Electron-Cyclotron Maser Instability Driven by a Loss-Cone Distribution

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(Received 2 August 1982)

It is shown that the electron-cyclotron maser instabilities may readily be excited in a plasma with a loss-cone distribution when the electron temperature exceeds a few tens of kiloelectronvolts. The growth rate is typically a few percent of the electron-cyclotron frequency. The appearance of the instability can be avoided by proper control of the plasma density.

PACS numbers: 52.35.Hr, 52.35.Py, 52.60.+h

The cyclotron maser mechanism, discovered in the late 1950's,[1-3] results from the relativistic mass dependence of the cyclotron frequency of the electrons. This relativistic effect leads to phase bunching in the electron-cyclotron orbit.[4] It becomes noticeable as the electrons reach an energy as low as a few kiloelectronvolts. This was demonstrated in an early experiment of Hirshfield and Wachtel.[5] These pioneering studies[1-3,5] heralded the recent development of gyrotrons[6-8] (also known as the electron-cyclotron masers), which employ electron beams with beam energy ranging from 20 to about 70 keV. As the beam energy is efficiently converted to radiation, gyrotron devices are able to provide radiation at millimeter wavelengths with unprecedented power levels. Such a source is currently exploited in plasma heating experiments.[9]

The fact that electrons with rotational energy at tens of kiloelectronvolts may radiate collectively suggests that a magnetized non-Maxwellian plasma with a high electron temperature would be subject to the cyclotron maser instabilities. Of particular interest is the case where the electrons assume a loss-cone distribution function, as in a magnetic mirror. If the emission of the radiation is efficient, like in the gyrotron, the energy loss in radiation may set a severe limit on the electron temperature. Furthermore, in the cyclotron maser mechanism, only the perpendicular energy of the electrons is converted to radiation,[6-8,10-12] the parallel energy being essentially unaffected. (Here perpendicular and parallel are with reference to the dc magnetic field.) As the perpendicular energy of the electrons is reduced, the loss cone effectively widens, leading to the loss of hot electrons. It is thus important to consider the condition under which the cyclotron maser instability would be excited.

In this paper, we provide a linear stability analysis of the cyclotron maser instability in a high-temperature plasma with a loss-cone distribution. Our analysis based on unsophisticated models shows that the cyclotron maser instability may easily be excited over a wide range of parameters. The growth rate is significant, being typically on the order of a few percent of the electron-cyclotron frequency.

Instabilities driven by a loss-cone distribution have been studied extensively in the past,[13-26] and are usually characterized by the electrostatic or electromagnetic nature of the wave. The cyclotron maser instability to be examined in the present paper is electromagnetic in nature, but is qualitatively different from the whistler instability[23-29] commonly examined for the loss-cone distribution. The cyclotron maser and whistler instabilities involve, respectively, the fast-wave (phase velocity $>c$) and slow-wave (phase velocity $<c$) branches of the right-hand-circularly-polarized wave. The former is driven by a relativistic bunching mechanism, while the latter by a nonrelativistic bunching mechanism. It was shown[11] that the two bunching mechanisms are always simultaneously present with one offsetting the other. Thus, a nonrelativistic formalism will tend to overestimate the growth rate of the whistler mode while completely missing the cyclotron maser instability.

The cyclotron maser instability occurs at the cyclotron frequency or its harmonics. Other possible mechanisms of cyclotron emission from a loss-cone plasma have been studied. Blanken, Stix, and Kuckes[18] examined the relativistic cyclotron instability for the electrostatic Bernstein mode. Bekefi, Hirshfield, and Brown[30] examined the electromagnetic cyclotron emission employing a single-particle model. Davidson[31] reported on the density threshold for the onset of electrostatic loss-cone modes. These papers consider cyclotron emission across the magnetic field rather than along the magnetic field as in the present case.

