Masses of Superpartners of Quarks, Leptons, and Gauge Mesons in Supergravity Grand Unified Theories

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(Received 6 December 1982)

A class of "realistic" supergravity grand-unified-theory models possessing two W-inos and two Z-inos which lie above and below $W^{\pm}$ and Z^0 mass, respectively, are examined. A lower bound on the W-ino mass of $\simeq 30$ GeV is obtained with an upper bound of $\simeq 250$ GeV on the Higgs mass. It is shown that the Z^0 decays prominantly into two W-inos if the mass of the W-ino $\leq M_W/2$. The superpartners of quarks and leptons are shown to be approximately degenerate and in some models have mass $\lesssim 100$ GeV.

PACS numbers: 11.30.Pb, 04.50.+h, 12.10.En, 14.80.Pb

Recently, Weinberg[1] has pointed out that supersymmetric grand unified models based on supergravity[2-4] imply the existence of charged fermions $\tilde{W}^{\pm}$ *lighter* than the $W^{\pm}$ gauge mesons of $SU(2)\otimes U(1)$, and a neutral fermion $\tilde{Z}$ lighter than the Z^0 gauge meson. Weinberg also calculated the branching ratio $\Gamma(W^{\pm}\to\tilde{W}^{\pm}+\tilde{\gamma})/\Gamma(W^{\pm}\to e^{\pm}+\nu)$ where $\tilde{\gamma}$, the photino, is the superpartner of the photon, and finds that it can be as large as 80%. These results are of particular importance since they are essentially model independent and represent predictions of supergravity grand unified theories (GUT's) which can be tested at currently existing high-energy accelerators. In this note we consider the general low-energy supergravity GUT fermion and boson mass matrices within the framework of a class of models based on the gauge group ($N=1$ supergravity)$\times$SU(5). These models are characterized by the superpotential $g(Z_A)=g_1(Z_a)+g_2(Z)$ where

$$g_1(Z_a)=\lambda_1(\tfrac{1}{3}\mathrm{Tr}\Sigma^3+\tfrac{1}{2}M\,\mathrm{Tr}\Sigma^2)+\lambda_2H_x'(\Sigma_y{}^x+3M'\delta_y{}^x)H^y+\lambda_3UH_x'H^x+\epsilon_{uvwxy}H^uM^{vw}f_1M^{xy}+H_x'M^{xy}f_2M_y'. \quad (1)$$

In Eq. (1), $\Sigma_y{}^x$, H^x, H_x', and U are chiral superfields in the $\underline{24}$, $\underline{5}$, $\underline{5}^*$, and singlet representations and Z is the super Higgs singlet. M^{xy} and M_y' are the $\underline{10}$ and $\underline{5}^*$ quark-lepton superfields and f_1 and f_2 are matrices in generation space. $M\sim 10^{16}$ GeV is the GUT mass. In Ref. 2, the super Higgs potential was chosen to have the Polony form[5] $g_2=m^2(Z+B)$ where $m\sim 10^{10}$ GeV was the intermediate mass and B a constant adjusted to make the cosmological constant vanish. Here we generalize these results to an *arbitrary* super Higgs potential of the form

$$g_2(Z)=(m^2/\kappa)f_2(\kappa Z);\quad \kappa=(8\pi)^{1/2}G^{1/2}, \quad (2)$$

where G is the gravitational constant [$\kappa=4.1\times10^{-19}$ (GeV)$^{-1}$]. The model of Ref. 2 was a "realistic" one in that it accounts for all the usual GUT predictions. Thus the Σ interactions break SU(5) to $SU(3)\otimes SU(2)\otimes U(1)$ at the GUT mass. The super Higgs potential spontaneously breaks supersymmetry and *simultaneously* leads to the breaking of $SU(2)\otimes U(1)$ to $U_\gamma(1)$ at the mass

$$m_s\equiv\kappa m^2\sim 100\ \text{GeV}, \quad (3)$$

and the tree-level gauge hierarchy is automatically achieved without dialing any masses.[6]

