vestigación Científica y Técnica is gratefully acknowledged.

[1] F. J. Ernst, Phys. Rev. 167, 1175 (1968).

[2] D. Kramer and G. Neugebauer, Commun. Math. Phys. 10, 132 (1968).

[3] R. Geroch, J. Math. Phys. (N.Y.) 13, 394 (1972).

[4] W. Kinnersley, J. Math. Phys. (N.Y.) 18, 1529 (1977).

[5] W. Kinnersley and D. M. Chitre, Phys. Rev. Lett. 40, 1608 (1978).

[6] D. Maison, Phys. Rev. Lett. 41, 521 (1978), and J. Math. Phys. (N.Y.) 20, 871 (1979).

[7] C. Hoenselaers, W. Kinnersley, and B. C. Xanthopoulos, Phys. Rev. Lett. 42, 481 (1978), and J. Math. Phys. (N.Y.) 20, 2530 (1979).

[8] V. A. Belinskiĭ and V. E. Zakharov, Zh. Eksp. Teor. Fiz. 75, 1955 (1978) [Sov. Phys. JETP 48, 985 (1978)].

[9] Bäcklund transformations for the Ernst equation of general relativity were independently introduced for the first time in the following: B. K. Harrison, Phys. Rev. Lett. 41, 1197, 1835(E) (1978); G. Neugebauer, J. Phys. A12, L67 (1979). Other expressions for Bäcklund transformations of the same problem, and their relation with gauge transformations, may be seen in the following: M. Omote and M. Wadati, J. Math. Phys. (N.Y.) 22, 961 (1981); F. J. Chinea, Phys. Rev. D 24, 1053 (1981), and 26, 2175(E) (1982), and Physica (Utrecht) 114A, 151 (1982).

[10] T. Lewis, Proc. Roy. Soc. London, Ser. A 136, 176 (1932); A. Papapetrou, Ann. Phys. (Leipzig) 12, 309 (1953).

[11] G. Neugebauer and D. Kramer, Exp. Tech. Phys. 28, 3 (1980); G. Neugebauer, J. Phys. A 13, L19 (1980); D. Kramer and G. Neugebauer, Phys. Lett. 75A, 259 (1980).

[12] B. K. Harrison, Phys. Rev. D 21, 1695 (1980).

[13] K. Pohlmeyer, Commun. Math. Phys. 46, 207 (1976).

[14] A permutability property for Bäcklund transformations of the Ernst equation was found by G. Neugebauer (Ref. 9).

Data on the Gross-Llewellyn Smith Sum Rule as a Function of q^2

T. Bolognese

Centre d'Etudes Nucléaires, Saclay, F-91190 Gif-sur-Yvette, France

and

P. Fritze and J. Morfin[(a)]

Physikalisches Institut der Technischen Hochschule, D-5100 Aachen, Germany

and

D. H. Perkins and K. Powell

Department of Nuclear Physics, University of Oxford, Oxford OX1 3RH, United Kingdom

and

W. G. Scott[(b)]

CERN, European Organization for Nuclear Research, CH-1211 Geneva 23, Switzerland

(Received 22 June 1982)

Data are presented on the Gross-Llewellyn Smith sum rule obtained from combined narrow-band neon and Freon bubble-chamber neutrino-antineutrino experiments. Remarkably no significant deviation from the parton-model prediction for the sum rule is observed at very low values of $q^2 \lesssim 1$ GeV^2. Limits on the effective QCD scale parameter Λ and on the magnitude of the twist-4 correction are set. The best fit, neglecting higher-twist contributions, gives $\Lambda = 92^{+20}_{-36}$ MeV.

PACS numbers: 13.15.Em, 12.35.Eq

In the quark-parton model the neutrino-nucleon structure function F_3 measures the difference of the x distributions of the quarks and the antiquarks in the nucleon:

$$F_3(x) \underset{q^2 \to \infty}{=} \frac{dN_q}{dx} - \frac{dN_{\bar{q}}}{dx}, \tag{1}$$

where x is the usual Bjorken scaling variable. The integral of F_3 measures the number of "valence quarks" per nucleon, equal to three in the quark model:

$$\int_0^1 F_3(x)\,dx \underset{q^2 \to \infty}{=} N_q - N_{\bar{q}} = 3 \text{ valence quarks}. \tag{2}$$

This relation, originally derived from current algebra, is known as the Gross–Llewellyn Smith (GLS) sum rule.[1]

In QCD the GLS sum rule remains valid in the leading log approximation, but in higher order it acquires a correction proportional to the strong-coupling constant α_s,[2,3]

$$\int_0^1 F_3(x)dx \underset{q^2\to\infty}{=} 3\left(1-\frac{\alpha_s}{\pi}+\frac{\kappa^2}{q^2}\right), \qquad (3)$$

where $\alpha_s = 12\pi/(27\ln q^2/\Lambda^2)$ and Λ is the effective QCD scale parameter, known to be of order a few hundred megaelectronvolts. In general (as in the case of the higher moments, $N \geqslant 2$) there will also be $1/q^2$ power corrections due to the presence of higher twist terms (twist >2). In Eq. (3) the term κ^2/q^2 represents the twist-4 correction. In the case of the $N=1$ moment (in contrast to the case of the higher moments), theoretical arguments[4,5] predict that the twist-4 correction will be particularly small. Thus it might be expected that measurements to test the validity of the GLS sum rule (particularly at low q^2) would provide definite evidence for the presence of the higher-order correction [Eq. (3)] and provide information on Λ.