To study the excitation of the right-hand-circularly-polarized wave [varying as $\exp(ik_z z - i\omega t)$] by mildly relativistic electrons, we assume in this paper that the wave frequency is close to the cyclotron frequency of the electrons. At such high frequencies, the ions are considered to be inactive dynamically. Following previous literature, we model the electron distribution function to be

$$f_0(p_\perp, p_z) = A(p_\perp)^{2j} \exp(-p^2/\Delta p^2), \tag{1}$$

where A is a normalization constant such that $\int d^3p f_0 = 1$ and $j > 0$ is the loss-cone index. In Eq. (1), $p_\perp$ and p_z are respectively the momentum of the electron perpendicular and parallel to the external magnetic field, $p^2 = p_\perp^2 + p_z^2$; Δp is a measure of the electron temperature T, where $T \equiv mc^2[1+(\Delta p)^2/m^2c^2]^{1/2} - mc^2$. Here, m is the electron rest mass and c is the speed of light. The electron distribution function peaks at $p_\perp = j^{1/2}\Delta p$. Hence, when $j \neq 0$, there is a population inversion in $p_\perp$. This is an essential requirement for the cyclotron maser instability.

The linearized relativistic Vlasov equation and the Maxwell equations lead to the following dispersion relationship (see, for example, Ref. 11):

$$\omega^2 - k_z^2c^2 = 2\pi\omega_{pe}^2 \int_0^\infty p_\perp dp_\perp \int_{-\infty}^\infty dp_z \frac{f_0}{\gamma}\left[\frac{\omega - k_z p_z/\gamma m}{\omega - k_z p_z/\gamma m - \Omega_e/\gamma} - \frac{p_\perp^2(\omega^2 - k_z^2c^2)}{2\gamma^2 m^2c^2(\omega - k_z p_z/\gamma m - \Omega_e/\gamma)^2}\right]. \tag{2}$$

In Eq. (2), ω_{pe} is the nonrelativistic electron plasma frequency, Ω_e is the rest-mass electron-cyclotron frequency, and $\gamma \equiv (1 + p_\perp^2/m^2c^2 + p_z^2/m^2c^2)^{1/2}$ is the relativistic mass factor. Shown in Fig. 1 is the normalized frequency $\omega/\Omega_e = (\omega_r + i\omega_i)/\Omega_e$ as a function of k_z, for the case $T = 50$ keV, $\omega_{pe}/\Omega_e = 0.5$, and $j = 1, 2$. We note that the growth rate reaches a maximum at $k_z = 0$ and remains significant even if the axial wave number is a substantial fraction of the free-space wave number ω/c. Figure 2 shows the normalized growth rate ω_i/Ω_e as a function of the electron temperature T for $k_z = 0$, $\omega_p/\Omega_e = 0.5$, and $j = 1, 2$. One observes here that the cyclotron maser instability has a moderately high threshold temperature, depending on the details of the distribution function. In Fig. 3, we show the range of plasma density over which the cyclotron maser instability would be excited, for three values of electron temperature, again setting $k_z = 0$ and $j = 1, 2$. It is interesting to note that, in the range of electron temperature surveyed, the cyclotron maser instability takes place over a limited range of plasma density. As one lowers or increases the plasma density such that $\omega_{pe} \lesssim 0.2\Omega_e$ or $\omega_{pe} \gtrsim 1.2\Omega_e$, respectively, the plasma becomes stable. This may provide a viable means for avoiding the cyclotron maser instability in high-temperature mirror devices. Figure 4 shows that the growth rates remain significant when the loss-cone index j is as low as 0.5.

