Fractional Charges, Monopoles, and Peculiar Photons

S. M. Barr, D. B. Reiss, and A. Zee
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 29 November 1982)

A grand unified model is proposed which incorporates a "peculiar" photon in addition to the familiar photon. The model is constructed to reconcile the possible observations of fractional charges and monopoles. One way to have the weak-mixing-angle prediction agree with the phenomenological value is to require the peculiar photon to couple more strongly than the regular photon.

PACS numbers: 12.10.En, 14.80.Hv, 14.80.Pb, 98.80.-k

Within recent years we have heard two experimental reports which, if confirmed, would have far-reaching implications. We refer, of course, to the observations of fractional charges by LaRue, Phillips, and Fairbanks[1] and of a magnetic monopole by Cabrera.[2] The fractional charges are in integral multiples of $q=\frac{1}{3}e$. The reported monopole appears to have precisely the magnetic charge $g=1/2e$ predicted by Dirac.[3] If both of these experiments are correct, then Dirac's quantization condition $gq=n/2$, with n an integer, is clearly contradicted. We will now suppose that both experiments are indeed correct and ask how one can reconcile them with Dirac's condition, which follows from exceedingly general considerations. One possibility[4] which immediately suggests itself is that there is, in addition to the U(1) of electromagnetism, another exact local U(1)$'$ symmetry of nature. Suppose then that the observed fractional charge also carries a U(1)$'$ charge q' and that the observed monopole has a magnetic charge under U(1)$'$ of g'. Dirac's arguments now lead to the requirement that $gq+g'q'=n/2$. This condition may then be satisfied without either gq or $g'q'$ being of the form $n/2$.

The existence of a hitherto undetected exact local U(1)$'$ symmetry and its associated photon (which we will refer to as the peculiar photon γ_P) would have exciting consequences for laboratory experiments and cosmology. Here we wish to construct a grand unified model which incorporates this phenomenon.

Considerable theoretical lore and experimental evidence favor the proposition that color is confined. Since there is no indication that the fractional charges observed by LaRue, Phillips, and Fairbank carry color[5] we will hold to the color-confinement doctrine. In particular it is easy to incorporate unconfined fractional charges into a grand unified model by extending the gauge group,[6,7] say, from SU(5) to SU(7). An attractive feature of the SU(5) model is that the correlation between quarks having three colors and their having electric charges in units of $\frac{1}{3}e$ is neatly realized. When one goes to SU(7), one pays the heavy price of losing this feature. We will describe below a model which maintains this correlation.

Monopoles occur[8,9] naturally in most grand unified models as a stunning consequence of non-Abelian gauge symmetry. Let us now set up the framework for our discussion and, at the same time, briefly remind the reader about monopoles. We start with some gauge group G which is broken down to $H=\mathrm{SU}(3)_c\otimes \mathrm{U}(1)_{\mathrm{em}}\otimes \mathrm{U}(1)'$. Outside the monopole core, the long-range gauge fields lie inside H and are described by the potential $A_\varphi=iQ(1-\cos\theta)$ in spherical coordinates in the appropriate gauge. Here Q denotes a generator of H which satisfies the constraint (Dirac quantization condition) $e^{i4\pi Q}=I$, where I is the identity of H. Monopole stability analysis[9,10] imposes additional constraints on the $\mathrm{SU}(3)_c$ components of Q.

We now have to find two U(1)'s in G with generators Q_{em} and Q_{em}' such that a Q satisfying the constraints mentioned above can be constructed as a linear combination of Q_{em}, Q_{em}', and a generator Y_c of $\mathrm{SU}(3)_c$. Furthermore, ordinary quarks and leptons are to have zero charge under Q_{em}' to avoid obvious conflicts with Coulomb's law and the Eötvös experiments. Of course, Q_{em} and Q_{em}' also have to be orthogonal to each other. Finally, the model is to contain the objects seen by Cabrera and LaRue, Phillips, and Fairbank.

