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# Measurement of the Decay Rate of the Positronium Negative Ion 

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#### Abstract

Positronium negative ions $\mathrm{Ps}^{-}$are formed by a beam of $420-\mathrm{eV}$ positrons impinging on a thin carbon film in vacuum. Measurements of the number of ions reaching a grid under different acceleration conditions are used to determine both the $\mathrm{Ps}^{-}$decay rate $\Gamma=2.09$ (9) nsec ${ }^{-1}$ and its initial kinetic energy $T=13_{-10}^{+19} \mathrm{eV}$. The decay rate is in good agreement with a recent calculation by Ho.


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Atoms with few electrons are relatively simple systems for testing many-electron calculation schemes. The classic variational method developed by Hylleraas ${ }^{1}$ for He and extended ${ }^{2,3}$ to $\mathrm{H}^{-}$ and $\mathrm{Ps}^{-}$(the negative ions of hydrogen and positronium) gives very precise theoretical values for the binding energies of these two-electron objects. The reason for this success is that the ground-state energy depends only quadratically on the difference between the amplitudes of the
exact and calculated wave functions. There is therefore interest in the measurement of properties which depend linearly on the wave functions. In the $\mathrm{Ps}^{-} \operatorname{system}^{4}\left(e^{+} e^{-} e^{-}\right)$the annihilation rate $\Gamma$ depends fairly strongly on electron-positron correlation and thus probes a particular aspect of the wave function.

In lowest order, the decay of $\mathrm{Ps}^{-}$is via two photons and $\Gamma$ is proportional to the density of electrons at the positron, ${ }^{5}$

$$
\boldsymbol{\Gamma}=\left(1+\sum \eta_{i}\right) 2 \pi r_{0}^{2} c \int \psi\left(\overrightarrow{\mathbf{r}}_{+}, \overrightarrow{\mathrm{r}}_{1}, \overrightarrow{\mathrm{r}}_{2}\right)^{2} \delta^{3}\left(\overrightarrow{\mathbf{r}}_{+}-\overrightarrow{\mathbf{r}}_{1}\right) d^{3} r_{+} d^{3} r_{1} d^{3} r_{2}
$$

where $\psi\left(\vec{r}_{+}, \vec{r}_{1}, \vec{r}_{2}\right)=\psi\left(\vec{r}_{+}, \vec{r}_{2}, \vec{r}_{1}\right)$ is the $\mathrm{Ps}^{-}$wave function, $\vec{r}_{+}$is the positron coordinate, and $\vec{r}_{1}, \vec{r}_{2}$ are the coordinates of the two relative singletstate electrons. The small quantities $\eta_{i}$ account for radiative corrections to the free-particle annihilation cross section ${ }^{6}\left[\eta_{i}=-\alpha(5 / \pi-\pi / 4)+\ldots\right]$, $3 \gamma$ annihilation ${ }^{7}\left[\eta_{2}=4 \alpha(\pi / 3-3 / \pi)+\ldots\right]$, and bound-state and relativistic wave-function effects $\left(\eta_{3}\right)$. The correction $\eta_{3}$ has not been calculated to
my knowledge. A recent calculation ${ }^{8,9}$ of $\Gamma$ by Ho gives $\Gamma=\left(1+\sum \eta_{i}\right) \times 2.0908 \times 10^{9} \mathrm{sec}^{-1}$ and should be accurate to about one part in $10^{3}$. In an effort to confront this theory with an experimental test, the first measurement of the $\mathrm{Ps}^{-}$decay rate has been made and is reported here.

The apparatus is shown in Fig. 1. A beam of $4-\mathrm{eV}$ positrons ( $10^{6} \mathrm{sec}^{-1}$ ) produced by a ${ }^{58} \mathrm{Co} \beta^{+}$


