Transition from Hydrodynamic to Ballistic Quasiparticle Behavior in a Fermi Gas: The Response of a Vibrating-Wire Resonator in a ^{3}He-^{4}He Solution from 0.3 to 10 mK

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Measurements of the frequency shift, Δf_1, and the damping, Δf_2, of a vibrating wire in a saturated ^{3}He-^{4}He solution are reported. A transition is observed in the ^{3}He quasiparticle Fermi gas from a hydrodynamic regime at 10 mK to a collisionless-excitation gas regime at the lowest temperatures, < 0.3 mK, where the mean free path becomes extremely long. The comparison between Δf_1 and Δf_2 supports the slip theory of Højgaard Jensen *et al.* with the recent modification proposed by Carless, Hall, and Hook.

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In a dilute solution of ^{3}He in ^{4}He at low temperatures the thermal and hydrodynamical properties are determined largely by the ^{3}He quasiparticles. Since these quasiparticles form a degenerate Fermi gas, the mean free path, l, for quasiparticle-quasiparticle scattering varies as T^{-2} and as a result l becomes very long at low temperatures. For example, for a saturated solution (6.4%) of ^{3}He in ^{4}He at zero pressure, l is 0.5 mm at a temperature of 1 mK. Thus it is straightforward to devise experiments with a length scale considerably smaller than the quasiparticle mean free path below 1 mK. Consequently these solutions provide an ideal laboratory for studying the transition, in a degenerate Fermi gas, from an essentially hydrodynamic regime at 10 mK to an almost collisionless-excitation gas regime at 0.3 mK.

In this paper we describe measurements of the damping and frequency shift of a resonant vibrating wire in such a solution through this transition region. The results give a dramatic verification of the theory of slip effects derived by Højgaard Jensen *et al.*[1], when modified by the method of Carless, Hall, and Hook[2] to give the correct expression in the long-mean-free-path limit.

As described in an earlier paper,[3] the helium solution is cooled by intimate contact with copper slabs coated with sintered silver powder, the copper nuclei being demagnetized from 6.5 T at 8 to 10 mK to a final field of 14–42 mT, in which field the experiments are performed. In the double-cell arrangement used, the final heat leak to the specimen is reduced below 15 pW and the lowest helium temperatures achieved are below 0.3 mK. (The Pt NMR thermometer used in this particular study had a large time-dependent heat leak, apparently from insulation on the wires, so that thermometry below 0.4 mK is not reliable. However, the variation of Δf_1 and Δf_2 indicates that the lowest temperatures were below 0.3 mK, where these two parameters saturate.)

The vibrating wire consists of a length of 0.124-mm-diam tantalum wire, bowed into a semicircle of radius 4 mm, situated in a small free volume of helium, the loop lying in the plane of the magnetic field. The wire is about 1.5 mm from the nearest walls. The flapping oscillation of the loop is excited by the Lorentz force as an ac current is passed through it in the static magnetic field. The oscillation is observed by the measurement of the voltage induced across the moving wire. The loop used in the present series of measurements oscillates in a vacuum at a frequency of $f_0^{\mathrm{vac}} = 897$ Hz with a Q value greater than 30 000. In a saturated ^{3}He-^{4}He solution at 0.5 bar (7% ^{3}He) the resonant width at the highest level of damping, i.e., at the lowest temperatures, is only ~50 Hz and thus analysis of the resonance is still a reasonably straightforward matter.

Tantalum was chosen as the wire material for two reasons. Firstly the high density (16.7 g/cm^3) gives a low resonant width, since the higher inertia of the wire is less affected by the drag from the liquid, and secondly since Ta is a superconductor the in-phase voltage has no component arising from the resistance of the wire. A small, but significant, signal from the self-inductance of the wire had to be subtracted from the quadrature voltage.

The measurements are controlled by a desk-top computer. The frequency is stepped in 100 steps (20 sec per step) through the resonance by a frequency synthesizer and the in-phase and quadrature voltages are measured at each step by a lock-in amplifier using a 3-sec time constant. The resonance curves obtained in this way fit very well to the expected Lorentzian type of line shape appropriate to a damped harmonic oscillator. The curves can be characterized by the two parameters Δf_1 and Δf_2 representing respectively the frequency shift and the frequency width. The width, Δf_2, is taken as the frequency interval between the two half-height points of the in-phase voltage. The resonant frequency f_0 is determined from both the maximum of the in-phase voltage and from the zero crossing point of the quadrature voltage and hence the frequency shift $\Delta f_1 = f_0^{\text{vac}} - f_0$ is obtained.

