Observation of a New Resonance in the Collective-Mode Spectrum of ^{3}He-A

M. W. Meisel, B. S. Shivaram, Bimal K. Sarma,[a] J. B. Ketterson, and W. P. Halperin
Department of Physics and Astronomy and Materials Research Center, Northwestern University, Evanston, Illinois 60201

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A high-resolution acoustic impedance technique has been used to investigate the collective-mode spectrum of ^{3}He-A near T_c. A new collective mode of the order parameter has been observed. The temperature at which the new mode appears is frequency, pressure, and magnetic field independent. The observed temperature dependence is very close to the predicted behavior of the super-flapping mode in the weak-coupling limit of the Anderson-Brinkman-Morel energy gap.

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It is now widely accepted that the A and B phases of superfluid ^{3}He are well described by the Anderson-Brinkman-Morel (ABM) state and the Balian-Werthamer state, respectively.[1] The existence of the A phase in zero magnetic field at high pressures is believed to be due to strong-coupling effects.[2] Magnetic fields of approximately 0.6 T are needed to stabilize the A phase at all pressures and temperatures.[3] One dramatic difference between the two superfluid phases is the presence of nodes in the energy gap of the A phase. A consequence of this unique energy gap is an anisotropic nature of this superfluid phase.

A variety of zero-sound experiments have been used to investigate the collective-mode spectra of both superfluid phases.[1,4] In particular, the anisotropic nature of zero sound in the A phase has been, theoretically[5,6] and experimentally,[7,8] established to depend upon the angle between the sound propagation direction, $\hat{q}$, and the ABM state energy-gap axis, $\hat{l}$. Three collective modes, the normal-flapping, the clapping, and the super-flapping (sfl) modes,[9] of the A-phase order parameter have been predicted to couple to zero sound[5,6] and to split in a magnetic field.[10] As a result of the nodes in the energy gap, pair breaking of the superfluid condensate occurs at all frequencies and provides a nontrivial background to the collective-mode spectrum. Near T_c, for instance, the sfl-mode frequency is nearly twice the maximum value of the energy gap and is therefore expected to be observable at frequencies exceeding 50 MHz.[5] An external magnetic field also plays an important role in determining which collective modes are excited by zero sound. For example, the free energy of the superfluid is minimized when $\hat{l}\perp\hat{H}$, except where $\hat{l}$ is normal to the walls (over a distance of a few microns).[11] Therefore, $\hat{H}\,||\,\hat{q}$ gives rise to $\hat{q}\perp\hat{l}$, and only the clapping mode is expected to be excited. For $\hat{H}\perp\hat{q}$, zero sound may couple to all three collective modes.

A number of experiments[1,7,8,12] have revealed large, broad attenuation peaks and phase velocity changes in the A phase immediately below T_c. From various orientations of $\hat{H}$ and $\hat{q}$, the existence of the clapping and the normal-flapping modes was inferred even though the experiments were unable to measure the precise temperature dependence of these excited states. In this Letter, we report the results of a high-resolution acoustic impedance investigation of ^{3}He-A. A new collective mode of the A phase has been identified.[13] Our experimental technique enables us to measure the precise temperature location of the resonance (Fig. 1). This new mode is T/T_c dependent but is not observed to have any pressure, frequency, or magnetic field dependence (Fig. 2). The T/T_c dependence is very close to the predicted behavior of the sfl mode which has been given by Wolfle[5] as

$$\nu_{\mathrm{sfl}}(T) = 2\Delta(T)\left\{1 - \frac{28}{5\pi^4}\zeta(3)\frac{h\Delta(T)}{k_B T}\right\}, \quad T \sim T_c, \tag{1}$$

where $\zeta(3)$ is the Riemann zeta function and $\Delta(T)$ is in frequency units. Equation (1) and the data are in best agreement (Fig. 2) when the weak-coupling limit of the ABM energy gap, $\Delta(T)$, is used. Following Wolfle, the maximum in the ABM energy gap has been approximated by

$$h\Delta(T) = \Delta_0 k_B T_c \tanh\{\pi/\Delta_0[\Delta C/C(T_c/T - 1)]^{1/2}\}, \tag{2}$$

where $\Delta_0 = 2.03$ and $\Delta C/C = 1.19$ in the weak-coupling limit.[14]

The details of our high-resolution acoustic impedance technique, experimental cell, and thermometry have been given elsewhere.[15,16] An important aspect of our acoustic impedance tech-