FIG. 1. Apparatus for measuring the $\mathrm{Ps}^{-}$decay rate. $e^{+}$, beam of positrons guided by a $100-\mathrm{G}$ axial magnetic field $B ; G_{1}$, pileup-reducing $\mathrm{grid}_{;} G_{2}$, $\mathrm{Ps}^{-}$-forming carbon film; $G_{3}$, acceleration grid; $D$, zero-electricfield drift region; $W$, vacuum chamber wall; $F$, graded $\mathrm{Pb}, \mathrm{Sn}, \mathrm{Cu}, \mathrm{Al}$ scattered- $\gamma$-ray filter for the $\mathrm{Ge}(\mathrm{Li})$ spectrometer.
source (initial activity 150 mCi ) and $\mathrm{W}(110)$ moderator ${ }^{10}$ is guided by a solenoidal magnetic field to a thin $(\sim 50-\AA)$ carbon film supported on a Ni grid $G_{2}$ (333 lines per in., $65 \%$ transmission) biased at a potential $V_{\mathrm{C}}=-416 \mathrm{~V}$. About half the positrons are transmitted by the film and roughly one positron in $10^{4}$ emerges from the film bound to two electrons as a Ps ${ }^{-}$ion. ${ }^{11}$ The ions are accelerated towards a second Ni grid $G_{3}$ ( 70 lines per in., $80 \%$ transmission) biased at a potential $V_{G}$ and separated from the carbon film $G_{2}$ by a distance $d$. Annihilation photons are detected by a Ge(Li) $\gamma$-ray spectrometer. The $\mathrm{Ps}^{-}$is traveling toward the $\mathrm{Ge}(\mathrm{Li})$ detector with velocities a few percent of the speed of light. Thus the annihilation photons from the decay of the $\mathrm{Ps}^{-}$in flight are blue-shifted and are easily distinguished from the many $511-\mathrm{keV} \gamma$ 's from positrons annihilating in the carbon film.

In the region $D$ beyond the grid $G_{3}$ the electric field vanishes and any $\mathrm{Ps}^{-}$in this region will have a constant velocity. The intensity $A^{-}$of the Dop-pler-shifted annihilation $\gamma$-ray peak resulting from motion with this velocity is proportional to the probability that the $\mathrm{Ps}^{-}$reaches $G_{3}$. The proper time interval between the emission of the $\mathrm{Ps}^{-}$and its arrival at $G_{3}$ is proportional to the distance traveled $d$ for a given potential difference $W=V_{G}-V_{C}$. This implies that the intensity $A^{-}$ will have an exponential dependence on'd that may


FIG. 2. $\gamma$-ray energy spectra obtained at various $G_{2}-G_{3}$ distances $d$ and two acceleration potentials $W$.
be used to determine the $\mathrm{Ps}^{-}$lifetime.
The total elapse of proper time during which the $\mathrm{Ps}^{-}$is moving from $z=0$ to $z=d$ in an electric field $W / d$ is

$$
\begin{equation*}
t=\frac{1}{c} \int_{0}^{d} \frac{d z}{\beta \gamma}=\frac{d}{\lambda c} \ln \left[1+\lambda+\left(2 \lambda+\lambda^{2}\right)^{1 / 2}\right] g(\lambda, \epsilon) \tag{1}
\end{equation*}
$$

where $\lambda=e W / 3 m c^{2}$ and $\epsilon=T / 3 m c^{2}$. The function $g(\lambda, \epsilon)$ is the correction for the nonzero initial Ps ${ }^{-}$kinetic energy $T$, and may be expanded as follows:

$$
\begin{equation*}
g(\lambda, \epsilon)=1-(T / W)^{1 / 2}+\frac{1}{2}(T / W)+\ldots \tag{2}
\end{equation*}
$$

The Ps ${ }^{-}$annihilation $\gamma^{\prime}$ s emitted in the direction of the $z$ axis have a Doppler-shifted energy $E$ $=m c^{2}(\gamma+\gamma \beta)$. The time spent by the Ps ${ }^{-}$in traveling from point $a$ to $b$ in the constant electric field may also be written $t_{a b}=(d / \lambda c) \ln \left(E_{b} / E_{a}\right)$. Using $d N \propto e^{-\Gamma t} d t$ we find that the energy spectrum of $\gamma$ 's detected in the forward direction should be

$$
\begin{equation*}
d N=N_{0} \exp \left[-\frac{d \Gamma}{\lambda c} \ln \left(\frac{E}{E_{0}}\right)\right]\left(\frac{d \Gamma \theta\left(E_{m}-E\right)}{\lambda c E}+\delta\left(E_{m}-E\right)\right) d E \tag{3}
\end{equation*}
$$