These two parameters characterize the behavior of the liquid. In the standard hydrodynamic treatment[4] for a high-Q resonance (which we expect to describe our data well at high T) Δf_1 and Δf_2 are given by

$$\frac{\Delta f_1}{f_0} = \frac{\rho}{2\rho_W} + \frac{\rho_n}{2\rho_W}(k-1), \tag{1a}$$

$$\frac{\Delta f_2}{f_0} = \frac{\rho_n}{\rho_W} k', \tag{1b}$$

where ρ is the total density of the helium, ρ_n the normal fluid density (i.e., ^{3}He density), and ρ_W the density of the wire. The functions k and k', first defined by Stokes, depend on the ratio of the wire radius, a, to the viscous penetration depth $\delta = (\eta/\rho_n\omega)^{1/2}$. In this regime we expect both Δf_1 and Δf_2 to increase together with decreasing temperature as the viscosity η rises. In physical terms, Δf_2 arises from the energy dissipated by the moving wire which increases with viscosity, and Δf_1 arises from the change in effective mass of the wire which also increases with the viscous penetration depth as more helium moves with the wire.

The experimental results[3] at low temperatures differ markedly from Eq. (1), since the hydrodynamic approximation of a local viscosity breaks down. As the temperature is reduced below 1 mK, Δf_2 saturates towards a limiting value, whose magnitude can be calculated from a simple model appropriate to the limit $l \gg a$. In the model, we regard the interaction between the wire and the helium as one of bombardment of the wire by the individual quasiparticles of the Fermi gas. The result is

$$\Delta f_2^{(0)} = Anp_{\text{F}}/2\pi^2 a\rho_W, \tag{2a}$$

where n is the ^{3}He quasiparticle density, p_{F} the Fermi momentum, and A a numerical constant (of order 2). The same philosophy gives no frequency shift from the bombarding quasiparticles, since the force on the wire is in phase with its velocity. Hence we expect a shift given by

$$\Delta f_1^{(0)}/f_0 = \rho/2\rho_W \tag{2b}$$

corresponding to the first (backflow) term only in Eq. (1a). Actually since the backflow is driven by the incompressibility of the fluid, it is possible that the small backflow contribution of the ^{3}He component also disappears in this limit. In this case one would have

$$\Delta f_1^{(0)}/f_0 = (\rho - \rho_3)/2\rho_W. \tag{2c}$$

Unfortunately the precision of our measurements is not quite sufficient to make a definitive test of this point.

A series of measurements of Δf_1 and Δf_2 are shown in Fig. 1 for a saturated (~7% ^{3}He) solution of ^{3}He in ^{4}He at a pressure of 0.5 bar, measured in a magnetic field of 42 mT. We have chosen to plot the resonant frequency f_0 (effectively a measure of the shift Δf_1) directly against the width Δf_2, a representation in which T enters only as a parameter, thus avoiding the difficulties of thermometry at the lowest temperatures. The form of the Δf_1 vs Δf_2 curves is broadly the same at all pressures up to 16 bars. However, at the higher pressures Δf_2 rises to a saturation value[3] approaching 100 Hz, at which width both Δf_1 and Δf_2 become hard to measure with the necessary precision if excitation currents are limited to values at which the liquid does not exhibit heating. Thus the curve of Fig. 1 is better characterized at the lower pressures. The measurements were made as the sample warmed up after demagnetization, a long process taking 7 d to reach 1 mK.

The form of the curve in Fig. 1 shows clearly the influence of the mean free path at the lower temperatures. With decreasing temperature, the resonance width Δf_2 rises gradually to a limit [cf. Eq. (2a)] while at the same time the shift Δf_1 first increases, as the fluid becomes more viscous, but then below 2 mK decreases again to the background value [cf. Eq. (2b)] as mean-free-path effects dominate. At the lowest temperatures f_0 is again almost at its high-temperature value, although Δf_2 is about 50 times greater than its value at 20 mK.

