[12]Knight shifts plotted in Fig. 1 are defined relative to the ^{133}Cs resonance position in a reference sample of 2.75*M* CsCl aqueous solution. Densities were determined from the measured values of temperature and pressure by using the equation of state data given by N. B. Vargaftik *et al.*, in *Proceedings of the Seventh International Conference on Thermophysical Properties, Washington, D. C., 1977*, edited by A. Cezairliyan (American Society of Mechanical Engineers, New York, 1977), p. 926; G. Franz, dissertation, University of Marburg, 1980 (unpublished), and Ref. 2.

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Decay and Regeneration of the Galactic Magnetic Field in the Presence of Magnetic Monopoles

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The resonant character of magnetic field damping by moving magnetic monopoles allows the field to survive indefinitely when the monopole plasma frequency is sufficiently large, even if the field is immersed in a conducting plasma. Dynamo growth of galactic field can still occur on time scales $\sim 10^7$–10^8 yr in the presence of a halo composed of monopoles. A linear stability analysis suggests that a monopole's mass is $m \sim 10^{18}$ GeV/c^2 and that the monopoles should have an anisotropic flux ~ 1 m^{-2} yr^{-1} sr^{-1}.

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Cabrera[1] has reported a possible candidate for a free magnetic monopole consistent with a Dirac charge $|g| = e/2\alpha_F \cong 3\times10^{-8}$ esu. Certain grand unification schemes[2] suggest that such monopoles should have a mass $m \sim 10^{16}$ GeV/c^2. (Hereafter mass in units of 10^{16} GeV/c^2 will be denoted by m_{16}.) When applied to models of the very early universe,[3] these theories indicate the possible existence of a large space density of free monopoles. One interesting possibility is that these monopoles could be the dominant mass contribution to the dark halo of our galaxy and of other spiral galaxies, which may be the origin of the observed stability of spiral galaxies' disks.[4] Such stabilizing influence requires $M_{\text{halo}}/M_{\text{galaxy}} \gtrsim 1$, and demands the existence of some nonluminous form of matter in the halo of our own galaxy with mass density[5] $\rho_h \gtrsim 10^{-24}$ g cm^{-3} and velocity dispersion $c_m \sim 200$ km/s. (Velocity in units 200 km/s are denoted by the subscript 200.) If the halo is composed of grand unified theory (GUT) monopoles, the expected flux at the earth would then be $\gtrsim 3[\rho_h/(10^{-24}\text{ g cm}^{-3})]\,m_{16}^{-1}$ m^{-2} sr^{-1} yr^{-1}, which is not inconsistent with Cabrera's event being a monopole of galactic origin.

However, several authors[6,7] have argued that such high fluxes are inconsistent with the persistence of the galactic magnetic field, with estimates for upper bounds on the monopole flux $\sim 10^{-5}$ m^{-2} yr^{-1} sr^{-1}. We show that persistence of the galactic magnetic field is consistent with higher fluxes of monopoles, provided that the monopole plasma frequency exceeds a certain *lower* bound. We also show that dynamo activity[8] in the interstellar medium *can* regenerate the magnetic field in the presence of a monopole-dominated halo, at a rate comparable to the orbital angular velocities of monopoles and stars in the halo and of stars and gas in the disk, $\sim(3\times10^7\text{ yr})^{-1}$.

We expect a monopole halo to form a nonrotating,[9] quasineutral, collisionless system with an approximately Maxwellian velocity distribution of dispersion speed c_m because of the effects of violent relaxation during galaxy formation.[10] The monopole plasma frequency $\omega_{pm} = (4\pi g^2 N_m/m)^{1/2}$ is related to the angular frequency of the galaxy by

$$\left(\frac{\omega_{pm}}{\Omega_K}\right)^2 = \frac{g^2}{m^2 G}\left(\frac{M_m}{M_g}\right) = b\left(\frac{M_m}{M_g}\right), \tag{1}$$