This set of constraints led us to the choice $G=\mathrm{SU}(5)\otimes\mathrm{SU}(5)'$. We embed $\mathrm{SU}(3)_c\otimes\mathrm{SU}(2)$ in SU(5) in the standard manner and find [in the fundamental representation $(\underline{5},\underline{1})\oplus(\underline{1},\underline{5})$][11]

$$\begin{aligned} Y_c &= (\tfrac{2}{3},-\tfrac{1}{3},-\tfrac{1}{3},0,0;\ 0,0,0,0,0),\\ Q_{\mathrm{em}} &= (\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},0,-1;\ \tfrac{2}{3},-\tfrac{1}{3},-\tfrac{1}{3},0,0),\\ Q_{\mathrm{em}}' &= (0,0,0,0,0;\ \tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},0,-1) \end{aligned} \tag{1}$$

[we are displaying the diagonal elements of various diagonal matrices, and the semicolon separates the components in the two separate SU(5) factors]. The desired matrix Q is given by $Q = \frac{1}{2}(Y_c + Q_{em} + Q_{em}')$.

Since the coefficient of Q_{em} in the above equation for Q is $\frac{1}{2}$, the monopole has exactly the Dirac value for its magnetic charge as is observed by Cabrera.

As is evident, our discussion is not tied specifically to $SU(5)\otimes SU(5)'$. There is a whole class of models that one can construct, of the form $G = SU(5)\otimes G'$. In particular, we can simply erase the ninth entries in Eq. (1) and go to the more economical model $G = SU(5)\otimes SU(4)'$. (Incidentally, this rank-7 group is the lowest-rank group which we have found for our purposes.) However, we rather like the suggestive symmetry of the generators displayed in Eq. (1). Q_{em}' is just the Georgi-Clashow charge for SU(5)'. We see that, by taking a direct-product structure, we maintain the correlation between the number of colors and the quark charges, since the generators must be traceless in each group factor.

We assign fermions to $f(\underline{5}^*,\underline{1})\oplus f(\underline{10},\underline{1})\oplus(\underline{1},\underline{5})\oplus(\underline{1},\underline{5}^*)$, where f is the number of families. A bare mass term $m_P(\underline{1},\underline{5})\cdot(\underline{1},\underline{5}^*)$ links $(\underline{1},\underline{5})$ and $(\underline{1},\underline{5}^*)$, giving five Dirac fermions with tree-level masses m_P. The Higgs representations include whatever Higgs fields are needed to break G to H. We could choose quantum numbers appropriately so as not to have any Higgs-field coupling to fermions besides the usual $(\underline{5},\underline{1})_H$ needed to give mass to ordinary matter. One could also obviously choose not to do this. [Another alternative is to have, instead of, or in addition to, fermions in $(\underline{1},\underline{5})\oplus(\underline{1},\underline{5}^*)$, an anomaly-free set such as $(\underline{1},\underline{5}^*)\oplus(\underline{1},\underline{10})$ to allow for a reflection symmetry between the two SU(5)'s.] One might then have a massless fermion ν_P, an analog of the neutrino, with zero Q_{em} and zero Q_{em}'. This particle will decouple early in the universe and would only affect the helium abundance through its contribution to the energy density.

Let SU(5) be broken to $SU(3)\otimes SU(2)\otimes U(1)_1$ at a mass scale M and SU(5)' to $U(1)_2\otimes U(1)_3$ at a mass scale M'. Subscripts distinguish various U(1)'s. At a mass scale $\tilde{M}$ the subgroup $U(1)_1\otimes U(1)_2$ is then broken to $U(1)_Y$. Finally, at ~300 GeV, $SU(3)\otimes SU(2)\otimes U(1)_Y\otimes U(1)_3$ is broken down to H. The group $U(1)_3$ is unbroken and is identified with $U(1)_{em}'$. In principle, m_P is arbitrary, but one might want to say[12] that m_P is of the same order as M'. [In contrast to the SU(7) theories,[6] mass terms for peculiar fermions do not break $SU(2)\otimes U(1)$ and so m_P is not constrained to be below $\sim 10^2$ GeV.] For the sake of simplicity and definiteness let us take M, M', and $\tilde{M}$ all to be the same order of magnitude, $M_{GUT}\sim 10^{14}$ GeV. Thus, below M_{GUT}, we have the symmetry $SU(3)\otimes SU(2)\otimes U(1)_Y\otimes U(1)_{em}'$.