According to Eq. (2a), the limiting value of Δf_2 should be independent of the resonant frequency. In the present series of experiments we had three independent wire resonators, one of which was placed between walls only a fraction of a millimeter away. This latter resonator was deformed by differential contraction during cooling

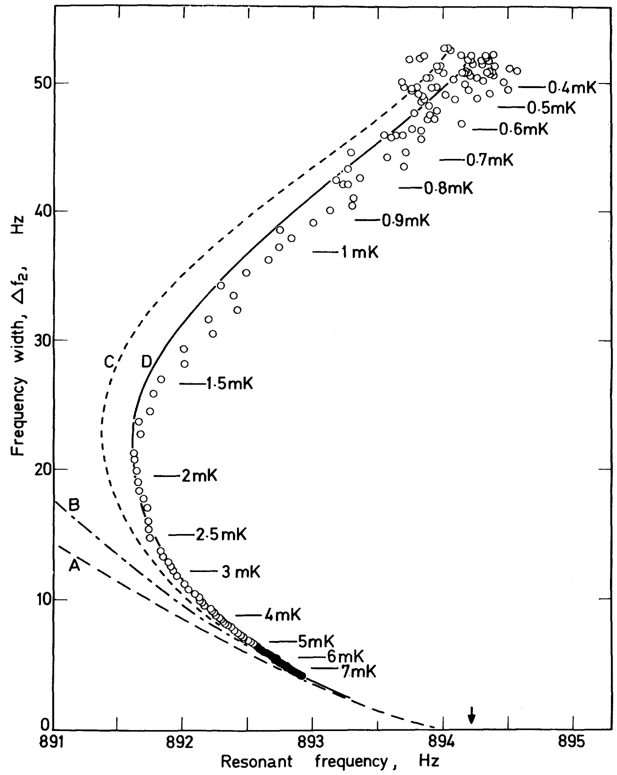

FIG. 1. The variation of the resonant frequency f_0 with the width Δf_2 for a vibrating-wire resonator in a saturated solution of ^3He in ^4He at 0.5 bar. Open symbols and filled symbols are results from two separate demagnetizations. The curve labeled A is a calculation using the Stokes theory of Eqs. (1) (see text). Curve B is a calculation using the slip theory of Højgaard Jensen *et al.* (Ref. 1). Curve C is based on the slip theory or Ref. 1 modified according to the iedas of Carless, Hall, and Hook (Ref. 2). Curve D represents a calculation similar to that of curve C, but with two modifications: (i) the expansion parameter β is defined in a different way (see text), and (ii) the ^3He backflow is assumed not to contribute to Δf_1 at low temperature [giving an upward shift of about 0.2 Hz at low temperatures, cf. Eqs. (2b) and (2c)]. The arrow marks the value, 894.21 Hz, chosen in all four calculations as the vacuum value of f_0 when corrected for backflow of the ^4He background only [i.e., as in Eq. (2c)].

to touch the walls at one point giving two lengths of loop with the high resonant frequencies of 4000 and 8000 Hz. The limiting frequency widths at the low temperatures for these two resonances were the same as those shown in Fig. 1 for the well-behaved resonator to within a few percent, giving further support to Eq. (2a).

A number of theoretical curves are also shown in Fig. 1. Curve A represents a purely hydrodynamic calculation based on the straightforward Stokes theory of Eq. (1). This curve is inadequate even at the very highest temperatures. Curve B is based on the slip theory of Højgaard Jensen *et al.*[1] [their Eqs. (113) and (114)]. This calculation uses an expansion parameter $\beta = 0.579l/(0.579l + a)$ modifying Stokes theory to take into account slip at the walls, as proposed by the authors. One can see that at the higher temperatures this gives a better fit to the experiment than the simple Stokes calculation showing that the slip theory gives a good account of the behavior for the case $l \ll a$. However, when l begins to become comparable to a (at around 3 mK) the curve no longer agrees with experiment. In particular, since the expansion parameter β is limited to values $\beta \leq 1$ by the definition given above, the theory does not predict the plateau in Δf_2 or the low-temperature collapse of Δf_1 suggested by the simple bombardment arguments in Eq. (2). In order to retain the limiting values of Eq. (2) we have therefore followed the ideas of Carless, Hall, and Hook[2] and have recalculated the theoretical curve using as an expansion parameter

