Deconfinement and Chiral Symmetry Restoration at Finite Temperatures in SU(2) and SU(3) Gauge Theories

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(Received 22 November 1982)

At zero temperature SU(2) and SU(3) gauge theories confine quarks and spontaneously break chiral symmetry. At some nonzero temperatures T_c (T_F) these gauge theories lose confinement (chiral symmetry breaking). The order of these phase transitions and the relation between T_c and T_F have been studied with use of simulation methods which neglect internal fermion loops. For SU(2) both transitions are second order and $1 \leq T_F/T_c < 1.30$. For SU(3) the transitions are first order and $1 \leq T_F/T_c \leq 1.05$, with $T_c \approx 200$ MeV.

PACS numbers: 11.15.Ex, 05.50.+q, 11.30.Jw

It is generally agreed that much can be learned about the interactions between elementary particles by studying them in unusual environments. In order to understand quantum chromodynamics better, we are engaged in a study of lattice gauge theories at different length scales, at variable temperatures, and in environments rich in baryon number. It is the purpose of this Letter to discuss some computer simulation results in SU(2) and SU(3) gauge theories at finite temperature.

These gauge theories confine quarks at zero temperature and they break chiral symmetry spontaneously.[1] However, at high temperature the gauge theories have neither property. Therefore, they experience two phase transitions in the intermediate temperature region. It is natural to wonder if these two transitions are intimately related. In a previous Letter[2] we presented evidence that for SU(2) theories with quarks in high-dimensional representations of the color group the chiral-symmetry-restoring temperature is many orders of magnitude larger than the deconfining temperature. We conclude, therefore, that if a theory has *sufficiently strong* short-distance (single-gluon) attraction between quarks and antiquarks, then chiral symmetry can be broken even in the absence of confinement. Here we wish to address similar questions in more quantitative detail with quarks in the *fundamental* representation of the gauge groups SU(N). We will consider SU(2) and compare with the physically relevant group SU(3). The questions we pose are the following: (1) Are the short-distance forces in these theories sufficient to break chiral symmetry, or is confinement necessary? (2) What are the orders of the chiral-symmetry-restoring and deconfinement phase transitions in each theory? To answer these questions we ran computer simulations at finite temperature and measured the deconfinement temperature T_c and the chiral-symmetry-restoration temperature T_F in a single computer simulation. Since various measurements reported in the literature are not in agreement,[3,4] we felt that another, independent measurement of T_c was necessary. For SU(2) we found that the T_F is very close to T_c, $1 \leq T_F/T_c \leq 1.30$, and the transitions could be coincident. For SU(3), we found that T_F and T_c are almost certainly *coincident,* $1 \leq T_F/T_c \leq 1.05$. The phase transitions for SU(2) appear to be continuous, but for SU(3) clear discontinuities in the order parameters were observed—both the chiral-symmetry-restoring transition and the deconfinement transition appear to be strongly *first-order* transitions. Our results imply that when quantum chromodynamics is heated above T_c both the string tension and the dynamical quark masses vanish discontinuously.[5]

Why should the finite-temperature transitions be qualitatively different in SU(2) and SU(3) gauge theories? At high temperature one expects SU(N) gauge theories to resemble three-dimensional spin systems.[6] The deconfining phase transition breaks a global $Z(N)$ symmetry.[6,7] Therefore, the SU(2) transition should be in the same universality class as the three-dimensional Ising model. It has a second-order transition. However, the SU(3) model should resemble a three-state Potts system.[7] Computer simulations[8] and $1/N$ expansions[9] suggest a first-order transition here.

Now consider our calculations. The gauge field degrees of freedom are SU(N) unitary matrices which are placed on the links of a four-dimensional Euclidean lattice. The fermions are represented by complex valued fields having N color indices and residing on the sites of the lattice. We use a lattice fermion action which couples only nearest neighbors together and which preserves remnants of chiral symmetry. The remnants include a continuous subgroup and various discrete elements of the familiar continuous group of chiral transformations.[10] These features of our lattice fermion method allow us to study spontaneous symmetry breaking and the Goldstone mechanism on the lattice. We do this by computer simulation methods which neglect internal fermion loops.[1] We hope that this approximation, the quenched or $N_f \to 0$ limit (N_f, number of flavors), is an adequate starting point to study the physics questions we pose. With this approximation, the computer simulation strategy is straightforward—generate a gauge field configuration at a certain coupling g^2, solve for the quark propagator in that configuration, and repeat this process so that one can average the quark propagator and gauge field matrix elements over many gauge field configurations. We obtained the quark propagator by the conjugate-gradient iterative procedure.[11] Typically, we made 75–150 calculations of the quark propagator in a sample of 3000 gauge configurations at each value of g^2.