with b = (magnetic Coulomb force)/(gravitational force) $= 5\times 10^7 m_{16}^{-2}$, N_m the total space density of monopoles of all charge, $M_m = 4\pi m N_m R^3$, R the distance of the sun from the galactic center [~8 kpc (pc stands for parsec)], and $\Omega_K \equiv (GM_g/R^3)^{1/2} \sim (3\times 10^7\ \mathrm{yr})^{-1}$. Numerically, $\omega_{pm} \sim 2\times 10^{-4} m_{16}^{-1} \times c_{200}(M_m/M_g)^{1/2}\ \mathrm{yr}^{-1}$. The associated magnetic Debye length is $h = c_m \omega_{pm}^{-1} \cong m_{16}$ pc. The galactic disk of stars and conducting interstellar gas rotates through the halo with velocity $u \cong c_m$.

The standard estimate[6] for the rate of field decay is as follows. Consider a region containing a magnetic field $\vec{B}$ which is correlated[11] over a length l. A monopole passing through this region with speed v will acquire a velocity perturbation $\Delta v \sim gBl/mv$ and an energy change $g\Delta v Bl/2v \sim g^2B^2l^2 2mv^2$ which must be withdrawn from the source of the field. The field therefore exchanges energy with the monopoles in a time $t_{\mathrm{exch}} \sim h/\omega_{pm} l \sim 10 m_{16}^2 [l/(500\ \mathrm{pc})]^{-1}$ yr which is shorter than the suspected regeneration time as long as $m < 2\times 10^{19}[l/(500\ \mathrm{pc})]^{1/2}\ \mathrm{GeV}/c^2$.

This argument is a correct estimate of the conservative rate of energy exchange, but is not in general a valid estimate of the *irreversible* rate of magnetic energy loss. Decompose the field into its Fourier components, each with frequency ω and wave number $\vec{k}$. For simplicity, assume that a monopole's orbit is almost a straight line, although the argument is more general. Then each monopole's motion in a Fourier component is an oscillation at the Doppler-shifted frequency $\omega - \vec{k}\cdot\vec{v}$ superposed on the straight line motion, and no net work is done if $\omega - \vec{k}\cdot\vec{v} \neq 0$. Irreversible transfer of energy from a field component with the wave number k and frequency ω to the particles occurs only for those particles with velocity equal to ω/k. In the frame of these particles, the field is constant and energy transfer continues in one direction until the particles travel at least one correlation length of the field (weak δB), are accelerated to a speed significantly different from ω/k (stronger δB), or leave the system.

We quantify this idea by computing the linear susceptibility tensor of the monopoles for a Fourier component of the field $\delta\vec{B} = \vec{B}_{k\omega}\exp i(\vec{k}\cdot\vec{r} - \omega t)$. We neglect the $g\vec{v}\times\vec{E}/c$ force on the monopoles which is $O(v^2/c^2)$ compared to the magnetic force in the circumstances of interest to us. The current induced in the monopoles by $\delta\vec{B}$ is $\delta\vec{J}_m = -i\omega\vec{\chi}(\vec{k},\omega)\cdot\delta\vec{B}$; this defines $\vec{\chi}$. Solution of the linearized Vlasov equation[12] yields

$$\vec{\chi} = \sum_\sigma \frac{n_\sigma g_\sigma^2}{m_\sigma \omega}\int d^3v\, \frac{\vec{v}\,\partial f_\sigma/\partial\vec{v}}{\omega - \vec{k}\cdot\vec{v}}, \tag{2}$$

where $f_\sigma(\vec{v})$ is the monopole distribution function in the absence of magnetic fields and the summation is over all monopole species. When the damping rate $\gamma = \mathrm{Im}\,\omega$ is small compared to $\omega_r = \mathrm{Re}\,\omega$, standard methods[12] applied to Poynting's theorem with the $\delta\vec{J}_m\cdot\delta\vec{B}$ work included yield