We assume here only that the super Higgs potential of Eq. (2) brings about a similar breakdown of supersymmetry and $SU(2)\otimes U(1)$. (We normalize f_2 so that to lowest order $f_2=\sqrt{2}$ at the minimum of the effective potential, as in the Polony example.) The conditions governing the spontaneous breaking for the vacuum expectation values (VEV's) of $H^5=H_5'=m_sy/(\sqrt{2}\lambda_3)$ and $U=-m_sx/\sqrt{2}\lambda_3$ are easily generalized from Ref. 2 and read

$$\begin{aligned}&y^2+x^2+\xi_1{}^2x+\xi_2{}^2=0,\quad \xi_2\equiv 1-3\lambda,\\ &\xi_1y^2+2xy^2+x=0,\quad \xi_1=(\tfrac{3}{2})^{1/2}\kappa\langle Z\rangle-6\lambda,\end{aligned} \quad (4)$$

where $\lambda\equiv\lambda_2/\lambda_1$, and $\langle Z\rangle$ is the VEV of Z determined from g_2 alone; for the Polony form, $\kappa\langle Z\rangle=\sqrt{6}-\sqrt{2}$. Equations (4), however, have solutions leading to minima of the effective potential for a wide range of values of $\langle Z\rangle$ and λ, and we leave these parameters arbitrary.

The Fermi and Bose mass matrices for the minimal theory were given in Ref. 4. In the gauge where the Goldstino has been absorbed by the gravitino so that it becomes massive, the Fermi mass matrices read

$$L^{\text{mass}} = -\tfrac{1}{2}\bar{\chi}^a m_{ab}\chi^b - \bar{\lambda}^\alpha m_a{}^\alpha \chi^b + \text{H.c.}, \tag{5}$$

where

$$m_{ab} = \exp(\tfrac{1}{4}\kappa^2)Z_c{}^2[g_{,ab} + (\tfrac{1}{6}\kappa^2)(Z_a g_{,b} + Z_b g_{,a}) + (\tfrac{1}{12}\kappa^4)Z_a Z_b g - \tfrac{2}{3}g_{,a}g_{,b}g^{-1}], \tag{6}$$

$$m_a{}^\alpha = i(\tfrac{1}{2}e_\alpha)Z_b{}^+(T^\alpha)_a{}^b;\quad g_{,a} \equiv \partial g/\partial Z_a, \text{ etc.} \tag{7}$$

T^α are the gauge group generators and e_α the gauge charge. In Eq. (5), $\{\chi^a\}$ are the set of left-handed fermions of the chiral superfields, $\{Z_a\}$ the VEV's of their scalar superpartners, and λ^α are the Majorana gauge fermion fields (in the adjoint representation).

We are interested here only in the low-energy SU(2)⊗U(1) sector. As discussed in Ref. 2, the superheavy fields $\Sigma_y{}^x$ can be eliminated in a power series in κ. If we keep only the leading terms of $O(m_s)$ [corrections are of order m_s/M, $\kappa M \kappa m_s$, and $(\kappa m_s)^2$ and hence quite small], Eq. (6) reduces to

$$m_{ab} = E g_{,ab};\quad E \equiv \exp\tfrac{1}{4}\kappa^2\langle Z\rangle^2, \tag{8}$$

where now in Eqs. (7) and (8), the indices a, b run only over the fields H^α, $H_\alpha{}'$, $\alpha = 4,5$, and U. T^α are now the SU(2)× U(1) matrices and $\lambda^\alpha = \{\lambda^i, i = 1,2,3; \lambda^0\}$ are the SU(2) and U(1) gauge fermions.

One may now diagonalize the Fermi mass matrix. There are nine low-energy Fermi fields (four Higgs fields $H_\alpha{}'$ and H^α, the "sliding singlet" field U, and four gauge fermions). Two combinations of the Higgs fields and U decouple from the gauge fermions, while the photino, $\lambda^\gamma \equiv \cos\theta_W\lambda^0 + \sin\theta_W\lambda^3$, remains massless. The remaining six fields combine to form two (charged) Dirac fields $\psi_{(+)}$ and $\psi_{(-)}$ with masses $\tilde{m}_+$ and $\tilde{m}_-$, one lying above and one lying below the $W^\pm$ mesons (i.e., two W-inos labeled $\tilde{W}_{(\pm)}$) and two Majorana fields $\chi_{(\pm)}$ with masses $\tilde{\mu}_{(\pm)}$, one above and one below the Z^0 meson (i.e., two neutral Z-inos labeled $\tilde{Z}_{(\pm)}$).[7] The W-ino masses are given by