where $E_{m}$ is the maximum $\gamma$-ray energy.
Figure 2 shows $\gamma$-ray energy spectra representative of the data obtained over a period of 60 d at different distances $d$ and two applied potentials $W$. The signal-to-noise ratio has been improved over that of Ref. 11 by turning off the positron beam with a $20-\mu$ sec $20-\mathrm{V}$ pulse on $G_{1}$ whenever the $\mathrm{Ge}(\mathrm{Li})$ detector starts to record an event. The spectra are fitted by linear least squares with use of the line shape of Eq. (3) [with independent amplitudes for the terms in $\theta\left(E_{m}-E\right)$ and $\left.\delta\left(E_{m}-E\right)\right]$ folded with a Gaussian resolution function having a 2.13 keV full width at half maximum (FWHM). Added to the theoretical line shape is a background spectrum obtained with $V_{G}=+13 \mathrm{~V}$ and $d=15.87 \mathrm{~mm}$. Under these conditions, the amount of $\mathrm{Ps}^{-}$and fast positronium formed and the number of positrons annihilating in the carbon film are unchanged. However, the low electric field between the carbon film and the grid gives us a spectrum having a negligible number of $\mathrm{Ps}^{-}$annihilation photons with energies in the region of the spectrum where the fits are made. The background spectrum was smoothed by folding it with a Gaussian having a 1.0 or 1.5 keV FWHM for $W=1$ or 4 keV . The value of $\chi^{2}$ for the fits to the line shape (see Table I) is not
very sensitive to $\Gamma$, which was taken to be 2.0 $\times 10^{9} \mathrm{sec}^{-1}$ in Eq. (3). The areas of the monoenergetic $\mathrm{Ps}^{-} \rightarrow 2 \gamma$ photopeaks $A^{-}$are divided by the areas of the $511-\mathrm{keV}$ photopeaks to obtain the relative monochromatic $\mathrm{Ps}^{-}$photopeak areas $f^{-}$. The inset in Fig. 2 shows the $\mathbf{P s}^{-}$photopeaks on an expanded scale with curves fitted over the ranges $522-542$ and $530-570 \mathrm{keV}$ at $W=1$ and 4 keV , respectively.

The lower left inset in Fig. 3 is a plot of the relative $511-\mathrm{keV}$ photopeak area count rate $R$ versus grid separation $d$. The count rate changes with $d$ because the solid angle subtended by the $\mathrm{Ge}(\mathrm{Li})$ detector decreases as $d$ increases. The data shown in the inset are well represented by a curve $R=d_{0}^{2} /\left(d_{0}+d-d_{1}\right)^{2}\left(d_{1}=2.99 \mathrm{~mm}, d_{0}\right.$ $\left.=180.3 \pm 3.7 \mathrm{~mm}, \chi^{2} / \nu=5.05 / 5\right)$ which has been used to correct the $511-\mathrm{keV}$ photopeak areas to a common distance $d_{1}$. It is important to note that since $G_{3}$ is fixed, the $\mathrm{Ps}^{-}$photopeak areas $A^{-}$need no such correction.

The position of the carbon film was controlled by micrometers on a long vacuum manipulator. The micrometers were read with a $10^{-3}-\mathrm{mm}$ precision and these readings were used to compute the manipulator displacement $\Delta$. The actual distance $d$ between $G_{2}$ and $G_{3}, d=c_{1}+c_{2} \Delta$, in-

TABLE I. Results of line-shape fits. $N$, run number; $d$, grid separation; $t / g$, computed delay time; $f^{-}$, relative monochromatic $\mathrm{Ps}^{-}$photopeak area; $\chi^{2} / \nu$, chi square per degree of freedom for fit; $W$, acceleration potential; $A$, total $511-\mathrm{keV}$ photopeak area.

| N | $\begin{gathered} d \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} t / g \\ (\text { psec }) \end{gathered}$ | $\begin{gathered} \mathrm{f}^{-} \\ \left(10^{-5}\right) \end{gathered}$ | $x^{2} / v$ | $\begin{gathered} \text { W } \\ \text { (volts) } \end{gathered}$ | $\begin{gathered} A \\ \left(10^{6}\right) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | 1.43 | 263 | $53.03 \pm 1.16$ | 100.25/77 | 1000 | 71.26 |
| 13 | 1.70 | 315 | $49.24 \pm 1.72$ | 87.38 | " | 30.88 |
| 19 | 2.35 | 433 | $37.12 \pm 1.42$ | 77.70 | " | 37.99 |
| 6 | 2.47 | 457 | $37.29 \pm 1.77$ | 82.96 | " | 25.70 |
| 11 | 2.99 | 552 | $32.14 \pm 1.62$ | 54.63 | " | 27.13 |
| 12 | 4.28 | 790 | $19.05 \pm 1.43$ | 76.42 | " | 26.41 |
| 17 | 4.92 | 909 | $15.77 \pm 0.99$ | 105.57 | " | 5]. 53 |
| 10 | 5.56 | 1028 | $11.85 \pm 1.24$ | 75.02 | " | 29.24 |
| 20 | 6.21 | 1147 | $11.37 \pm 0.76$ | 72.49 | " | 74. 79 |
| 8 | 8.14 | 1505 | $4.36 \pm 0.88$ | 84.95 | " | 45.97 |
| 23 | 1.70 | 158 | $78.68 \pm 1.44$ | 158.14/158 | 3936 | 52.82 |
| 14 | 2.17 | 202 | $73.47 \pm 1.90$ | 145.00 | " | 28.68 |
| 2 | 2.99 | 278 | $63.65 \pm 1.78$ | 174.74 | " | 29.53 |
| 25 | 4.28 | 398 | $49.23 \pm 1.66$ | 169.72 | " | 28.78 |
| 4 | 5.56 | 518 | $40.25 \pm 1.48$ | 166.23 | " | 30.08 |
| 1 | 8.14 | 759 | $25.50 \pm 1.24$ | 129.48 | " | 32.31 |
| 5 | 10.72 | 998 | $16.00 \pm 1.17$ | 169.11 | " | 28.90 |
| 15 | 12.01 | 1113 | $12.40 \pm 0.62$ | 168.96 | " | 92.52 |
| 3 | 13.30 | 1238 | $9.23 \pm 0.97$ | 181.19 | " | 31.30 |
| 24 | 14.58 | 1358 | $6.09 \pm 0.54$ | 179.03 | " | 99.89 |
| 7 | 15.87 | 1477 | $5.78 \pm 0.71$ | 151.63 | " | 48.50 |
| 9,16,18,22 |  | background runs |  |  | 429 | 136.19 |