To examine further the properties of the instability, we have used the Briggs-Bers criterion[32] to determine that the instability is absolute. In the presence of axial variations of the magnetic field, we find that unstable, nonlocal eigenmodes which satisfy the radiation boundary conditions[33]

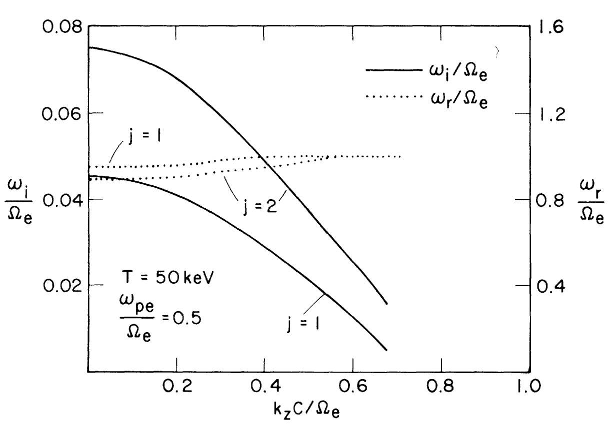

FIG. 1. The real and imaginary parts of the normalized (complex) frequency ω/Ω_e as a function of $k_z c/\Omega_e$ for $T = 50$ keV, $\omega_{pe}/\Omega_e = 0.5$, and $j = 1, 2$.

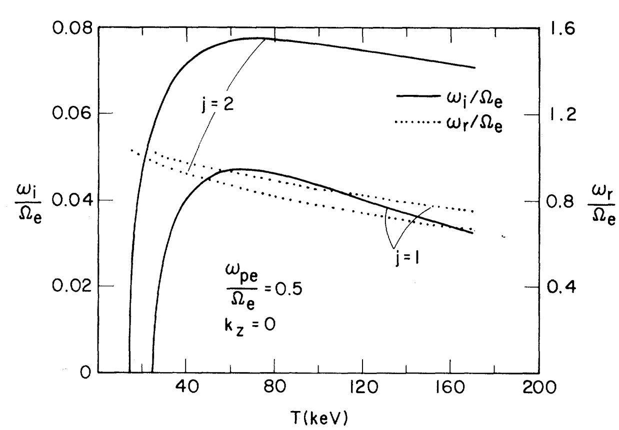

FIG. 2. The normalized complex frequency as a function of the electron temperature T for $k_z = 0$, $\omega_{pe}/\Omega_e = 0.5$, and $j = 1, 2$.

