Aggregates: A Renormalization-Group Approach

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The first renormalization-group approach for irreversible growth models of randomly branched aggregates is presented. The main result is that the Witten-Sander diffusion-limited aggregation model, a discrete version of a dendritic growth model, is in a different universality class than "equilibrium" lattice animals. Also calculated is the fractal dimension for the Witten-Sander model and the Eden model (a model developed for the study of biological structures).

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There has been considerable recent interest in the physical mechanisms governing the structure of randomly branched aggregates or clusters formed by an irreversible kinetic process. Much of this interest is due to the applicability of these mechanisms to a variety of problems such as branched polymers and the sol-gel transition,[1] coagulation of smoke particles,[2,3] turbulence,[4,5] the early stages of nucleation,[6] and the growth of tumors.[7,8] Much of our understanding of the structure of clusters formed by irreversible growth processes has been obtained by computer simulation. Although such a procedure can yield accurate results and many insights into the structure of such clusters, it nevertheless is not usually sufficient to establish the universality classes for kinetic aggregation nor to determine how the structure of such clusters might differ from clusters formed by a random "equilibrium" process[9]; e.g., percolation clusters which model gelation and random lattice animals which model dilute branched polymers. Here we develop a renormalization-group method applicable to kinetic aggregation problems.