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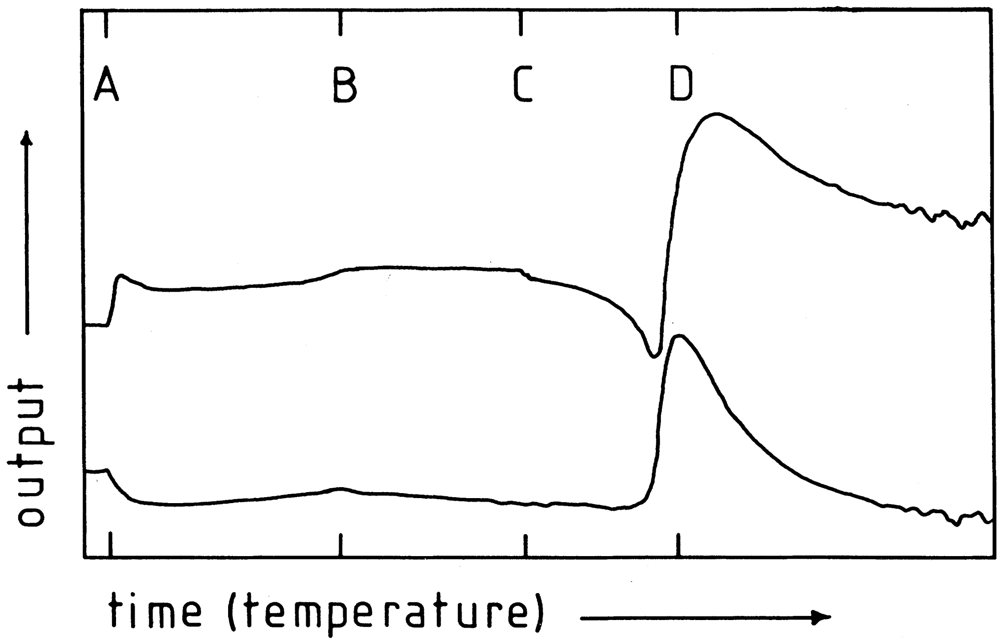

FIG. 1. The detected output as a function of time (temperature) is shown for 38.2 MHz, 1.35 bars, and $H = 0.15$ T. The upper (lower) curve represents the change in the dispersive imaginary (absorptive real) component of the acoustic impedance. Temperatures A, B, C, and D label the normal fluid to superfluid transition ($T_c = 1.242$ mK), the new A-phase mode ($T_{\nu A} = 1.166$ mK), the A- to B-phase transition ($T_{AB} = 1.102$ mK), and the B-phase squashing mode ($T_{sq} = 1.033$ mK), respectively. The time elapsed between A and D is 56 min.

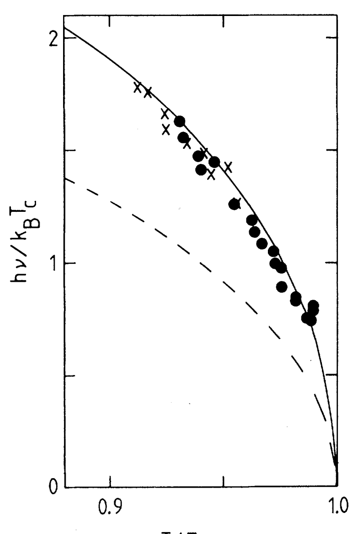

FIG. 2. The temperature dependence of the new A-phase collective mode is shown. The circles (crosses) are for data taken at 38.2 MHz, 0.94 to 18.57 bars (63.8 MHz, 5.28 to 12.40 bars). All the data shown are for $H = 0.15$ and $\hat{H} \perp \hat{q}$. The super-flapping mode in the weak-coupling limit, Eqs. (1) and (2), is given by the solid curve. The clapping mode in the weak-coupling limit is represented by the broken curve.

nique is that it is particularly sensitive to regions of high attenuation and dramatic phase velocity changes. When the zero-sound frequency crosses the collective-mode frequency, the acoustic loading of the transducer changes and gives rise to well-defined changes in the detected output (Fig. 1). Our measurements were performed at 38.2 and 63.8 MHz over a pressure range from 0 to 18 bars. Magnetic fields up to 0.2 T perpendicular to $\hat{q}$ were used to stabilize the A phase.

The observed temperature dependence of this new mode is shown in Fig. 2. For 63.8 MHz, the mode could only be seen for the pressure range $5 < P < 13$ bars. The low-pressure limit is established by our ability to stabilize the A phase while maintaining the reliability of our thermometry. The upper-pressure limit arises from the decrease in the attenuation at higher pressures[5] such that our technique is no longer sensitive enough to identify the mode above the noise background. For 38.2 MHz, the mode was observed at all pressures.[17] Because of the A-phase pair breaking background, only a semiquantitative comparison of the coupling strength of this new mode can be made by comparing it to the B-phase squashing mode.[4] At 38.2 MHz, 1.35 bars, the coupling of this new A-phase mode is ~10 times weaker at $T/T_c = 0.939$ than the B-phase squashing mode at $T/T_c = 0.832$ (Fig. 1). A complete magnetic field study at all T/T_c values is not possible since the field is required for the stabilization of the A phase; however, this new mode was observed to be magnetic field independent for $0.09\ \text{T} \leq H \leq 0.15$ T at 38.2 MHz, 7.03 bars. A decrease in the applied rf power by approximately one-half caused the signal to drop by a corresponding amount. Consequently, all of our measurements fall in the linear transport regime. Finally, no detectable changes in the output were observed in the predicted[5] vicinity of the clapping mode (Fig. 2).

