of the oscillations in real space is somewhat longer on the trailing edge than on the leading edge. Therefore, interference effects are only observable when the two wave packets are nearly coincident as discussed by Klein, Opat, and Hamilton.[10] To our knowledge, this is the first experiment in which the detailed longitudinal shape of a neutron wave packet has been observed, and the uncertainty relation for neutrons in the longitudinal direction explicitly verified.

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Longitudinal Coherence in Neutron Interferometry

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The proposition that the coherence length of de Broglie wave packets remains unchanged even though the length of the packets increases upon propagation is discussed and demonstrated.

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An interesting conceptual difference between classical interferometry and neutron interferometry is brought into evidence by the experiment of Kaiser, Werner, and George.[1] It concerns the question of longitudinal coherence and is due to the intrinsically dispersive propagation of massive de Broglie waves. The problem may be illustrated with reference to a Gaussian wave packet propagating in accord with the Schrödinger equation. The width of the packet, as shown in Fig. 1, increases according to the expression[2]

$$\sigma_x^2(t) = \sigma_x^2(0) + [\hbar t/2m\sigma_x(0)]^2 . \qquad (1)$$

Neutrons, which may be represented by such a wave packet, are coherently split in a neutron interferometer and are later recombined after traveling along unequal optical paths. It is obvious that no interference is to be observed if the path difference, Δx, exceeds the spatial extent of the wave packet. However, if the interferometer is a long way downstream from the monochromator (in which the initial packet was prepared), the partial packets will now overlap, as shown in Fig. 2. May we now expect an observable interference pattern?

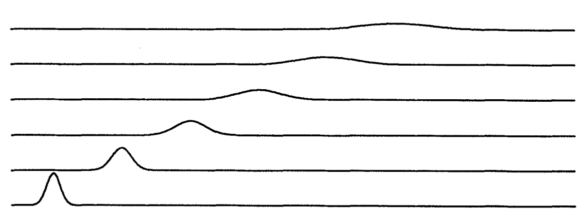

FIG. 1. Evolution of a freely propagating Gaussian wave packet.

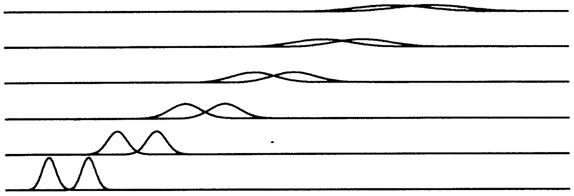

FIG. 2. Evolution of two parts of a wave packet separated by a distance Δx. They eventually overlap as a result of the spreading upon propagation.

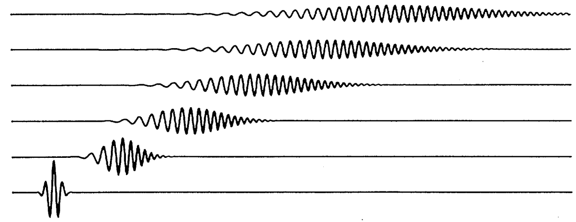

FIG. 3. The real part of the wave function for a propagating wave packet, showing the effect of dispersion.

The answer, as shown by experiment,[1] is no! In spite of the fact that the wave packets spread, the contrast (analogous to the fringe visibility function in classical optics), $V(\Delta x) = (I_{max} - I_{min})/(I_{max} + I_{min})$ remains unchanged. Its width, which is the longitudinal coherence length, is constant and proportional to $\sigma_x(0)$, no matter how long the wave packet has evolved since its initial preparation. Before giving a formal proof that this fact is consistent with theoretical expectations, we present the following heuristic argument: By its very nature, an interferometer reveals the relative phases of wave packets. To see how the phases behave, consider the plots of the real part of the wave function for a propagating wave packet, shown in Fig. 3. Clearly, both the spatial and temporal frequencies change along the evolved packet. The shorter wavelengths, representing faster motion, precede the longer wavelengths representing slower motion, as expected. (This fact is obscured if we consider only the square of the wave function as in Figs. 1 and 2.) Consider now the superposition of the evolved wave packet with a replica of itself, shifted by a length Δx. Obviously, if the path difference, Δx, is too great, the overlapping parts of the wave packets will not have a stationary phase relationship with each other and will produce "beats" which move through the detector and average to zero. A stationary interference pattern will result only when the path difference is small enough, and this turns out to be independent of the distance traveled by the wave packet, i.e., of the time t for which it has evolved. A formal proof of this statement follows.