$$\gamma = -\frac{\vec{B}_{k\omega}\cdot[4\pi\omega_r\vec{\chi}^a(k,\omega_r)]\cdot\vec{B}_{k\omega}^*}{\vec{B}_{k\omega}\cdot(\partial/\partial\omega_r)(\omega_r\vec{\epsilon}^h)\cdot\vec{B}_{k\omega}^*} \tag{3}$$

where $\vec{\epsilon} = \vec{I} + 4\pi\vec{\chi}$ is the dielectric tensor, $\vec{I}$ is the unit tensor, and the superscripts a and h refer to the anti-Hermitian and Hermitian parts, respectively. Here

$$4\pi\chi_{ln}{}^a = -\frac{1}{8}\frac{\omega_{pm}^2}{\omega}\int d^3v\left(v_l\frac{\partial f}{\partial v_m} + v_m\frac{\partial f}{\partial v_l}\right)\times\delta(\omega_r - \vec{k}\cdot\vec{v}), \tag{4}$$

with $l, n = 1, 2, 3$. We have assumed that two species of monopoles are present with equal mass, equal but opposite charge, and equal unperturbed density.

The standard argument for damping is recovered and generalized as we consider a transverse mode, $\vec{k}\cdot\vec{B}_{k\omega} = 0$, and a monopole plasma frequency sufficiently low to yield $\epsilon^h \cong I$. Then

$$\gamma = -\pi(\omega_{pm}^2/k)\int d^2v_\perp f(v_\parallel = \omega_r/k, \vec{v}_\perp). \tag{5}$$

Equation (5) tells us that all transverse modes are damped by the monopoles, whatever the shape of the distribution function. For an isotropic Maxwellian $f = \pi^{-3/2}c_m^{-3}\exp(-v^2/c_m^2)$, we have

$$\gamma = -\pi^{1/2}(\omega_{pm}^2/kc_m)\exp(-\omega^2/k^2c_m^2). \tag{6}$$

When the magnetic disturbance moves slowly compared to the monopoles' dispersion speed, we recover the standard estimate. When $\omega/k \gg c_m$, however, the damping is exponentially small. Such weak damping is the case for fields frozen in the solar wind moving through monopoles trapped in the solar system, for then $\omega/k \sim 300\ \mathrm{km\ s^{-1}}$ and $c_m \sim 30$ km/s. We therefore disagree with the damping rate used by Dimopolous *et al.*[13] For the same reason, the magnetic fields of supernova remnants are unaffected through most of the life of these systems.

In order to find the damping rate for fields of a given wavelength $2\pi k^{-1}$, we need a dynamical model which relates ω to $\vec{k}$. This is found by solving the induction equation $-c\nabla\times\delta\vec{E} = 4\pi\delta\vec{J}_m + \partial\vec{B}/\partial t$. The interstellar gas acts as a perfect conductor for fields of length and time scale of interest here,[14] and so the electric field vanishes in the frame comoving with the plasma. In the monopole frame, we have $\delta\vec{E} = -c^{-1}\vec{u}\times\delta\vec{B}$. Then the dispersion relation is

$$\mathrm{Det}[(\omega - \vec{k}\cdot\vec{u})\delta_{ln} + u_l k_n + 4\pi\chi_{ln}\omega] = 0. \qquad (7)$$

If $\omega_{pm} \neq 0$, the waves are damped, but with unusual dispersion properties when ω_{pm} is large.

Specialization to an isotropic distribution function yields

$$\chi = \chi_{\parallel}\hat{k}\hat{k} + \chi_{\perp}(I - \hat{k}\hat{k}) \qquad (8)$$

with $\hat{k} = \vec{k}/k$. The dispersion relation (7) then factors into $1 + 4\pi\chi_{\parallel} = 0$, corresponding to longitudinal magnetic plasma oscillations, and yields a transverse mode with

$$\omega = \vec{k}\cdot\vec{u}/(1 + 4\pi\chi_{\perp}). \qquad (9)$$

The longitudinal dispersion relation is identical to that of longitudinal plasma oscillations, and again gives damped solutions as long as the Penrose criterion[15] is satisfied, as is certainly the case for a nonrotating halo. For a Maxwellian distribution, we have

$$4\pi\chi_{\parallel} = -(\omega_{pm}^2/k^2c_m^2)W'(z_m),$$
$$4\pi\chi_{\perp} = (\omega_{pm}^2/kc_m\omega)W(z_m), \qquad (10)$$

where $z_m = \omega/kc_m$ and W is the plasma dispersion function.[16]