The model has a global U(1) symmetry corresponding to opposite phase rotations on the fermions $(\underline{1},\underline{5})$ and $(\underline{1},\underline{5}^*)$. We call the associated conserved quantum number "peculiarity." Ordinary quarks and leptons have peculiarity $P=0$. The object seen by LaRue, Phillips, and Fairbank is then a peculiar fermion.[13] There are, in addition, in the model, two peculiar fermions with integral Q_{em} and zero Q_{em}'.

Peculiar particles interact among themselves by grand-unified–level feeble interactions and by the two electromagnetic interactions. They interact with ordinary matter solely via ordinary electromagnetism, always conserving peculiarity. Thus, the Universe does not develop a net peculiarity.

The experimental situation on the allowable abundance of peculiar matter is not totally clear. One might naively deduce[14] a limit of the order 10^{-20} for the ratio of densities of peculiar matter and of ordinary matter n_P/n_B if one ignores various complicated effects (concentration on niobium spheres, for instance). There is a spectrographic limit for anomalous particles which is relevant only if the peculiar particles are light.[15]

In order to discuss the cosmological abundance of peculiar matter we first have to decide whether or not we subscribe to the inflationary-Universe scenario.[16] Suppose we do. Then any density of peculiar matter existing prior to inflation will have been diluted to essentially zero by the time the Universe reheats itself back to the temperature T_{rh}. Peculiar particles are regenerated in pairs by processes like $\gamma\gamma\to P\bar{P}$, $o\bar{o}\to P\bar{P}$ (o denotes ordinary particles and P, peculiar particles). But, if $m_P > T_{rh}$, then these processes are suppressed by the Boltzmann factor $\exp(-2m_P/T_{rh})$. The ratio of the present density of peculiar matter to that of photons is then essentially controlled by this factor (modulo corrections such as subsequent reheatings of the photon gas). With $2m_P/T_{rh}\sim 50-100$ the abundance of peculiar matter to ordinary matter could be made sufficiently small, say of order 10^{-20}, but this number is clearly extremely sensitive to the precise value of m_P/T_{rh}. This is very similar to the recent discussion of monopole generation.[17] (For

$m_P < T_{\rm rh}$ see below.)

In a noninflationary universe, at high temperature, $T > m_P$, peculiar matter maintains a reaction rate of $n\langle v\sigma\rangle_{\rm thermal} \sim T^3(\alpha^2/T^2)$. Comparing this with the Hubble expansion rate of $\sim g^{1/2}T^2/M_{\rm Pl}$ [$g(T)$ is the number of degrees of freedom at temperature T; $M_{\rm Pl}$ is the Planck mass] we find that, as the universe cools, peculiar matter comes into equilibrium with ordinary matter at a temperature $T_* \sim \alpha^2 M_{\rm Pl}/g^{1/2} \equiv M_* \sim 10^{14-15}$ GeV. As the universe cools below m_P the reaction rate between the two kinds of matter becomes suppressed by the Boltzmann factor and so peculiar matter eventually decouples from ordinary matter at a temperature T_D determined approximately by

$$(m_P T_D)^{3/2} \exp\left(\frac{-m_P}{T_D}\right)\frac{\alpha^2}{m_P^2} \sim \frac{g^{1/2}T_D^{\,2}}{M_{\rm Pl}}. \tag{2}$$