To study the chiral phase structure of these models we computed $\langle\bar{\psi}\psi\rangle$ from the fermion propagator.[12] The confining phase structure was monitored with the expectation value of the Wilson line W,

$$W = \langle \textstyle\prod_C U_\mu(n)\rangle, \tag{1}$$

where $U_\mu(n)$ is the SU(N) matrix on the link $n \to n + \mu$ and the contour C is over a path of timelike links which transverses the entire width of the lattice. The path is closed by the periodic boundary conditions imposed on the gauge fields. Physically, the matrix element Eq. (1) represents the presence of a static quark interacting with the gauge fields. For SU(2) gauge theory the absolute value of the Wilson line is essentially the exponential of minus the excess free energy in the system due to a heavy quark.[7] So, in the confined phase the Wilson line is zero and in the deconfined phase it is not. For SU(3) the Wilson line has a $Z(3)$ symmetry. In that case one must project W against the nearest cube root of unity and use the real part of the result as a measure of the static quark energy.[13]

Finally, recall how finite temperatures are simulated. One considers an asymmetric lattice, $N_\tau \times N^3$, with the width in the "time" direction related to the physical temperature T,

$$aT = N_\tau^{-1}, \tag{2}$$

where a is the lattice spacing, and the size of the box N should be much larger than N_τ. The temperature should scale according to asymptotic freedom at weak coupling,

$$aT \propto (\beta_0 g^2)^{-\beta_1/2\beta_0^2} \exp(-1/2\beta_0 g^2), \tag{3a}$$

where

$$\beta_0 = \tfrac{11}{3}N/16\pi^2, \quad \beta_1 = \tfrac{34}{3}N^2/(16\pi^2)^2 \tag{3b}$$

in SU(N) gauge theory without internal fermion loops. To set the scale in Eq. (3), one can write

$$T = C\sqrt{\sigma}, \tag{4}$$

where C is a dimensionless constant and $\sqrt{\sigma}$ is the zero-temperature string tension of the model. One uses Eqs. (2)–(4) as follows. The order parameters $\langle\bar{\psi}\psi\rangle$ and W are computed as functions of g^2 on a given $N_\tau \times N^3$ lattice. The couplings where $\langle\bar{\psi}\psi\rangle$ and W vanish are measured and if the coupling lies in the scaling region of the theory

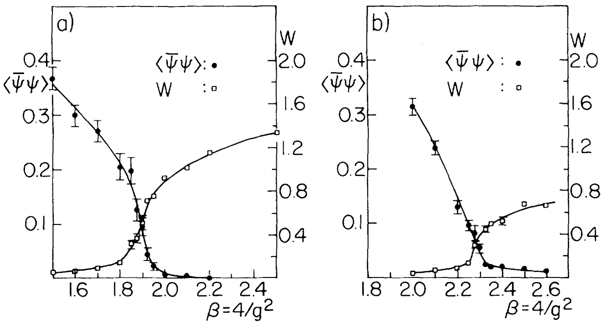

FIG. 1. $\langle\bar{\psi}\psi\rangle$ and W vs $\beta=4/g^2$ for SU(2) gauge theory on (a) 2×8^3 and (b) 4×8^3 lattices. The curves are meant to guide the eye; they are not precise fits.

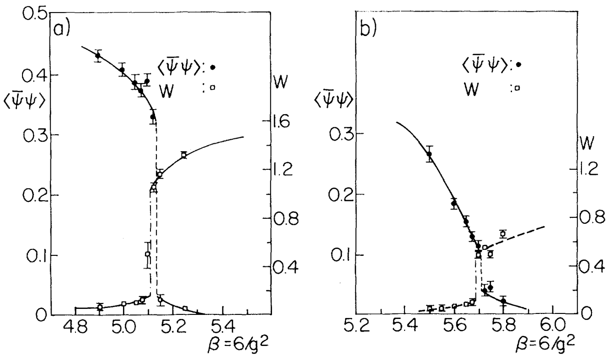

FIG. 2. $\langle\bar{\psi}\psi\rangle$ and W vs $\beta=6/g^2$ for SU(3) gauge theory on (a) 2×8^3 and (b) 4×8^3 lattices.

