Quark Distributions in Nuclei

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(Received 18 October 1982)

Recent deep-inelastic muon-scattering data from iron and deuterium targets are interpreted as evidence that the distribution of quarks in iron differs markedly from that in isolated nucleons. Some sources of the difference are suggested.

PACS numbers: 12.35.Ht, 13.60.Hb, 21.90.+f, 25.30.Mr

At asymptotically large four-momentum transfer, deep-inelastic lepton scattering from a hadronic target measures the distributions of quarks and gluons within the target. The data are parametrized in terms of one or more structure functions, $F_i(x, Q^2)$, which are functions of the invariant four-momentum transfer from the lepton to the target, $-Q^2$, and the energy transfer in the laboratory, q^0, or $x = Q^2/2Mq^0$ where M is the nucleon mass. Quantum chromodynamics (QCD) perturbation theory predicts logarithmic Q^2 variation for $F_i(x, Q^2)$ at fixed x and very large Q^2. The x dependence of $F_i(x, Q^2)$ is not predicted by QCD perturbation theory. It measures the momentum distribution of the constituents of the target.

Generally it has been assumed that a nucleus differs little from a collection of isolated nucleons so far as deep-inelastic scattering at very large Q^2 is concerned. Recently, however, the European Muon Collaboration (EMC) has reported[1] that the structure function for deep-inelastic muon scattering off iron differs significantly from the structure function of deuterium at very large Q^2 ($Q^2 \cong 50$ GeV^2). Corrections for Fermi motion and for the neutron excess in iron do not account for the difference. The former have the wrong sign and the latter are much too small.

In this Letter I wish to point out that the difference of structure functions, if it can be measured precisely as a function of Q^2, x, and A (A is the atomic number), is a potentially rich source of information about nuclear structure, and about the response of quarks and nucleons to the presence of a nuclear medium.

For our purposes it is much more convenient to analyze the difference,

$$\Delta^A(x, Q^2) \equiv F_2{}^A(x, Q^2) - F_2{}^{\mathrm{D}}(x, Q^2),$$

rather than the ratio $F_2{}^A/F_2{}^{\mathrm{D}}$ given in Ref. 1. Here and henceforth $F_2{}^A$ denotes the structure function per nucleon. If the label Q^2 or A is omitted from Δ, $Q^2 \cong 50$ GeV^2 or $A = 56$ may be assumed. $\Delta(x)$ obtained from Ref. 1 and the published EMC data on iron[2] are shown in Fig. 1. $\Delta(x)$ should be corrected for the neutron excess in iron and for Fermi motion in both iron and deuterium. I have not made these corrections in Fig. 1. They do not change the qualitative features of my analysis.

At $Q^2 = 50$ GeV^2 higher-twist [i.e., $O(1/Q^2)$] effects in inelastic lepton scattering are thought to be negligible.[3] I will therefore assume $\Delta(x, Q^2)$ to be a leading, twist-2 effect. This assumption can be checked by measuring the Q^2 dependence of Δ, which should be logarithmic. The twist-4 corrections to $\Delta(x, Q^2)$ can be estimated with the methods of Ref. 4 and appear to be small. In fact they are interesting in themselves because the dominant ones can be predicted in asymptotic perturbative QCD.[5] The leading twist-2 effects are summarized in the quark-parton model with logarithmically evolving quark, antiquark, and gluon distribution functions.[6] Using standard, elementary parton model methods I will show that $\Delta(x)$ shown in Fig. 1 implies the following.

(1) The "valence" quarks in iron are degraded relative to those in deuterium, i.e., there is a

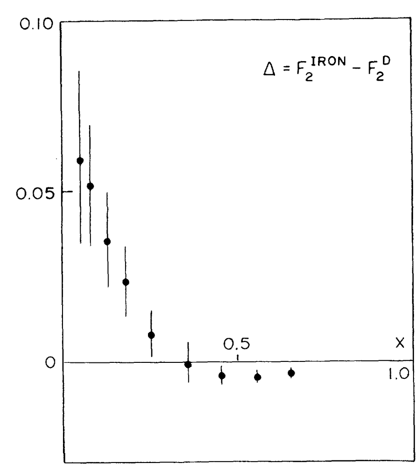

FIG. 1. EMC data on $\Delta(x) = F_2{}^{\mathrm{iron}}(x) - F_2{}^{\mathrm{D}}(x)$.

