# Upper Bound on Gauge-Fermion Masses

Steven Weinberg
*Department of Physics, University of Texas, Austin, Texas 78712*

(Received 22 November 1982)

A large class of broken supersymmetry theories is shown to imply the existence of fermions $\lambda^{\pm}$ and $\lambda^0$, lighter than or nearly degenerate with the $W^{\pm}$ and $Z^0$ gauge bosons, and with vanishing baryon and lepton number. If the $\lambda^{\pm}$ is appreciably lighter than the $W^{\pm}$ it can be readily produced in $W^{\pm}$ decay, as well as in $e^+$-$e^-$ collisions.

PACS numbers: 11.30.Pb, 14.80.Er, 14.80. Pb

Because supersymmetry if valid at all is surely broken, we do not in general know how high are the masses where we should search for the superpartners of known particles. This note will show that for a large class of broken supersymmetry theories, each massive gauge boson must be accompanied by a fermion with the same conserved quantum numbers, a mixture of superpartners of gauge bosons and chiral scalars, that is lighter than (or nearly degenerate with) the gauge boson. In particular, there is a *charged* fermion, the $\lambda^{\pm}$, which is lighter than the $W^{\pm}$ gauge bosons and hence should be readily produced in electron-positron colliding-beam accelerators such as LEP, as well as a neutral fermion $\lambda^0$ lighter than the $Z^0$.

The key assumption is that whatever breaks supersymmetry, it does not affect the formulas for gauge-fermion masses in terms of scalar vacuum expectation values (VEV's), except through small radiative corrections. This will of course be the case if the effective Lagrangian that describes physics at ordinary energies is supersymmetric, with supersymmetry broken spontaneously by scalar VEV's; in such theories supersymmetry breaking affects the tree-approximation spinor and vector masses only through the values of the scalar VEV's. Unfortunately theories of this sort lead to severe conflicts with experiment.[1]

However, our assumption is also valid in the more promising class of theories[2] in which supersymmetry is intrinsically broken in the effective Lagrangian, through supergravity interactions with a "hidden" sector of superfields in which supersymmetry is spontaneously broken. Inspection of the results of Ref. 3 show that although supergravity interactions change the form of the scalar-field potential, they do not enter in mass terms involving those gauge fermions that do not mix with the gravitino.

Strictly speaking, this holds for theories with only minimal kinematic terms for the Yang-Mills superfields. Nonminimal terms of the form $[f_{\alpha\beta}(S)W_\alpha W_\beta]_F$, arising perhaps from gravitational radiative corrections, would produce gauge-fermion mass terms in tree approximation.[4] Because supersymmetry is assumed to be directly broken only in the hidden sector, any dependence of $f_{\alpha\beta}(S)$ on other "observable" superfields could produce only negligible gauge-fermion masses of order $m_g^2/M_{\rm GU}$, where $m_g \approx 100$ GeV is the gravitino mass and $M_{\rm GU} \approx 10^{17}$ GeV is a grand-unified mass. Terms in $f_{\alpha\beta}(S)$ that depend on the hidden-sector superfields could produce gauge-fermion masses as large as $m_g$, but any such terms that are produced by gravitational radiative corrections would be suppressed by a factor $m_g/M_{\rm Pl}$, where $M_{\rm Pl} = 1.2\times 10^{19}$ GeV is the Planck mass. This is because purely gravitational effects respect a U($n$) symmetry among the $n$ chiral superfields of the hidden sector, while the function $f_{\alpha\beta}(S)$ depends on left-chiral superfields but not their adjoints, so that any terms in $f_{\alpha\beta}(S)$ that depend on the hidden-sector superfields cannot respect this symmetry, and must therefore be proportional to the hidden-sector superpotential which breaks the U($n$) symmetry, and hence be proportional to a factor $m_g$. Such terms would yield a negligible gauge-fermion mass, of order $m_g^2/M_{\rm Pl}$. It is of course possible that the Lagrangian contains nonminimal Yang-Mills terms that are much larger than could be produced by radiative corrections; we assume here that this is not the case.

There are also ordinary nongravitational radiative corrections. These are expected[5] to contribute masses to the electroweak and color gauge fermions of order $\alpha m_g N/2\pi \approx (\lambda/e)\times(1\text{ GeV})$ and $\alpha_{\rm QCD} m_g N/2\pi \approx (\lambda/e)\times(15\text{ GeV})$, respectively. (Here we take $m_g \approx m_W\lambda/e$, where $\lambda$ is a typical coupling in the superpotential, presumably not very different from $e$, and $N\approx 10$ is a Casimir factor depending on the numbers of nonneutral superheavy supermultiplets.) Hence we do not need to invoke gravitational radiative corrections or nonminimal Yang-Mills terms to explain why color gauge fermions ("gluinos") have not been seen. Also for $m_g \approx m_W$ the estimated mass of roughly 1 GeV for the photon superpartner ("photino") is close to the cosmological lower bound[6] of about 2 GeV, which is set by the condition that such heavy, neutral, weakly interacting, stable fermions should annihilate fast enough to reduce their present mass density to acceptable levels. Thus there is no clear conflict with cosmology, and we have a hint that photinos may provide an important "dark" contribution to the cosmic mass density. We shall assume here that all radiative corrections contribute no more than a few gigaelectronvolts to electroweak gauge-fermion masses.