Eqs. (2)–(4) are used to predict the critical temperatures in physical units. The simulations are run for different values of N_τ (2, 4, and 6) which permits the scaling law Eq. (3) to be verified explicitly.

Now consider our results in Fig. 1. We show plots of $\langle\bar{\psi}\psi\rangle$ and W for the gauge group SU(2) on lattices having $N_\tau=2$ and 4 with $N=8$. Both order parameters appear to vanish continuously.[14] Using Eq. (3), we infer that the difference in couplings at which W and $\langle\bar{\psi}\psi\rangle$ vanish, $\Delta\beta\leq 0.10$, implies a ratio of temperatures $1\leq T_F/T_c\leq 1.30$.[15] We also have data on lattices with $N_\tau=6$, and the $N_\tau=4$ and 6 data satisfy asymptotic freedom, Eq. (3). Our measurement of T_F agrees with our earlier data,[2] and our measurement of T_c is closer to that reported in Ref. 3 than that of Ref. 4 which we accepted previously.

Now consider the analogous SU(3) data presented in Fig. 2. Both the chiral-symmetry-restoring and deconfinement transitions are very sharp and suggest a strongly first-order transition. Note that $\langle\bar{\psi}\psi\rangle$ varies from 0.324 ± 0.015 at $\beta=6/g^2=5.125$ to 0.023 ± 0.015 at $\beta=5.150$. According to Eq. (3) this change in β, $\Delta\beta=0.025$, corresponds to a fractional change of temperature of only 3%. The ratio of the critical temperatures is very near unity, $1\leq T_F/T_c\leq 1.05$. One can verify that the $N_\tau=2$ and 4 data are in good agreement with asymptotic freedom. We also have data at $N_\tau=6$ which are consistent with asymptotic freedom. Comparing with string-tension calculations, we find

$$T_c=(0.46\pm0.10)\sqrt{\sigma}=(0.93\pm0.10)\Lambda^{\mathrm{mom}}. \qquad (5)$$

So, if $\sqrt{\sigma}\cong 450$ MeV as usually assumed,[16] then $T_c\approx 200$ MeV.

We have developed additional evidence for the first-order character of these transitions by finding evidence of coexisting states, hysteresis, and metastability in long simulation runs. In Fig. 3 we show W as a function of Monte Carlo sweep number for $\beta=5.075$ (lower curve) and $\beta=5.125$ (upper curve) with both runs beginning with identical hot (small β) gauge configurations. The sharp difference between the two runs and the lack of large fluctuations is evidence for a strongly first-order transition. Note in Fig. 3 that the first 600 sweeps through the lattice at $\beta=5.125$ do not reach equilibrium although the SU(3) program used was a standard Metropolis routine in which 20 hits were made per link on

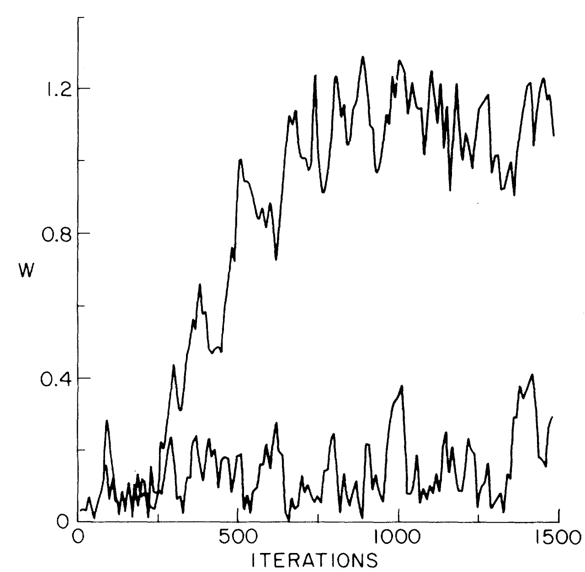

FIG. 3. W vs Monte Carlo sweep number for SU(3) gauge theory on a 2×8^3 lattice. Both curves have identical disordered starts; the lower curve is at $\beta=5.075$ and the upper curve is at $\beta=5.125$.

each sweep. We, therefore, discarded typically 500–600 sweeps in our runs before taking averages of $\langle\bar{\psi}\psi\rangle$ and W.