(We neglected the fact that there are many reaction channels.) The density of peculiar matter relative to the entropy density at that time, $(n_P/s)_D$, is very roughly of order $m_P^2/(g^{1/2}\alpha^2 T_D M_{\rm Pl})$ (if we assume $m_P > T_D$). The present value of $(n_P/n_B)_o$ is thus approximately

$$\sim \left(\frac{n_\gamma}{n_B}\right)_o\left(\frac{n_P}{s}\right)_D \gtrsim \frac{10^{10} m_P}{(g^{1/2}\alpha^2 M_{\rm Pl})} \sim \frac{10^{-6} m_P}{(1\,{\rm GeV})}.$$

We have used the fact that T_D/m_P as determined by Eq. (2) is a very slowly increasing function of m_P. (For $m_P \sim 10^2$ GeV, $T_D/m_P \sim \frac{1}{3}$.) (Peculiar photons maintain thermal contact by interacting with peculiar matter only. We will suggest later that the peculiar electromagnetic coupling is stronger than ordinary electromagnetic coupling. Thus, after decoupling from ordinary matter, peculiar matter and peculiar photons continue to interconvert until reaching a lower decoupling temperature T_D'. The present density of peculiar matter is thus suppressed somewhat further.) We conclude that, without inflation, if $m_P > M_* > 10^{14-15}$ GeV peculiar matter would never have been in equilibrium[18] and thus we could not calculate its abundance. For m_P much below M_*, we see that even for m_P as low as order 10^2 GeV n_P/n_B is too large. (Goldberg[13] has, however, argued that subsequent concentration and annihilation of P and $\bar{P}$ in massive stars may reduce the abundance to acceptable levels.) We favor m_P to be large, of grand unified scale.

After decoupling from ordinary matter, peculiar particles drift in the Universe. Inside galaxies they attain a typical virial velocity $v \sim 10^{-3}$. To estimate the stopping distance of these fractionally charged particles in terrestrial matter we use Bohr's ionization energy-loss formula and obtain a stopping length roughly of the order $\sim (10\ {\rm km})[m_P/(10^{15}\ {\rm GeV})]$. We expect that most of the negatively charged peculiar particles, after coming to rest, would attach themselves to atomic nuclei by electrostatic attraction. For $m_P \lesssim 10^{18}$ GeV we find that the Earth's gravitational pull on this peculiar ion is overwhelmed by the electrostatic binding force in matter, which we take to be roughly of order $\sim e^2/r^2$ where r is the average interatomic separation. Thus, at least some peculiar particles are expected to stay near the Earth's surface.

With the symmetry-breaking scales M, M', and M_1, and the mass of peculiar fermions, m_P, chosen to be all of the order 10^{14-15} GeV, the running of the couplings below this scale obeys the same equations as in the standard analysis.[19] However, the starting value of $\sin^2\theta$ is slightly different. We find that at low energies, μ,

$$\sin^2\theta \simeq \frac{1 + [\frac{10}{3} + (2 - \frac{4}{11}f)(g/g')_{\rm GUT}^2][\alpha(\mu)/\alpha_s(\mu)]}{6 + (3 - \frac{4}{11}f)(g/g')_{\rm GUT}^2}. \tag{3}$$

The factor α/α_s denotes the ratio of electromagnetic coupling to strong coupling at low energies. [Here g and g' denote coupling at the grand unification scale normalized by specifying the couplings of the normalized gauge fields W_3, B, C, and D to their respective generators:

$$gW_3T_3 + g_1BY/2 + (g'/\sqrt{2})CQ_{\rm em}' + g'DY_c'/2.$$

$Y/2$ and $Y_c'/2$ are the generators of the group $U(1)_1$ and $U(1)_2$ mentioned earlier.]