Under our assumptions, the Majorana fermion mass terms connecting the $\alpha$th gauge fermion with the $n$th chiral scalar and the $\beta$th gauge fermion are given to a good approximation by the well-known formulas[7]

$$m_{\alpha n} = -\sqrt{2}\,(t_\alpha \langle z\rangle)_n \tag{1}$$

$$m_{\alpha\beta} = 0. \tag{2}$$

Here $\langle z\rangle$ is the column of left-chiral scalar VEV's, suitably renormalized for a general Kähler metric, and $t_\alpha$ is the representation on the scalars of the $\alpha$th Hermitian gauge generator, including coupling-constant factors. We will not need the formula for $m_{nm}$, which depends on the superpotential.

The squares of the fermion masses are the eigenvalues of the matrix $mm^\dagger$. The gauge-gauge elements of this matrix can be calculated from (1) and (2):

$$(mm^\dagger)_{\alpha\beta} = 2(\langle z\rangle^\dagger t_\beta t_\alpha \langle z\rangle). \tag{3}$$

In comparison, the gauge boson mass-squared matrix is

$$\mu_{\alpha\beta}{}^2 = (\langle z\rangle^\dagger \{t_\beta, t_\alpha\} \langle z\rangle). \tag{4}$$

If $u_\alpha$ is a (real) eigenvector of $\mu_{\alpha\beta}{}^2$ with eigenvalue $\mu^2$, and we define $u_n \equiv 0$, then

$$\textstyle\sum_{NM}(mm^\dagger)_{NM} u_N u_M / \sum_N u_N{}^2 = \mu^2 \tag{5}$$

the indices $N$ and $M$ running over all $\alpha$ and $n$. It follows that the lowest eigenvalue of $mm^\dagger$ must be lower than or equal to $\mu^2$. That is, the lightest fermion with the same conserved quantum numbers as a given gauge boson must be lighter than or degenerate with the gauge boson (and the heaviest, degenerate or heavier). The fermions lighter than but with the same quantum numbers as the $W^\pm$ and $Z^0$ bosons will be called $\lambda^\pm$ and $\lambda^0$. Radiative corrections will shift the $\lambda^\pm$ and $\lambda^0$ masses, but presumably not by more than a few gigaelectronvolts.

The production and decay of these fermions is governed by the multiplicative conservation of an "$R$-parity",[8] that can be taken as

$$R = (-)^{F-3B-L}, \tag{6}$$

where $F$, $B$, and $L$ are fermion, baryon, and lepton numbers. The $\lambda^\pm$ and $\lambda^0$ fermions have $B = L = 0$, like the $W^\pm$ and $Z^0$, so that they are $R$ odd, and can therefore only decay into states containing at least one other $R$-odd particle: a Higgs fermion, a scalar quark or lepton, or a "photino" or "gluino" (the fermionic counterparts of the photon and gluon). Of these, the only $R$-odd particles that are definitely expected to lie below the $\lambda^\pm$ and $\lambda^0$ are the photino and the gluino, and so the dominant decay modes of the $\lambda^\pm$ and $\lambda^0$ are into three jets (quark plus antiquark plus gluino) or with smaller branching ratio into photino plus two jets (quark plus antiquark) or photino plus lepton plus antilepton. The $\lambda$ lifetime is very short, of order $10^{-19}$ sec. In addition to the production of $\lambda^+ + \lambda^-$ pairs in $e^+ + e^-$ collisions, the $\lambda^\pm$ would be produced in the decay process $W^\pm \to \lambda^\pm + \text{photino}$, with relative rate

$$\frac{\Gamma(W^\pm \to \lambda^\pm + \text{photino})}{\Gamma(W^\pm \to \nu + e^\pm)} = 4\Phi \cos^2\varphi \sin^2\theta,$$

where $\varphi$ is the mixing angle of the $W^\pm$ gauge fermions with the $\lambda^\pm$; $\theta$ is the usual $Z^0$-$\gamma$ mixing angle; and $\Phi = (1-x)^2(1+x/2)$ is a phase-space factor, with $x = m_\lambda{}^2/m_W{}^2$. It is easy to see that for just one pair of left-chiral Higgs doublet superfields $\{H^0, H^-\}$ and $\{H^{+\prime}, H^{0\prime}\}$, the factor $\cos^2\varphi$ is given by

$$\cos^2\varphi = m_W{}^2/(m_W{}^2 + m_\lambda{}^2)$$

and $\varphi$ behaves similarly as a function of $m_\lambda/m_W$ for any number of charged chiral superfields. The $\lambda^+/e^+$ ratio in $W^+$ decay is therefore 84% for $m_\lambda \ll m_W$ (where $\Phi = \cos\varphi = 1$), and it drops to about 10% for $m_\lambda = 0.8 m_W$. Thus the $\lambda^\pm$ may be discovered in $W^\pm$ decay even before it can be produced in $e^+$-$e^-$ collisions.