Let the wave function $\psi(x,y,z,t)$ represent the state of a neutron before entering the interferometer, having evolved from the initial state $\psi(x,y,z,0)$. The state emerging from the interferometer is given by

$$\Psi(\vec{r},t) = \tfrac{1}{2}\{\psi(x,y,z,t) + \psi(x+\Delta x,y,z,t)\}, \quad (2)$$

where Δx is the extra path length inserted in one arm of the interferometer relative to the other. In analogy with photon optics[3,4] we define the normalized correlation function $\gamma(\Delta x)$ as the ensemble average of the overlap integral of the time-dependent wave packets:

$$\gamma(\Delta x) = \int \langle\langle \psi^*(x,y,z,t)\psi(x+\Delta x,y,z,t)\rangle\rangle\, dx\, dy\, dz\,. \quad (3)$$

The magnitude of $\gamma(\Delta x)$ may be shown to correspond to the fringe visibility function, defined earlier, i.e., $|\gamma(\Delta x)| = V(\Delta x)$. It is not immediately obvious that Eq. (3), in which time appears explicitly, is actually independent of time. However, if we consider the wave packet in Fourier space, i.e., if we set

$$\psi(x,y,z,t) = \int_{-\infty}^{\infty} A(\vec{k}) \exp i(\vec{k}\cdot\vec{r} - \hbar k^2 t/2m)\, d^3k/(2\pi)^{3/2}, \quad (4)$$

then, by applying Parseval's theorem to Eq. (3), we get

$$\gamma(\Delta x) = \int \langle\langle |A(\vec{k})|^2\rangle\rangle \exp i(k_x \Delta x)\, d^3k\,, \quad (5)$$

in which the right-hand side is clearly independent of time.

A more direct proof that this is indeed the case follows if we introduce the time displacement operator for the x direction, $D_x(\Delta x)$. Then

$$\int \psi^*(x,y,z,t)\psi(x+\Delta x,y,z,t)\, dx\, dy\, dz = \langle\psi(t)|D_x(\Delta x)|\psi(t)\rangle \quad (6)$$

and

$$i\hbar \frac{d}{dt}\langle\psi(t)|D_x(\Delta x)|\psi(t)\rangle = \langle\psi(t)|[D_x(\Delta x),H]|\psi(t)\rangle = 0 \quad (7)$$

since the displacement operator commutes with the Hamiltonian, i.e.,

$$[D_x(\Delta x),H] = 0\,.$$

The particular case shown in Figs. 1–3 was calculated on the basis of a Gaussian wave packet but we see that the argument is completely general. The special feature of a pure sinusoidal wave with a Gaussian envelope is that it has a minimum uncertainty product, i.e., $\sigma_x \sigma_k = \frac{1}{2}$. With the evolution of time such a wave packet retains its Gaussian profile but not its sinusoidal form, as seen in Fig. 3, and thus $\sigma_x \sigma_k > \frac{1}{2}$. (In fact σ_k stays constant and σ_x grows monotonically.) However, since $\gamma(\Delta x)$ is independent of time, the coherence length remains equal to its initial value which is proportional to $\sigma_x(0)$. The experiment thus demonstrates that $\sigma_x(0)\sigma_k = \frac{1}{2}$.

Similar conclusions will apply to electron interferometry[5] and also to photon interferometry[6] in a dispersive medium. In fact, there seems to be a strange gap in the optical literature concerning coherence in dispersive media. We speculate that the reason for this is that the most elegant statements, such as the Zernike–Van Cittert theorem[3,4] or the proposition that the correlation function obeys the wave equation, are not immediately applicable to dispersive media. Thus, the substitution $\Delta x = c\Delta t$ is not applicable and longitudinal coherence is no longer synonymous with temporal coherence.

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