Since the coupling for $U(1)_{\rm em}'$ does not run below $M_{\rm GUT}$ in our particular version of the model, $g'/\sqrt{2} = e'$ measures the coupling of the peculiar photon at low energies. It is easily checked that the standard neutral-current phenomenology is maintained, in accordance with general theorems.[20]

The standard value for $\sin^2\theta$ corresponds to setting $(g/g')^2 = 0$. The present experimental value[21] of $\sin^2\theta_W$ is 0.215 ± 0.014. The deviation of $\sin^2\theta_W$ from the standard value can be made less than the experimental error bar if $(g/g')_{\rm GUT}^2 \lesssim \frac{1}{3}$. The unification scale M and the coupling $\alpha_{\rm GUT}$ are modified only very slightly.

Note added.—Our interest in this subject was inspired by a talk given by J. Preskill at the Wingspread Conference[22] and also by a private communication from him. We understand that there is a related work by H. Georgi and J. Preskill. After this work was completed we received a paper by J. Pantalone on the same subject. We understand that there is also a paper by S. Aoyama, Y. Fujimoti, and Z.-y. Zhao (which we have not seen).

[1]G. S. LaRue, J. D. Phillips, and W. M. Fairbank, Phys. Rev. Lett. 46, 967 (1981).

[2]B. Cabrera, Phys. Rev. Lett. 48, 1378 (1982).

[3]P. A. M. Dirac, Proc. Roy. Soc. London, Ser. A 133, 60 (1931).

[4]See, for example, A. Strominger, to be published.

[5]See, for example, A. Zee, Phys. Lett. 84B, 91 (1979).

[6]H. Goldberg, T. Kephart, and R. Vaugh, Phys. Rev. Lett. 47, 1429 (1981); L. F. Li and F. Wilczek, Phys. Lett. 107B, 64 (1981).

[7]For earlier non-GUT discussions of fractional charges, see F. Wilczek and A. Zee, Phys. Rev. D 16, 860 (1977); G. Karl, Phys. Rev. D 14, 2374 (1976); C. B. Dover, T. K. Gaisser, and G. Steigman, Phys. Rev. Lett. 42, 1117 (1970); A. Zee, Ref. 5.

[8]G. 't Hooft, Nucl. Phys. B79, 276 (1974); A. M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. 20, 430 (1974) [JETP Lett. 20, 194 (1974)]; see also C. Dokos and T. Tomaras, Phys. Rev. D 21, 2940 (1980).

[9]For a masterful review see S. Coleman, in Proceedings of the "Ettore Majorana" Summer School, 1981, Harvard University Report No. HUTP-82/A032 (to be published).

[10]R. Brandt and F. Neri, Nucl. Phys. B161, 253 (1979).

[11]We note that there are other options even within our SU(5) $\otimes$ SU(5)' model, such as interchanging the last five entries between Q_{em} and Q_{em}'.

[12]H. Georgi, Nucl. Phys. B156, 126 (1979).

[13]H. Goldberg, Phys. Rev. Lett. 48, 1518 (1982).

[14]It is not necessary that the lightest peculiar particle is a fermion. It is possible that it is a scalar. We only require that it have electric charge $e/3$.

[15]R. A. Muller, L. W. Alvarez, W. Holley, and E. J. Stephenson, Science 196, 521 (1977); T. Alvager and R. Naumann, Phys. Lett. 24B, 647 (1967).

[16]A. Guth, Phys. Rev. D 23, 347 (1981).

[17]M. S. Turner, Enrico Fermi Institute Report No. 82-12 (to be published).

[18]D. Toussaint, S. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D 19, 1036 (1976).

[19]H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).

[20]H. Fritzsch and P. Minkowski, Nucl. Phys. B103, 61 (1976); J. C. Pati, S. Rajpoot, and A. Salam, Phys. Rev. D 17, 131 (1978); H. Georgi and S. Weinberg, Phys. Rev. D 17, 275 (1978); J. Kim and A. Zee, Phys. Rev. D 21, 1939 (1980).

[21]W. J. Marciano and G. Senjanović, Phys. Rev. D 25, 3092 (1982).

[22]J. Preskill, Harvard University Report No. HUTP-82/A059 (to be published).