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substantial depletion of valence quarks at high x and a corresponding enhancement at low x. I will argue that this may be attributed to "percolation" of quarks from nucleon to nucleon in iron or, equivalently, to the existence of aggregates of more than three quarks (with small probability) in iron.

(2) There is a large (of order unity) increase in the number of "sea" quarks in iron over deuterium. This may be due to pion or other meson or mesonlike components in the nuclear wave function.

(3) The fraction of momentum (per nucleon) on gluons is significantly less in iron than in the deuteron.

The derivation of these results is elementary. $F_2{}^A(x)$ and $\Delta^A(x)$ are sums over quark (and antiquark) distributions which measure the probability of finding a quark (or antiquark) carrying a momentum $k^+ = xM/\sqrt{2}$ $[k^+ = (k^0 + k^3)/\sqrt{2}]$ in the target rest frame. In an isoscalar target we have $u^A(x) = d^A(x)$, $\bar{u}^A(x) = \bar{d}^A(x)$, where u^A $(\bar{u}^A)$ and d^A $(\bar{d}^A)$ are distributions of up and down quarks (antiquarks) per nucleon. I assume a flavor-SU(3)-symmetric "sea": $u^A(x) = d^A(x) \equiv q^A(x) + o^A(x)$, $\bar{u}^A(x) = \bar{d}^A(x) = s^A(x) = \bar{s}^A(x) \equiv o^A(x)$. s^A $(\bar{s}^A)$ is the distribution of strange quarks (antiquarks). The opposite extreme assumption, $s^A = \bar{s}^A = 0$, changes the analysis very little. I ignore heavier quarks. $q^A(x)$ is the distribution of "valence" quarks, normalized to $\int_0^A dx\, q^A(x) = \frac{3}{2}$. It is expected to behave like $1/\sqrt{x}$ for $x \sim 0$. $o^A(x)$ is the distribution of "sea" quarks, expected to behave like $1/x$ for $x \sim 0$. In the nucleon $o(x)$ $\sim (1-x)^p/x$ with $p \sim 8$.[7] $q(x)$ and $o(x)$ refer to an isolated nucleon. Since the deuteron is large and weakly bound, I believe that it is reasonable to regard it as a superposition of an essentially isolated proton and neutron. Then $\delta q^A(x)$ and $\delta o^A(x)$ are the differences $\delta q^A = q^A - q$ and $\delta o^A = o^A - o$, which contribute to $\Delta^A(x)$. Note that because $q^A(x)$ is normalized, we have

$$\int_0^1 dx\, \delta q^A(x) = 0, \tag{1}$$

if one ignores the region $1 < x < A$ where $q(x)$ vanishes and $q^A(x)$ is extremely small. In this model

$$\Delta^A(x) = x[\tfrac{5}{9}\, \delta q^A(x) + \tfrac{4}{3}\, \delta o^A(x)]. \tag{2}$$

Now consider $\Delta(x)$ shown in Fig. 1. For $x > 0.35$, $o(x)$ (in the isolated nucleon) is known to be negligible, and thus $\delta o^A(x) \geq 0$ for $x \gtrsim 0.35$. Since $\Delta(x) < 0$ for $x \gtrsim 0.35$, Eq. (2) requires $\delta q^A(x)$ to be negative for this range of x. The valence-quark distribution is conserved [viz. Eq. (1)] and so the depeletion of $q^A(x)$ at large x must be accompanied by an enhancement in $q^A(x)$ at small x (point 1). This effect is substantial: If we compare $\Delta(x)$ with $F_2{}^D(x)$ the depletion (uncorrected for Fermi motion) for the largest measured value of x appears to be 15%–20%. In the following I will assume that $\delta q^A(x)$ does not change sign twice but instead remains positive at small x.

Now consider the integral of $\Delta(x)/x$ over the interval $0.05 \lesssim x \lesssim 0.65$. It is safe to assume that the integral of $\delta q^A(x)$ over the interval $0.65 \lesssim x \leq A$ is negligible. The region $x < 0.05$, however, cannot be ignored because it is likely that $\delta q \sim 1/\sqrt{x}$ for small x. With use of Eq. (1), the integral of $\Delta(x)/x$ may be rewritten as

$$\int_{0.05}^{0.65} dx\, \delta o^A(x) = \tfrac{3}{4}\int_{0.05}^{0.65} (dx/x)\, \Delta^A(x) + \tfrac{5}{12}\int_0^{0.05} dx\, \delta q^A(x) \geq \tfrac{3}{4}\int_{0.05}^{0.65} (dx/x)\, \Delta^A(x), \tag{3}$$