A striking feature of Fig. 2 is the pressure independence of this new mode. For $0.930 < T/T_c < 0.955$, the 38.2-MHz, 0–3-bars data overlap the 63.8-MHz, 7–13 bars results. Thus the overlapping region of the two sets of data corresponds to a pressure difference of ~10 bars. However, the data follow a smooth curve independent of frequency and pressure. The temperature dependence appears to closely follow the sfl mode, Eq. (1), when the weak-coupling ABM energy gap, Eq. (2), is used. This result is significant in view of the fact that strong-coupling effects are thought to be important at (and above) the intermediate-pressure range, 7–13 bars, of this work. The low-pressure regime should be well modulated by the weak-coupling ABM gap, $\Delta_0 = 2.03$ and $\Delta C/C = 1.19$, Eq. (2). At intermediate pressures, the heat-capacity jump may be best approximated by $\Delta C/C = 1.48$.[3,18,19] When this value is used in Eqs. (1) and (2), $h\nu_{\text{sfl}}/k_B T_c$ becomes ~9% larger than the weak-coupling result (Fig. 2). This shift is approximately three times larger than the scatter in the data where the low and intermediate pressures overlap. This phenomenon is similar to the behavior of the real squashing collective mode of the B phase[4,15] and indicates that Fermi liquid corrections, which have not yet been incorporated into the A-phase theory, may significantly change Eq. (1).

Recently, Piche, Rouff, Varoquaux, and Avenel[11] have interpreted acoustic attenuation measurements in the A phase as indicating that the low-pressure maximum of the ABM energy gap, Eq. (2), might be approximately one-half the weak-coupling value. This result conflicts with the thermodynamic analysis of Feder *et al.*[3] who find the low-pressure heat-capacity jump at T_c, $\Delta C/C$, to be essentially the weak-coupling result. Our identification of the newly observed mode as the predicted sfl mode would require small, if any, Fermi liquid corrections to Eq. (1) at low pressures and corrections of ~9% at intermediate pressures. This interpretation is inconsistent with the existence of low-pressure anomalies in the ABM energy gap.

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[a]Present address: Materials Science and Technology Division, Argonne National Laboratory, Argonne, Ill. 60439.

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Critical Velocity for a Self-Sustaining Vortex Tangle in Superfluid Helium

K. W. Schwarz
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
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A recent treatment of superfluid turbulence is extended to the study of turbulence in a channel. It is found that as the flow velocity is reduced, a critical velocity is reached such that the rate at which new vortex singularities are created by line-line reconnections becomes insufficient to balance the loss of vortices at the channel walls, and the vortex tangle ceases to be topologically self-sustaining. Comparison with experiment indicates that this approach provides a reasonable explanation of observed critical velocities.

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One of the more interesting properties of superfluid ^{4}He is that it can flow without dissipation. It is well known, however, that this is true only up to some maximum flow velocity, above which the fluid undergoes a transition to a kind of turbulent state in which it becomes filled with a tangle of quantized vortex lines. Numerous elegant experiments have established the basic properties of this instability,[1-8] yet its origin has remained an enduring mystery.[9] In this paper, a recently developed method for treating superfluid turbulence[10] is used to investigate the vortex tangle in a channel, with the particular aim of finding out what happens to the tangle as the average flow velocity is reduced. This approach yields a simple qualitative explanation of the transition, and allows one to calculate its properties.

If the curve $\vec{s}=\vec{s}(\xi,t)$ specifies the instantaneous configuration of vortex-line singularities which make up the tangle, the instantaneous motion of the line with respect to the local average superfluid velocity is to a good approximation given by[11]

$$\partial\vec{s}_0/\partial t=\vec{s}_0'\times\vec{s}_0''+\alpha\vec{s}_0'\times(\vec{v}_0-\vec{s}_0'\times\vec{s}_0''), \qquad (1)$$

where $\vec{s}_0'$ is the vector tangent and $\vec{s}_0''$ the vector curvature at the point in question, $\vec{v}_0=\vec{v}_{n0}-\vec{v}_{s0}$ is the local relative velocity between the normal fluid and the superfluid, and α is the coefficient which measures the force exerted by the normal fluid on the vortex line.[12] The zero subscripts signify that Eq. (1) is given in reduced units, such that length is expressed in terms of the characteristic dimension D, velocities in terms of the corresponding Feynman velocity β/D,[13] and time in terms of D^2/β. Equation (1) breaks down on the relatively infrequent occasions when vortex lines try to cross. It is assumed[14] that whenever this occurs, the vortex singularities will undergo a topology-changing reconnection. As was pointed out previously,[10] this new idea is of central importance in developing an understanding of superfluid turbulence, since it is through this process that new vortex singularities are generated.

Equation (1) plus the reconnection *Ansatz* give a complete, although slightly idealized, prescription for calculating the time development of the vortex tangle. The combination of Eq. (1) acting on randomly curving singularities and the occa-

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