We find our SU(3) results particularly intriguing. We hope to investigate the thermodynamics of the transition further in the near future.

This work was supported in part by the National Science Foundation under Grants No. PHY-82-01948 and No. DMR 81-17182, and in part by the U. S. Department of Energy under Grant No. DE-AC02-81ER10957. The SU(3) simulations required 200 CPU hours on the three CDC-7600 research machines at Los Alamos National Laboratory. We thank D. K. Campbell and The Center for Nonlinear Studies for their excellent support, without which this project could not have been accomplished. We thank the Research Board of the University of Illinois for 200 hours of CYBER 175 CPU time for the SU(2) simulations.

[1]H. Hamber and G. Parisi, Phys. Rev. Lett. 47, 1792 (1981); E. Marinari, G. Parisi, and C. Rebbi, Phys. Rev. Lett. 47, 1795 (1981); D. Weingarten, Phys. Lett. 109B, 57 (1982).

[2]J. Kogut, M. Stone, H. W. Wyld, J. Shigemitsu, S. H. Shenker, and D. K. Sinclair, Phys. Rev. Lett. 48, 1140 (1982). The calculations reported in this paper have been refined and a sequel is in preparation.

[3]J. Engels, F. Karsch, H. Satz, and I. Montvay, Nucl. Phys. B205, [FS5] 545 (1982).

[4]J. Kuti, J. Polonyi, and K. Szlachany, Phys. Lett. 98B, 199 (1981).

[5]If the deconfinement transition is first order and confinement is necessary for chiral-symmetry breaking, then the chiral-symmetry-restoring transition is expected to be first order in calculations which ignore internal fermion loops.

[6]A. M. Polyakov, Phys. Lett. 72B, 477 (1978); L. Susskind, Phys. Rev. D 20, 2610 (1979).

[7]L. G. Yaffe and B. Svetitsky, Phys. Rev. D 26, 963 (1982).

[8]S. J. Knak Jensen and O. G. Mouritsen, Phys. Rev. Lett. 43, 1736 (1979).

[9]J. B. Kogut and D. K. Sinclair, Solid State Commun. 41, 187 (1982).

[10]T. Banks, S. Raby, L. Susskind, J. Kogut, D. R. T. Jones, P. Scharback, and D. K. Sinclair, Phys. Rev. D 15, 1111 (1977); L. Susskind, Phys. Rev. D 16, 3031 (1977); N. Kawamoto and J. Smit, Nucl. Phys, B192, 100 (1981).

[11]M. R. Hestenes and E. Stiefel, J. Res. Nat. Bur. Stand. 49, 409 (1952). This paper's second-order formalism must be applied to insure convergence in our application.

[12]$\langle\bar{\psi}\psi\rangle$ is obtained from the limit of the quark propagator as the bare fermion mass m_0 is taken to zero. A numerical study of this procedure will appear in the sequel to Ref. 2.

[13]With this calculational procedure W is always positive and is not a truly satisfactory order parameter. However, on a finite lattice where there is tunneling between vacuum states, this calculational procedure is necessary. This procedure introduces "rounding" into the curves of W vs β. This effect and the mass extrapolation for $\langle\bar{\psi}\psi\rangle$ are our two largest systematic uncertainties.

[14]For $N_T=4$, the data for the Wilson line are fitted well with $W=1.03(\beta-2.25)^{0.34}$. The critical index agrees with the three-dimensional Ising model. The chiral transition will be studied in greater detail in the sequel to Ref. 2.

[15]Since the deconfining phase transition is second order, the analysis of Ref. 6 suggests that heavy quarks interact with a logarithmic potential at large distances for $T=T_c$. This attraction could be strong enough to preserve the $\langle\bar{\psi}\psi\rangle$ condensate for temperatures slightly above T_c.

[16]M. Creutz, Phys. Rev. Lett. 45, 313 (1980); E. Pietarinen, Nucl. Phys. B190, 349 (1981).