I am grateful for discussions with Richard Arnowitt, Howard Georgi, Lawrence Hall, Joe Lykken, Joe Polchinski, Lenny Susskind, and Xerxes Tata. This research was supported in part by the Robert A. Welch Foundation.

[1]See, e.g., L. Alvarez-Gaumé, M. Claudson, and M. Wise, Nucl. Phys. B207, 96 (1982); G. Farrar and S. Weinberg, Rutgers University–Texas University Report No. RU-82-38 (to be published).

[2]A. H. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. 49, 970 (1982); P. Nath, R. Arnowitt, and A. P. Chamseddine, to be published; A. H. Chamseddine, P. Nath, and R. Arnowitt, to be published; R. Barbieri, S. Ferrara, and C. A. Savoy, CERN Report No. TH. 3365 (to be published); L. Ibañez, to be published; H. P. Nilles, M. Srednicki, and D. Wyler, CERN Report No. TH. 3432 (to be published); J. Ellis, D. V. Nanopoulos, and K. Tamvakis, CERN Report No. TH. 3418 (to be published); L. Hall, J. Lykken, and S. Weinberg, University of Texas Report No. UTTG-1-83 (to be published); M. Gaillard and B. Zumino, to be published. In the earlier work of B. A. Ovrut and J. Wess, Phys. Lett. 112B, 347 (1982), supersymmetry was broken in the hidden sector intrinsically rather than spontaneously, through the discard of a cosmological constant.

[3]E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen, CERN Report No. TH. 3312 (to be published); J. Bagger, to be published; Chamseddine, Arnowitt, and Nath, Ref. 2.

[4]Ellis, Nanopoulos, and Tamvakis, Ref. 2.

[5]R. Arnowitt, private communication; L. Alvarez-Gaumé, J. Polchinski, and M. B. Wise, to be published.

[6]B. W. Lee and S. Weinberg, Phys. Rev. Lett. 39, 165 (1977).

[7]B. de Wit and D. Z. Freedman, Phys. Rev. D 12, 2286 (1975).

[8]G. Farrar and P. Fayet, Phys. Lett. 76B, 575 (1978); Farrar and Weinberg, Ref. 1.

# Analysis of $e^+e^-$ Charged-Particle Inclusive Cross Section Including the Higher-Order Correction of Quantum Chromodynamics

Kiyoshi Kato
*Department of Physics, Kogakuin University, Tokyo 160, Japan*

and

Kohei Kitani
*Sagami Women's University, Kanagawa 228, Japan*

and

Tomo Munehisa
*National Laboratory for High Energy Physics, Ibaraki 305, Japan*

and

Hidehiko Okada
*Department of Physics, Tokyo Institute of Technology, Tokyo 152, Japan*

and

Yoshimitsu Shimizu
*Department of Physics, University of Tokyo, Tokyo 113, Japan*
(Received 4 October 1982)

The new data on charged-particle inclusive cross sections obtained by the TASSO and MARK II detectors in the $e^+e^-$ annihilation process are analyzed in the framework of perturbative QCD. In the leading logarithmic order there results $\Lambda = 0.9$–$1.5$ GeV for the scale parameter, but this reduces to $\Lambda = 0.4$–$0.6$ GeV if the higher-order correction in the $\overline{\mathrm{MS}}$ scheme is included. The obtained value of $\Lambda_{\overline{\mathrm{MS}}}$ is slightly larger than but still consistent with that determined from the data of deep-inelastic electron and neutrino scatterings.

PACS numbers: 12.35.Eq., 13.65.+i

Recently it has been reported that scaling violation is observed in the charged-particle inclusive cross section in $e^+e^-$ annihilations.[1,2] The experiments were done by two groups, one with the TASSO detector[1] at PETRA and the other with the MARK II detector[2] at PEP in the center-of-mass energy range $5.0 < \sqrt{s} < 35$ GeV. They measured the cross section in which all charged hadrons are detected without specifying the kind of particles—$\pi^\pm$, $K^\pm$, and so on. They found that the cross section grows for small $z$ and decreases for large $z$ as $\sqrt{s}$ increases. Here $z = 2p/\sqrt{s}$ and

 