Three-Dimensional Simulation of Spheromak Creation and Tilting Disruption

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Spheromak dynamics is studied for a zero-β plasma by a three-dimensional magnetohydrodynamics simulation code. The growth rate of the tilting instability is found to be of the order of $10\tau_A$ (Alfvén transit time) and, more interestingly, once tilt occurs, the spheromak field reconnects three dimensionally with the vertical field, thus leading to a disruptive deformation of the spheromak.

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The study of dynamic behavior of magnetically confined plasmas necessitates essentially the analysis in a three-dimensional geometry.[1-3] Self-reversion of the magnetic field in reversed-field pinches and tilting instability in spheromaks provide such examples.

With the intent of numerically studying dynamic behaviors of such plasmas in three dimensions we have developed a compressible, resistive magnetohydrodynamics (MHD) code. In this work we apply this code to the tilting instability of the Princeton spheromak,[4] whose formation was already studied for an axisymmetric (two-dimensional) geometry.[5-7]

Although the present code includes the adiabatic change of pressure, we assume in this particular simulation a zero-temperature plasma to save computer memory and time, so that we can increase the grid number and do our analysis in a system with a reasonably fine-grained mesh size.

The integration method we used is the two-step Lax-Wendroff method. As is well known, this method requires some smoothing procedure in order to suppress numerical artifacts. In our code we perform an average over four neighboring grid points for the increment of each variable at every time step. In addition, we carry out the same average procedure at every four time steps for the difference $f-f_0$ where f is a dependent variable and f_0 is its reference value updated at every 200 time steps. This average method is employed because we already proved the validity of this scheme by a two-dimensional code.[8]

The simulation model of spheromak creation is shown in Fig. 1. It consists of a cylindrical vacuum vessel, a toroidal flux core with an octagonal cross section, and one pair of vertical-field coils. The walls of the vacuum vessel are located at $r=6L$ and $z=\pm 3L$ [a right-handed cylindrical coordinate system (r,θ,z) is used]. The center position of the flux core is $(r,z)=(4L,0)$ and the width of the flux core is $6L/5$ in both r and z directions. The vertical coil is placed at $r=6L$ and $z=\pm 3.3L$. In the actual simulation L is assumed to be 0.25 m.

We assume that a long time ago a current of 220 kA was applied to the vertical-field coils in the negative θ direction, so that at present $(T=0)$ the vertical field is supposed to have fully penetrated inside the vacuum vessel. We also assume that at this time a toroidal current is flowing inside the flux core in such a way that its surface is kept to hold a constant magnetic flux ψ_{p0} ($\psi_{p0}=-0.3$ Wb and the corresponding current was roughly 350 kA). The initial axisymmetric poloidal-field configuration thus calculated is shown in the two cross sections of the simulation model (Fig. 1).

Starting from this initial condition, we supply the toroidal flux and the poloidal flux from the flux core surface in such a way that $B_T{}^c = 0.4(4L/r)\sin(\pi T/88)$ T for $T \leq 88$ and $B_T=0$ for $T>88$ and

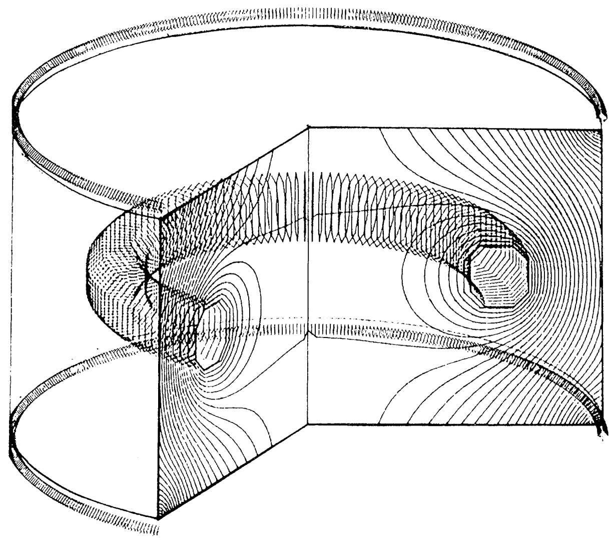

FIG. 1. Three-dimensional geometry of the simulation model and the cross-sectional view of the initial poloidal-field configuration.

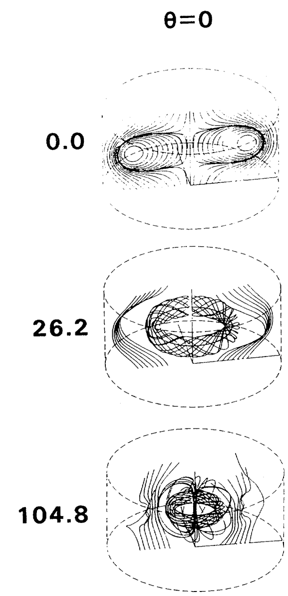

FIG. 2. Magnetic field line trajectories in formation stage of spheromak.

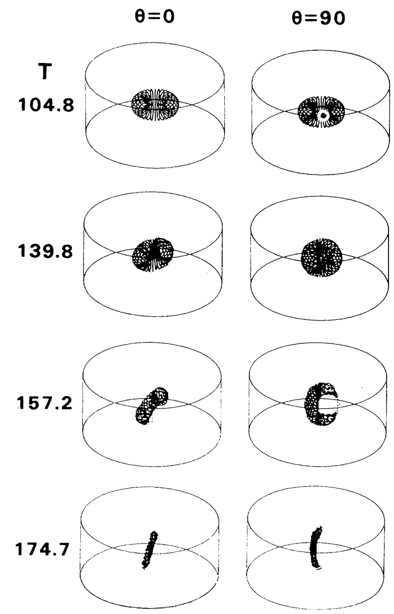

FIG. 3. A three-dimensional display of spheromak tilting and disruption. Plotted are the contours of the azimuthal component ($B_\theta = 1$ kG). The left column is the view seen from the direction of the tilt axis ($\theta = 0$) and the right one is the view from $\theta = 90°$. Note the asymmetric deformation.