When damping is weak and $\omega_r/kc_m \gg 1$, the longitudinal wave has frequency

$$\omega \cong \omega_{pm}\left\{1 - i\pi^{1/2}\left(\frac{\omega_{pm}}{kc_m}\right)^3 \exp\left[-\left(\frac{\omega_{pm}}{kc_m}\right)^2\right]\right\} \qquad (11)$$

and in the transverse case,

$$\omega_r = \tfrac{1}{2}\{\vec{k}\cdot\vec{u} + [(\vec{k}\cdot\vec{u})^2 + 4\omega_{pm}^2]^{1/2}\} \qquad (12a)$$

with

$$\gamma = -\pi^{1/2}\frac{\omega_r}{[(\vec{k}\cdot\vec{u})^2 + 4\omega_{pm}^2]^{1/2}}\frac{\omega_{pm}^2}{kc_m} \times \exp\left[-\left(\frac{\omega_r}{kc_m}\right)^2\right]. \qquad (12b)$$

If the halo has an anisotropic distribution function, as is certainly possible, growing magnetic fields could occur, in principle. We find that this is possible only if the $g\vec{v}\times\vec{E}$ force is retained. Then an analog of the Weibel[17] instability exists but growth occurs only for wavelengths exceeding $c^2/\omega_{pm}u$ which is likely to be larger than the galaxy.

From (12), we recover the standard analysis for the damping rate when $\vec{k}\cdot\vec{u} \gg 2\omega_{pm}$, reduced by a factor $\exp[-(\vec{k}\cdot\vec{u}/c_m)^2]$. In this regime, our results agree with previous upper limits on the density of monopoles consistent with the survival of the galactic field. However, if ω_{pm} is sufficiently large, (14) shows that ω/kc_m becomes large and the damping of the magnetic field is exponentially small. The damping time exceeds the regeneration time t_r if

$$\omega_{pm}^2 > 4k^2u^2[\ln\Lambda_r - \beta_m(\ln\Lambda_r)^{1/2}], \qquad (13)$$

with $\beta_m = u/c_m$ and $\Lambda_r = \pi^{1/2}\omega_{pm}^2 t_r/kc_m$. If $k \sim 10^{-21}$ cm^{-1}, $u \sim 10^{-3}c$, and $\beta_m \sim 1$, then $\omega_{pm}^2 > 0.9\times10^{-27} \times u_{200}^2 l_{500}^{-2}$ s^{-2}, with $l_{500} = l/(500\text{ pc})$. The flux is a more readily measured quantity. The isotropic flux measured in the halo frame then has a *lower* bound

$$J_m > 2\times10^{-15} m_{16} c_{200} u_{200}^2 l_{500}^{-2}\ \mathrm{cm^{-2}\,s^{-1}\,sr^{-1}}.$$

The basic point is quite general: High phase velocities are negligibly damped, and high phase velocity is guaranteed if the monopole plasma frequency is sufficiently large.

While magnetic fields can survive an arbitrarily long time if ω_{pm} is sufficiently large, some arguments[8] suggest that the magnetic field is regenerated in the time $t_r \sim 10^7$–10^8 yr. Since the damping can be negligibly small, dynamo activity[8] still can occur in the gaseous medium, which can amplify the field faster than it decays.