with the assumption that $\delta q^A \geq 0$ for $0 < x < 0.05$. From Fig. 1 I estimate $\frac{3}{4}\int_{0.05}^{0.65} dx\, \Delta(x)/x \cong 0.072 \pm 0.013$. For a comparison I calculate the same quantity in the nucleon using a popular parametrization of the sea-quark distribution, $o(x) \sim (1-x)^8/x$, normalized so that the sea carries 13% of the nucleon's momentum.[7] I find $\int_{0.05}^{0.65} o(x) \times dx \cong 0.126$. The sea in iron in the same range of x is enhanced by a factor of 0.072/0.126 or nearly 60%[8] (point 2).

Finally, consider the momentum sum rule for iron and the deuteron (in the valence model),

$$\delta\epsilon^A = 2\int_0^A x\, dx[\delta q^A(x) + 3\,\delta o^A(x)]. \tag{4}$$

ϵ^A is the fraction of the target p^+ $[p^+ = (p^0 + p^3)/\sqrt{2}]$ on quarks and antiquarks in the target rest frame and $\delta\epsilon^A = \epsilon^A - \epsilon^D$. From Eq. (2) I obtain

$$\delta\epsilon^A = \tfrac{18}{5}\int_0^A dx\, \Delta^A(x) + \tfrac{6}{5}\int_0^A x\, dx\, \delta o^A(x). \tag{5}$$

If we ignore small contributions from $x \gtrsim 0.65$ and $x \lesssim 0.05$ the data in Fig. 1 give $(2.01 \pm 0.56) \times 10^{-2}$ for the first term on the right. The second term is almost undoubtedly positive [see Eq. (3)] and so we obtain a bound $\delta\epsilon \gtrsim (2.01 \pm 0.56) \times 10^{-2}$. In fact the second term in Eq. (5) is likely to be comparable to the first: A crude estimate gives $\approx 1.3 \times 10^{-2}$. Thus (point 3) the momentum fraction on the gluons drops by at least 2% and perhaps by 3.3% from its value of $\approx 50\%$ in the isolated nucleon.

Clearly, a much more informative analysis could be made from more complete data. It would be especially useful to have measurements of $F_3^{\nu A}$ since F_3 distinguishes between quarks and antiquarks.

The effects I have found may be attributed to the presence within the nucleus of objects whose quark distributions differ from those of an isolated nucleon. Two obvious candidates are multinucleon correlations and pions or other mesons. This picture implicitly assumes that the constituents interact incoherently with the scattered lepton, which may not be so. There may be no constituents above the level of the quarks themselves for which incoherence is a good assumption. Nevertheless the model may guide our interpretation of the data. Let $G_{T/A}(y)$ $(0<y<A)$ be the probability of finding some constituent T in nucleus A with $k_T^+=yM/\sqrt{2}$. Let $a_T(z)$ $(0<z<1)$ be the probability of finding a quark of flavor a in T with $k^+=zk_T^+$. Then this contribution to the distribution of a quarks in nucleus A is a convolution:

$$a_{T/A}(x)=\int_x^A (dy/y)\,G_{T/A}(y)\,a_T(x/y). \qquad (6)$$

If the constituents are all nucleons, Eq. (6) reproduces the Fermi smearing model[9] which fails to fit the data.

The degradation of the valence quarks can be explained if quarks in iron are at least partially deconfined, i.e., free to move from nucleon to nucleon. This should enhance q_A at long range in the nucleus' rest frame. Long range is known to be associated with low x.[10] Coupled with Eq. (1) this requires a depletion at large x.

To put this argument on somewhat firmer ground consider a simple model. Suppose that, on average, iron contains a small number N of spherical six-quark bags of mass M' and radius R' larger than the nucleon, representing two-nucleon correlations. I take them to be at rest so that $G_{Q^6/A}(y)=N\delta(y-M'/M)$. Quark distributions in a spherical bag are easily computed.[11] The distribution function for a quark in a fixed orbital is independent of the number of quarks in the bag and scales simply with its radius and mass,

$$q^{\mathrm{bag}\,\prime}(z)=(M'R'/MR)\,q^{\mathrm{bag}}((M'R'/MR)z). \qquad (7)$$