$$\beta = \frac{0.579l}{a}\left(\frac{1+\alpha l/a}{1+l/a}\right), \tag{3}$$

where α is a constant of order 2. This leads to the same result as Ref. 1 when $l \ll a$ but (with suitable adjustment of the constant α) to the same result as Eqs. (2) when l is very long, with a smooth (albeit somewhat arbitrary) interpolation between these limits. In the discussion of their results in superfluid ^{3}He-B, Carless, Hall, and Hook propose $\alpha = 2.46$, which corresponds to $A = 9\pi/16 = 1.77$ in Eq. (2a).

The curve C in Fig. 1 is based on such a calculation using a value of 2.3 for the constant α. Since the adjustment of α changes the limiting value of Δf_2 on which the curve terminates at low T, this constant is rather accurately determined by the experiment. Our data at higher pressures are consistent with the same value of α.

The remarkable agreement between experiment and this calculation is all the more striking since, apart from this slight adjustment of α (or A) to give the correct limit of Δf_2, all the other quantities are measured and thus not negotiable. The wire parameters ρ_W and a are measured directly. The value of ηT^2 is obtained from the variation of Δf_2 with T in the hydrodynamic regime (we find $\eta T^2 \approx 0.3 \times 10^{-7}$ SI units for the saturated solution at 0.5 bar consistent with values of earlier workers at higher temperatures[5]). The value of ρ_n is taken from the solubilities and molar volumes of Watson, Reppy, and Richardson.[6] The magnitude of lT^2 is then fixed (since $\eta=\frac{1}{5}np_{\rm F}l$, and n and $p_{\rm F}$ are derived from ρ_n only). This is a particularly significant feature of the agreement since the form of the Δf_2-Δf_1 curve in the intermediate region depends strongly on the scale of l/a.

In spite of the success of the calculation, some important questions remain. Firstly, Eq. (3) is somewhat empirical in form; in fact we can make our good agreement even more precise by instead using

$$\beta=\frac{0.579l}{a}\left[\frac{1+\alpha^2 l/a}{1+l/a}\right]^{1/2},$$

represented by curve D in Fig. 1. Clearly these empirical ideas need a firmer physical basis. Secondly, the theory is appropriate to an infinite volume of fluid in the low-frequency limit ($l/\delta \ll 1$, or $\omega\tau \ll 1$ with $l=v_{\rm F}\tau$). Although this is a reasonable starting point in our work, it is by no means an accurate assumption since a calculation for our configuration suggests that $\omega\tau \sim 0.08$ at 1 mK and ~ 1 at 0.3 mK. However, at 0.3 mK the calculated values of l and δ are both about 3 mm, a length greater than the container size. Hence it seems possible that as T falls the small container size may take the vibrating wire with unseemly haste into a low-temperature limit given by Eqs. (2), and that $\omega\tau$ effects do not play a significant role. [Presumably the value of $\omega\tau$ is irrelevant in the limit of Eqs. (2), since the only assumption is that the wire is bombarded by quasiparticles from an equilibrium background. When l is greater than the container size then presumably these quasiparticles come from the walls and the behavior of an infinite fluid is of little importance.]

In summary, we have made the first measurements of the damping and frequency shift of a vibrating wire in a saturated ^{3}He-^{4}He solution below 0.4 mK. The damping and frequency shift vary together in a very distinctive way, as shown in Fig. 1, with behavior characteristic of a gradual transition in the degenerate Fermi gas from a local hydrodynamic regime at 10 mK to a ballistic quasiparticle regime at the lowest temperatures, less than 0.3 mK.

It is worth noting in this context that ^{3}He-^{4}He solutions are unique. The full range of this transition can be followed since the mean free path varies by a factor of 1000 over the available temperature range in the degenerate Fermi fluid. Corresponding effects in pure ^{3}He are more difficult to observe since in the normal fluid the mean free path is more than 10 times shorter at the same temperature and the superfluidity at the lower temperatures introduces additional complications of its own.

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