Consider a model for field regeneration in which the velocity field of the gaseous disk includes a component of helical turbulence $\delta\vec{v}$ with length scale $l_T \ll l$. Some aspects of the origin and maintainence of such turbulence have been discussed by Parker.[8] If turbulence is dissipative and helical, it can support a component of electric field parallel to the magnetic field on the length scale l. Thus

$$\delta\vec{E} = -c^{-1}\vec{u}\times\delta\vec{B} + c^{-1}\alpha\delta\vec{B} + (c\eta_T\nabla\times\delta\vec{B}/4\pi), \qquad (14)$$

where α is the dynamo coefficient and η_T is the turbulent resistivity.[18] For the long wavelengths of interest here, the effect of the resistivity η_T is negligible. The dispersion relation (8) now is modified, and for the transverse mode now is

$$k^2\alpha^2 + [\vec{k}\cdot\vec{u} - \omega - 4\pi\chi_{\perp}\omega]^2 = 0, \qquad (15)$$

corresponding to a circularly polarized wave. The longitudinal mode is unaffected. In the long-wavelength limit $kh < 1$, the real part of the frequency is still given by (12a), while the imaginary part now is

$$\gamma = \frac{\omega_r}{(k^2u^2 + 4\omega_{pm}^2)^{1/2}} \times\left\{k\alpha - \pi^{1/2}\frac{\omega_{pm}^2}{kc_m}\exp\left[-\left(\frac{\omega_r}{kc_m}\right)^2\right]\right\}. \qquad (16)$$

Expression (16) shows that the magnetic field can grow with a maximum rate $\gamma_m \cong k_m \alpha$. Here $k_m h \cong (\ln\Lambda)^{1/2}$ and $\Lambda \cong 2\pi^{1/2}(c_m/\alpha)$. For $\alpha \sim 1$ km s^{-1} and $u/c_m \sim 1$, we find $k_m h \cong 0.4$, $\omega_r \cong 1.24\omega_{pm}$, and $\omega/k_m c_m \cong 3.21$. (Our numerical values include thermal corrections in computing ω_r and k_m.)

If we now hypothesize the galactic magnetic field to be these transverse dynamo waves in the monopole-plus-gas plasma we can identify πk_m^{-1} with the correlation length l, which yields $\omega_{pm} \cong 10^{-13} c_{200} l_{500}^{-1}$ s^{-1}. Equation (1) then implies $m = 10^{18} l_{500}^{-1}(M_m/M_g)^{1/2}$ GeV$/c^2$, and the predicted monopole flux is then $J_m \cong 0.6 l_{500}^{-3}(M_m/M_g)^{1/2}$ m^{-2} sr^{-1} yr^{-1}.

Because of the resonant character of the damping, the halo is undisturbed by the energy and angular momentum absorbed by the monopoles from the disk for times up to and including the life of the galaxy. For our example obtained with $\pi k_m^{-1} = l$, we find the halo lifetime to be $\sim 10^{14}$ yrs, in contrast to the nonresonant estimates made by Salpeter, Shapiro, and Wasserman.[7]

Our dynamo has several simplifications which prevent us from applying it to the observed spatial structure of the galactic field in detail. The magnetic polarization of each dynamo mode is circular, generally yielding a component of $\vec{B}$ normal to the galactic disk. A better model requires the inclusion of the vertical structure of the disk and differential stretching of the field lines parallel to $\vec{u}$ because of the shear in the disk velocity.

If Cabrera's event is corroborated by future experiment, it will be of interest to explore the halo model for the site of the monopoles in more detail. Our dynamo model certainly suggests an observable flux ($\sim 1-10/\text{m}^2$ yr for an isotropic detector with no velocity threshold), but one which is ~ 3 orders of magnitude below a naive interpretation of Cabrera's event as a measured flux. Fortunately, this whole class of models for the site of the monopoles makes a clear qualitative prediction. Since the solar motion is comparable to the monopoles' dispersion speed, the flux should be highly anisotropic. Independent of the origin of the monopoles, our analysis shows that galactic (and solar system) magnetic fields can coexist with monopole densities greatly in excess of previous upper limits, provided the monopole plasma frequency is sufficiently large to cause the phase speed of the field to exceed the dispersion speed of the monopole velocity distribution.

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