Neglecting the six-quark component in the deuteron one obtains

$$\delta q^{\mathrm{bag}}(x)=(2N/A)[(R'/R)q^{\mathrm{bag}}(R'x/R)-q^{\mathrm{bag}}(x)]$$

from Eqs. (6) and (7). If $R'>R$, δq^{bag} shows the expected degradation. The bag virial theorem $(M=4BV)$ puts a bound on R'/R: $R'/R=(M'/M)^{1/3}\geq 2^{1/3}$. To illustrate the size of this effect $(A/2N)x\,\delta q^{\mathrm{bag}}(x)$ is shown in Fig. 2 for $R'/R=2^{1/3}$. This estimate should not be taken too seriously. The model lacks Regge behavior near $x=0$ and the proper threshold at $x=1$.[11,12] Furthermore, a spherical six-quark bag is not necessarily a good approximation to the configurations responsible for the nuclear effect on $q(x)$. I expect them to be deformed.

One may well ask what has happened to the momentum which has been taken from the valence quarks. It is easy to show that the contribution to the momentum sum rule for quarks in an object of mass M_T and baryon number A_T is $2\int x\,dx\,q_T(x)=(M_T/MA_T)\epsilon_T$ where ϵ_T is the fraction of the target's p^+ on the valence quarks. In a naive six-quark bag model (no gluon exchange, no zero-point energy) $\epsilon_T=1$ but $M_T<2M$. In more realistic models $M_T>2M$ but $\epsilon_T<1$ because other fields (e.g., gluons) carry some of the target momentum.

Pions or other mesons or mesonlike components in the nucleus are a possible source of the enhancement in $o^A(x)$ at small x. Valence quarks and antiquarks in a meson contribute to the sea distribution in the nucleus. The valence-quark distribution in the pion is "hard," $q(z)\sim(1-z)$, but the pion distribution in the nucleus is "soft": $G_{\pi/A}(y)$ is peaked near $y=0$ because $p_\pi^+\ll M/\sqrt{2}$. Consequently $a_{\pi/A}(x)$ is shifted to small x where the enhancement is seen. The size and shape of $a_{\pi/A}(x)$ depends on the detailed form of $G_{\pi/A}(y)$ which requires further study.

There is important information contained in the A dependence of $\Delta^A(x, Q^2)$. If (as the model above suggests) the shift in the valence distribution is a consequence of two (or, rarely, more) nucleon

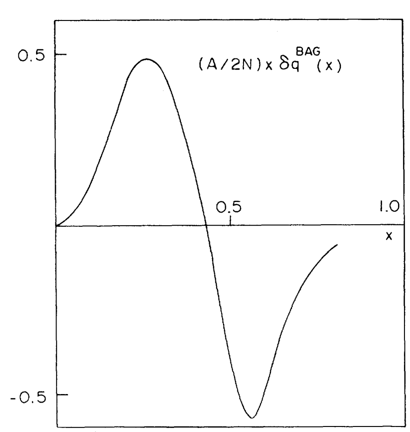

FIG. 2. Bag-model estimate for $(A/2N)x\delta q(x)$.

correlations, it should saturate with A: i.e., at large A, $\delta q^A(x)$ should approach a universal function $\delta\bar{q}(x)$ with corrections due to the nuclear surface which fall like $A^{-1/3}$. In light nuclei effects peculiar to individual nuclei (e.g., He^4) may dominate the steady approach of $\delta q^A(x)$ to saturation. The variation of $\Delta^A(x, Q^2)$ with A may provide important information about short-range correlations, especially in light nuclei. The A dependence of the sea component, $\delta o^A(x)$, is less certain. As $x \to 0$ longer and longer distances are probed. $\delta o^A(x)$ may display nonuniform behavior for large A and small x; i.e., at any fixed x, $\delta o^A(x)$ saturates for large enough A, but at any fixed A there is an x below which saturation has not occurred. The same may occur for $\delta q^A(x)$ but the effect would be less striking since $\delta o^A(x)$ dominates at low x.

In summary there seems to be a wealth of information to be obtained from precise measurement of nuclear structure functions at large Q^2 as functions of x, Q^2, and A. It should be of interest to nuclear physicists who wish to understand the role of quarks in nuclei and to particle theorists who need more information on long-range, confining phenomena in QCD.

I wish to thank Erwin Gabathuler for drawing my attention to and providing me with the data of Ref. 1. I am grateful to my colleagues in the Center for Theoretical Physics for discussions and suggestions and to C. H. Llewellyn Smith and N. N. Nikolaev for pointing out an error in an early version of the manuscript. This work was supported in part by the U. S. Department of Energy under Contract No. DE-AC02-76ER03069.

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