FIG. 3. Logarithm of the relative amount of $\mathrm{Ps}^{-}$surviving for a time $t / g=t /\left[1-(T / W)^{1 / 2}\right]$ computed from Eqs. (1) and (2). Lower inset: the relative $511-\mathrm{keV}$ photopeak count rate $R$ vs grid separation $d . \quad(R=250$ $\mathrm{sec}^{-1}$ at the start of the experiment.) Upper inset: the extrapolation of the measured $\mathrm{Ps}^{-}$decay rate to infinite acceleration potential $W$.
volves two constants which were determined by traveling microscope observations of the two grids through a vacuum window. The additive constant has no effect on the measurement of $\Gamma$. The multiplicative constant is $c_{2}=0.906 \pm 0.004$.
The logarithms of the relative $\mathrm{Ps}^{-} \rightarrow 2 \gamma$ photopeak areas $f^{-}$are plotted in Fig. 3 versus the time $t$ computed from Eqs. (1) and (2). The data are linear-least-squares fitted by straight lines $\ln \left(10^{6} f^{-}\right)=a-(g \Gamma)(t / g)$ [where $g$ is from Eq. (2)] with the following results. At $W=1 \mathrm{kV}(1000 \pm 1$ V), $a=4.48 \pm 0.03, g \Gamma=(1.851 \pm 0.058) \mathrm{nsec}^{-1}$, and $\chi^{2} / \nu=6.22 / 8$; at $W=4 \mathrm{kV}(3936 \pm 4 \mathrm{~V}), a=4.70$ $\pm 0.02, g \Gamma=(1.971 \pm 0.034) \mathrm{nsec}^{-1}$, and $\chi^{2} / \nu=8.47 /$ 9. The values of $a$ are not the same for the two sets of data because at $W=4 \mathrm{kV}$ the $\mathrm{Ps}^{-}$decays
closer to the $\mathrm{Ge}(\mathrm{Li})$ detector than at 1 kV . The fitted values of $g \Gamma$ are unaffected by this because the spatial distribution of $\mathrm{Ps}^{-}$contributing to the monoenergetic peak does not depend on $d$ for a given value of $W$. It is to be noted that although the angular distribution of annihilation $\gamma$ 's depends on $W$, this does not affect the fitted values of $g \Gamma$ because the angular distribution is independent of $d$. The two decay rates are plotted versus $W^{-1 / 2}$ in the upper right inset of Fig. 3. As shown by Eq. (2), $g \Gamma$ should depend approximately linearly on $W^{-1 / 2}, g \Gamma=\Gamma\left[1-(T / W)^{1 / 2}\right]$. The straight line through the two measurements of $g \Gamma$ gives the extrapolated value $\Gamma=(2.09 \pm 0.09)$ $\mathrm{nsec}^{-1}$ and a mean value (actually the square of the mean of the square root) of the $\mathrm{Ps}^{-}$kinetic energy at the time of formation of $T=13_{-10}^{+19} \mathrm{eV}$. At the present level of precision, the errors are completely dominated by counting statistics.

In conclusion, the first measurement of the decay rate of the positronium negative ion has been made and is in good agreement with the calculation by Ho. An order-of-magnitude increase in the experimental precision appears feasible and would allow us to test the radiative corrections and bound-state contributions to the decay rate.

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