$\psi_p{}^c = -0.3 + 0.5\sin(\pi T/44)$ Wb where T is normalized by the Alfvén transit time τ_A which is approximately 2.3 μsec and r is the radial distance of a core surface point. We note, however, that ψ_p is so programmed that the toroidal-flux core current is crowbarred at -100 kA once it reduces to -100 kA (the crowbar time was roughly $35\tau_A$ in our case). The wall of the vacuum vessel is assumed to be a conductor so that the perpendicular magnetic field component is fixed at the wall. The simulation domain (r, θ, z) is divided into $37 \times 16 \times 37$ meshes.

First, we show the formation stage of the spheromak. Figure 2 shows some field line trajectories at three different times. It is seen that a thin closed magnetic surface (torus) formed just inside the flux core develops into a fat torus (spheromak) with a gradual movement of the magnetic axis toward the central axis. The formation is completed at about $90\tau_A$ and the created spheromak keeps its shape thereafter. We have continued the run until $140\tau_A$ but it is found that its shape is maintained without any appreciable change. This indicates that the numerical diffusion time is much longer than $50\tau_A$.

With this success in creating a spheromak, we then go on to study the tilting instability. We have applied a small $n = 1$ tilting perturbation at $T = 105$. With intent to see the dynamic behavior of the spheromak in response to the perturbation,

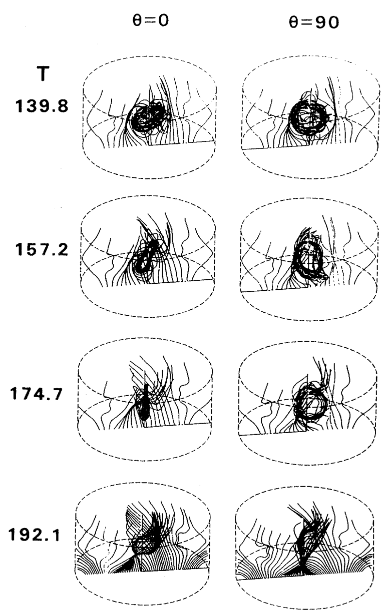

FIG. 4. Magnetic field line trajectories in tilting stage seen from $\theta = 0$ (left) and $\theta = 90°$ (right). Note reconnection of the spheromak field with the vertical field.

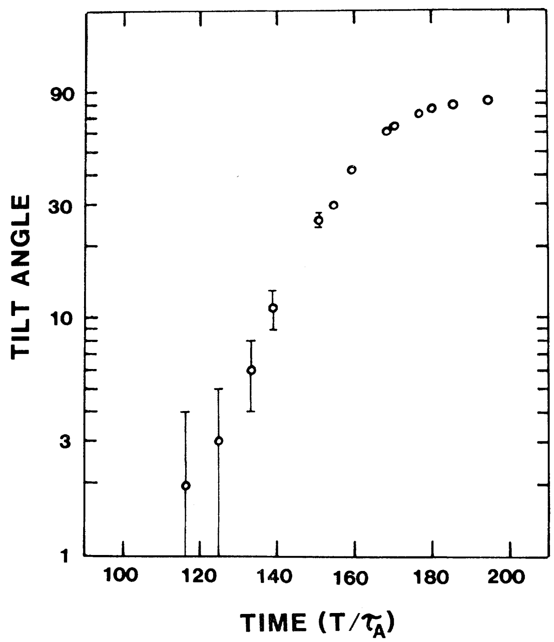

FIG. 5. Temporal evolution of the tilt angle of the spheromak.

we first show the contour plots of the azimuthal component of the magnetic field ($B_\theta = 1$ kG) in Fig. 3. The left column is the view seen from the direction of the tilt axis ($\theta = 0$) and the right column is the view from the perpendicular direction ($\theta = 90°$).

From this result, it is clearly seen that the spheromak can certainly suffer from a tilting instability. Besides, we have discovered a more interesting nonlinear process associated with tilting. Namely, as it tilts, the spheromak can no longer maintain a toroidal symmetry but gets severely distorted like a nibbled doughnut and completely disappears when it tilts about 90°. More specifically, as one side of the spheromak ring (the right side in this figure) is eroded, the other side moves toward the central axis, and the tilted spheromak turns into a thin cylinder when the erosion is completed. It then completely disappears.

Shown in Fig. 4 are some examples of field-line trajectories of the tilting spheromak. On detailed examination of the field-line trajectories, we find that the spheromak field does reconnect with the vertical field. This is interpreted as follows: As the tilt angle grows, the spheromak toroidal field generates more and more vertical components. On one side the vertical component is parallel to the original vertical field but opposite on the other side. Thus it is expected that reconnection occurs three dimensionally on the antiparallel side. Our result certainly shows that the observed erosion is associated with a three-dimensional asymmetric reconnection between the spheromak field and vertical field. Once reconnection occurs, then particles trapped by the spheromak field would rapidly escape into the wall through reconnected field lines, thus leading to disruption.

The time evolution of the tilt angle is shown in Fig. 5. It is found that the characteristic time of the tilting instability is roughly $20\tau_A$ which is in good agreement with the experimental results.[9,10] It is also interesting to note that the tilt tends to saturate at 90°. Stabilization effects of the wall and the magnetic index, as well as the finite-β effect, are being studied and will be published elsewhere.

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