$$\tilde{m}_{(\pm)} = (M_W{}^2 + m_1{}^2)^{1/2} \pm m_1, \tag{9}$$

where M_W is the W mass. The corresponding Dirac fields are

$$\psi_{(\pm)} = f_\pm \chi_+ \mp f_\mp \lambda_+;\quad f_\pm \equiv [m_\pm/(m_+ + m_-)]^{1/2}, \tag{10}$$

where $\sqrt{2}\lambda_+ = (\lambda^1 + i\lambda^2)$ and $\chi_+ = i(H_4{}' + H^{4c})$. (Here $H_\alpha{}'$ stands for the left-handed Weyl Higgsino partner of the $\underline{5}^*$ of Higgs scalars and $H^{\alpha c}$ is the charge conjugate of the Weyl Higgsino in the $\underline{5}$ representation.) The Z-ino masses $\tilde{\mu}_{(\pm)}$ obey Eq. (9) with M_W replaced by the Z^0 mass M_Z. m_1 is given by

$$m_1 = \tfrac{1}{2}m_g(3\lambda - x);\quad m_g = E(m_s/\sqrt{2}), \tag{11}$$

where m_g is the gravitino mass, $\lambda = \lambda_2/\lambda_1$, and x is the solution of Eq. (4).

Equation (10) shows that there is mixing between the gauge fermions and Higgsinos, the amount being determined by m_1. This parameter measures the amount of supersymmetry breaking and hence governs the splitting between the W and W-ino masses (and Z^0 and Z-ino masses). Elimination of m_1 leads to the relations

$$\tilde{m}_{(+)}\tilde{m}_{(-)} = M_W{}^2,\quad \tilde{\mu}_{(+)}\tilde{\mu}_{(-)} = M_Z{}^2, \tag{12}$$

$$\tilde{m}_{(+)} - \tilde{m}_{(-)} = \tilde{\mu}_{(+)} - \tilde{\mu}_{(-)}. \tag{13}$$

Decay of Z^0 ($\tilde{m}_{(-)} < M_Z/2$).—To determine more information requires the fixing of the model more precisely, e.g., choosing $\lambda \equiv \lambda_1/\lambda_2$ and $\langle Z\rangle$. Before doing this, we first consider the class of models where $\tilde{m}_{(-)} < M_Z/2$. This situation is of particular interest in that it allows the Z^0 to decay into two $\tilde{W}_{(-)}$ fermions, and we now calculate this decay rate.

The SU(2) ⊗ U(1) gauge mesons, $V_\mu{}^i$ and B_μ, couple to the Majorana gauge fermions λ^i and the Higgsino Weyl fermions H^α, $H_\alpha{}'$ according to

$$L = -\tfrac{1}{2}g\epsilon_{ijk}\bar{\lambda}^i\gamma^\mu V_\mu{}^j\lambda^k + (\tfrac{1}{2}i)gV_\mu{}^i[\bar{H}^\alpha\gamma^\mu\tau^i H^\alpha - \bar{H}_\alpha{}'\gamma^\mu\tau^i H_\alpha{}'] + (\tfrac{1}{2}i)g'B_\mu[\bar{H}^\alpha\gamma^\mu H^\alpha - \bar{H}_\alpha{}'\gamma^\mu H_\alpha{}']. \tag{14}$$

When we reexpress L in terms of the physical W^μ, Z^μ, $\psi_{(\pm)}$, etc. fields, the part of the interaction coupling the Z^0 to the $\tilde{W}_{(-)}$ fermion reads

$$L_{Z\text{-}\tilde{W}} = ie\cot\theta_W[1 - (\tfrac{1}{2}f_-{}^2)(\cos\theta_W)^{-2}]\bar{\psi}_{(-)}\gamma^\mu\psi_{(-)}. \tag{15}$$

Equation (15) implies a branching ratio R for $\Gamma(Z^0 \to \tilde{W}_{(-)}{}^+\tilde{W}_{(-)}{}^-)/\Gamma(Z^0 \to e^+e^-)$ of

$$R = 4(\cos\theta_W)^4 \left[1 - \frac{1}{2}\left(\frac{f_-}{\cos\theta_W}\right)^2\right]^2 \left(1 - 4\frac{\tilde{m}_{(-)}{}^2}{M_Z{}^2}\right)^{1/2} \left(1 + 2\frac{\tilde{m}_{(-)}{}^2}{M_Z{}^2}\right). \qquad (16)$$