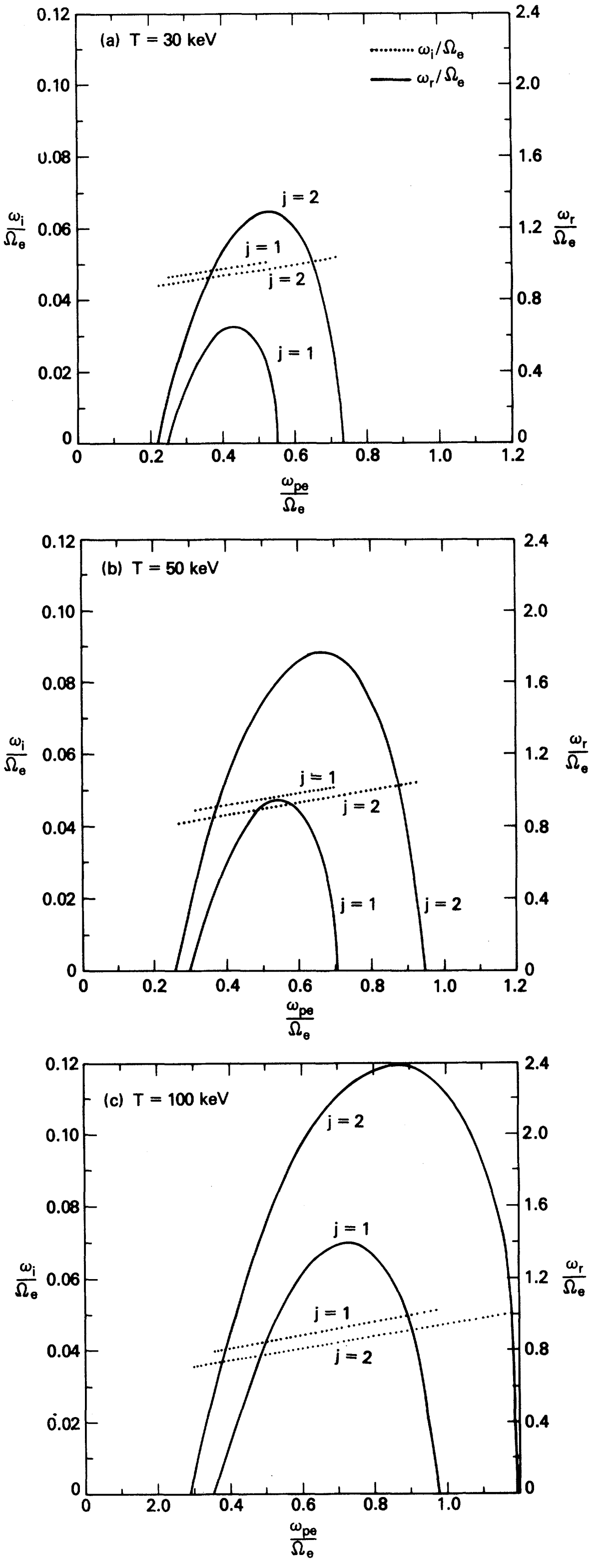

FIG. 3. The normalized complex frequency as a function of the plasma density (ω_{pe}/Ω_e) for $k_z = 0$, and $j = 1, 2$ at various electron temperatures: (a) $T = 30$ keV, (b) $T = 50$ keV, and (c) $T = 100$ keV.

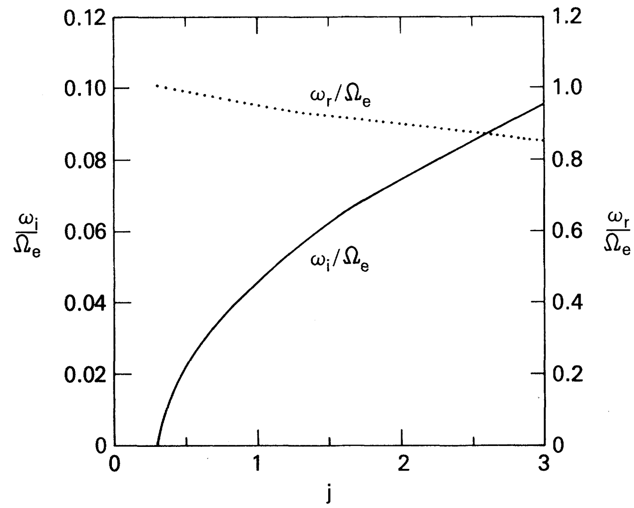

FIG. 4. The normalized complex frequency as a function of the loss-cone index j for $k_z = 0$, $T = 50$ keV, and $\omega_{pe}/\Omega_e = 0.5$.

are admissible solutions.

In summary, the electromagnetic cyclotron maser instability is found to occur over a wide range of parameters of a model loss-cone distribution function [Eq. (1)]. The relatively large growth rate and the electromagnetic nature of the instability are potentially dangerous in that they may make a hot loss-cone plasma behave as an effective "radiator" similar to the gyrotron. A strong burst of cyclotron emission has been observed in a recent mirror experiment[34] characterized by 100-keV electron temperature. While the observed radiation was attributed to the whistler instability, the cyclotron maser mechanism could not be ruled out in light of its lenient onset conditions. The instability is absent, however, in the low-density ($\omega_{pe} \lesssim 0.2\Omega_e$) or high-density ($\omega_{pe} \gtrsim 1.2\Omega_e$) regimes. Hence, the planned MFTF-B mirror experiment at Lawrence Livermore Laboratory ($T_e \simeq 100$ keV, $n_e \simeq 5 \times 10^{12}$ cm^{-3}, $B = 20-60$ kG) could be either stable (if $B > 35$ kG) or unstable ($B < 35$ kG) on the basis of the present model. A refined distribution function which realistically models the heated plasma may be needed for a more definitive prediction.

The authors are grateful to Dr. T. M. Antonsen, Dr. B. H. Hui, and Dr. W. M. Manheimer for helpful discussions. The work was supported by the U. S. Office of Naval Research.

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