Equation (16) shows that R is quite large. Thus for a $\tilde{W}$ mass $\tilde{m}_{(-)} = 30$ GeV, one finds $R = 1.9$, and for $\tilde{m}_{(-)} = 40$ GeV one has $R = 0.9$. Thus if the $\tilde{W}$ mass is sufficiently small that the Z^0 decay into two W-inos is energetically possible, this decay mode will be important, and in fact the dominant one in supergravity GUT models for $\tilde{m}_{(-)} < M_W/2$.

s-quark, s-lepton, and Higgs masses.—Associated with the quarks and leptons are their Bose superpartners, the s-quarks and s-leptons. The Bose mass matrices[4] for the low-mass particles may be diagonalized. Each boson member of a chiral multiplet gains a model-independent mass equal to the gravitino mass m_g, plus additional model-dependent contributions due to the Yukawa interactions, etc. For the case of the s-quarks and s-leptons, the Yukawa interactions scale by the corresponding quark and lepton masses and hence are quite small.[8] Thus to first approximation, all the s-quarks and s-leptons are degenerate with mass equal to m_g (and in higher approximation with mass splitting proportional to the quark and lepton masses).

The H^α, H_α', and U multiplets have five complex and hence ten Hermitian scalar fields in the low-mass sector. Upon diagonalizing their mass matrices, one finds three massless Goldstone particles (which are absorbed by the $W^\pm$ and Z^0 to grow their masses), three degenerate states of mass m_H (formed from components of H_α' and H^α) where

$$m_H = m_g\sqrt{2}\,[1 + (3\lambda - x)^2]^{1/2}, \qquad (17)$$

and four additional states which mix components of H^5, H_5', and U. From Eqs. (9), (11), and (17) one may derive a sum rule relating the gravitino, Higgs, and gauge sectors:

$$m_g{}^2 + (M_W{}^2 - \tilde{m}_{(-)}{}^2)^2/\tilde{m}_{(-)}{}^2 = \tfrac{1}{2}m_H{}^2. \qquad (18)$$

Equation (18) implies two bounds: $m_g \leq m_H/\sqrt{2}$ (the upper limit occurring when $\tilde{m}_{(-)} = \tilde{m}_{(+)} = M_W$) and

$$\tilde{m}_{(-)} \geq [M_W{}^2 + m_H{}^2/8]^{1/2} - m_H/(2\sqrt{2}). \qquad (19)$$

We now make use of the requirement that the renormalization-group equations do not become singular below the GUT mass M. This puts a strong limit on the Higgs mass[9]: $m_H \lesssim 250$ GeV. This constraint now implies $m_g \lesssim 177$ GeV (which then is also the upper bound on s-quark and s-lepton masses) and a *lower bound on the W-ino mass of* $\tilde{m}_{(-)} \gtrsim 31$ GeV. Thus for this case the $\tilde{W}_{(-)}$ W-ino is constrained to lie between about 30 and 80 GeV.

More detailed information can be obtained by specifying the model more completely. Thus for the simple Polony model for the super Higgs potential, $g_2 = m^2(Z + B)$, Eq. (4) has solutions for λ in the range $1.05 \lesssim \lambda \lesssim \infty$. The upper bound on m_g and the lower bound on $m_{(-)}$ are given in Table I, again with the assumption that $m_H < 250$ GeV. The parameter $\lambda = \lambda_2/\lambda_1$ measures the effects that the heavy-mass GUT couplings have on the low-mass sector. Though their presence is essential in order that there be spontaneous breaking of $SU(2) \otimes U(1)$, the bounds on m_g and $\tilde{m}_{(-)}$ are remarkably insensitive to the value of λ over its entire range. Thus in the Polony model, the s-quark and s-lepton masses will lie *below* about 105 GeV and the W-ino mass above about 35 GeV. Particles with these masses should be observable at current accelerators and at those currently under construction.

In this note we have examined a class of realistic supergravity GUT models and have shown that associated with the $W^\pm$ and Z^0 mesons there are two charged W-inos lying on either side of the $W^\pm$ meson, and two neutral Z-inos lying on either side of the Z^0 meson.[7] If the mass of the low-lying W-ino is $\lesssim M_W/2$, the $Z^0 \to \tilde{W}^- \tilde{W}^+$ mode should be fairly significant. Since the W-ino would have decays such as $\tilde{W} \to e + \nu + \tilde{\gamma}$ ($\tilde{\gamma}$ = photino) one would expect to see $Z^0 \to e^+ + e^- + \nu + \bar{\nu} + 2\tilde{\gamma}$ events. These supergravity GUT models also predict that the s-quarks and s-leptons

TABLE I. Upper bounds on the gravitino mass m_g and lower bounds on the W-ino mass $\tilde{m}_{(-)}$ in the Polony model when $m_H < 250$ GeV.

λ	$(m_g)_{\max}$	$(\tilde{m}_{(-)})_{\min}$
1.05	100.4	35.4
3.00	106.2	36.1
5.00	106.5	36.1
∞	106.9	36.2

should be approximately degenerate with mass equal to the gravitino mass m_g. A sum rule, Eq. (18), relates the gravitino mass to the W, $\tilde{W}$-ino, and Higgs masses, and the renormalization-group constraint on the Higgs mass gives a lower bound to the W-ino mass of 31 GeV. For the special case of the Polony super Higgs model the $\tilde{W}$-ino mass is constrained to be $\gtrsim 35$ GeV and s-quarks and s-leptons to lie below ~ 105 GeV. s-quarks should be strongly produced (and hence at these masses possibly observable at CERN) while s-leptons should be electromagnetically produced (and hence observable at the Stanford Linear Collider and the Large Electron-Positron machine). Thus supergravity GUTS make a number of striking low-energy predictions which can be experimentally tested. Finally, we note that all results given here, except Table I, are independent of the form of the super Higgs potential, $g_2(Z)$, of the "hidden sector." Further, Eqs. (12), (13), and (16) do not depend on the detailed choice of Eq. (1) for the GUT sector $g_1(Z_a)$ but rather on there being just two Higgs multiplets (a $\underline{5}$ and $\underline{5}$*) which develop VEV's in the electroweak sector. Equations (4), (11), and (17) depend on the explicit form of Eq. (1), in particular on the existence of the single singlet multiplet U (though one may expect similar formulas if there is more than one "sliding" singlet field present). Thus our results apply to a reasonably wide range of models.

We are pleased to acknowledge discussions with M. Machacek and S. Weinberg. This work is supported in part by the National Science Foundation under Grants No. PHY77-22864 and No. PHY80-08333.

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[1]S. Weinberg, "Upper Bound on Gauge Fermion Masses" (to be published).

[2]A. H. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. 49, 970 (1982).

[3]S. Weinberg, Phys. Rev. Lett. 48, 1176 (1982); P. Nath, R. Arnowitt, and A. H. Chamseddine, Northeastern University Report No. NUB 2565 (to be published); J. Ellis, D. V. Nanopoulos, and K. Tamvakis, CERN Report No. TH 3418-CERN, 1982 (unpublished).

[4]R. Arnowitt, A. H. Chamseddine, and P. Nath, Northeastern University Report No. NUB 2569 (to be published).

[5]J. Polony, Central Energy Research Establishment, Budapest, Report No. KFKI-1977-93, 1977 (unpublished); E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello, and R. van Nieuwenhuizen, Nucl. Phys. B147, 105 (1979).

[6]P. Nath, R. Arnowitt, and A. H. Chamseddine, Harvard/Northeastern University Report No. HUTP 82/A/NUB 2579 (to be published).

[7]The existence of such W-inos and Z-inos for models of this type had previously been obtained by S. Weinberg (private communication).

[8]An exception may be the t quark, should its mass turn out to be large.

[9]N. Cabibbo, L. Maiani, G. Parisi, and R. Petronzio, Nucl. Phys. B158, 295 (1979); M. Machacek and M. Vaughn, Phys. Lett. 103B, 29 (1981). The bound stated on m_H should be viewed as a rough estimate for our model as the published calculations have been done for SU(5) GUT and not for supersymmetric SU(5).