Figure 1 shows the data on the GLS sum rule from the combined BEBC (Big European Bubble Chamber) narrow-band neon and the GGM-PS (Gargamelle–proton synchrotron) freon neutrino/antineutrino experiments plotted versus q^2. The errors shown are statistical only and do not include various systematic errors which are discussed below. Details of the event sample together with a complete account of the methods and assumptions used to extract F_3 are published elsewhere.[6]

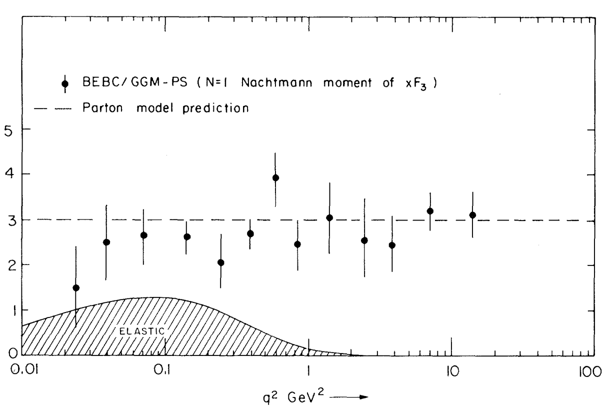

FIG. 1. Data on the Gross–Llewellyn Smith sum rule from the combined BEBC narrow-band neon and GGM-PS freon neutrino/antineutrino experiments plotted vs q^2 (errors shown are statistical only). The quantity plotted is the $N=1$ Nachtmann moment of F_3. The broken line is the parton-model prediction and the solid curve represents the QCD prediction [Eq. (3)].

In Fig. 1 the quantity plotted is actually the $N=1$ Nachtmann moment of F_3 as given by Wandzura,[7]

$$\int_0^{\xi_{\max}} F_3 \frac{(3+m^2\xi^2/q^2)}{3}\frac{(1+m^2\xi^2/q^2)}{(1-m^2\xi^2/q^2)}d\xi, \qquad (4)$$

computed in terms of the Nachtmann variable, $\xi = 2x/[1+(1+4m^2x^2/q^2)^{1/2}]$. For comparison with theoretical predictions, the Nachtmann definition [Eq. (4)] is preferred to the simpler Cornwall-Norton definition [Eq. (2)] because Nachtmann moments correctly account for kinematical scaling violations due to the target nucleon mass m. For $q^2 > 1$ GeV^2 this choice has essentially no effect on the experimental result for the sum rule. For lower q^2, however, the two definitions differ appreciably and for $q^2 \sim 0.1$ GeV^2, for example, the Cornwall-Norton definition would give a larger result for the sum rule by about a factor of 2.

Elastic events, for which $\xi = \xi_{\max} = 2/[1+(1+4m^2/q^2)^{1/2}]$, have been included in the evaluation of the integral [Eq. (4)] with the assumption of dipole forms for the vector and axial-vector form factors F_V and F_A: $xF_3 = F_V F_A$, $F_V = 4.71/(1+q^2/0.71)^2$, $F_A = 1.22/(1+q^2/0.90)^2$. In Fig. 1 the contribution of the elastic events is shown separately. The contribution of Δ production is qualitatively similar and the elastic and Δ-production events together contribute most of the integral at $q^2 \sim 0.1$ GeV^2.

In Fig. 1 the broken line indicates the parton-model prediction [Eq. (2)]. On the basis of Fig. 1 we conclude that the data are broadly consistent with the prediction from the parton model. Similar conclusions have been drawn from previous data.[6,8] More remarkably we observe that the data show very little evidence for appreciable deviations from the parton-model prediction even at very low values of q^2 ($q^2 < 1$ GeV^2) where elastic and quasielastic processes make important contributions to the cross section. Before drawing quantitative conclusions, however, we discuss some experimental problems to which attention should be drawn.

Firstly, a determination of the absolute magnitude of the neutrino structure functions requires a knowledge of the incident neutrino flux. The neutrino flux determination depends on measurements of muon fluxes in the neutrino shielding performed with solid-state counters and on measurements of the K/π ratio for the parent meson beam. For the BEBC narrow-band runs the uncertainty in the flux is 4%–5% (Ref. 9) while for the GGM-PS runs in the wide-band beam the corresponding uncertainty is 7%–12% (Ref. 10) depending on the energy. Thus in addition to the statistical errors shown in Fig. 1 there is an overall systematic error on the inelastic contribution of ~±10% at low q^2 falling to ~±5% for $q^2 \gtrsim 1$ GeV^2.

Apart from flux errors, one other fundamental difficulty arises when an attempt is made to test the Gross–Llewellyn Smith sum rule experimentally. The point is that at fixed q^2 there is a minimum value of ξ, $\xi_{\min} \sim q^2/2mE_{\max}$, which is accessible experimentally ($E_{\max}$ is the highest available neutrino energy). While it is possible to give an experimental estimate for the integral between the limits $\xi_{\min}$ and $\xi_{\max}$, any estimate for the complete integral [Eq. (4)] necessarily involves a model-dependent assumption about the behavior of F_3 at small x. In this analysis we have replaced the integral [Eq. (4)] by a summation over bins of width $\Delta\xi = 0.1$ and quote an estimate for the complete integral for those values of q^2 for which $\xi_{\min}$ is appreciably less than 0.1. The contribution to the integral coming from a particular bin is computed from the data multiplied by a correction factor which takes account of the distribution of the events within a bin. These correction factors are computed from the neutrino spectrum and the cross-section formulas with the assumption of specific functional forms for

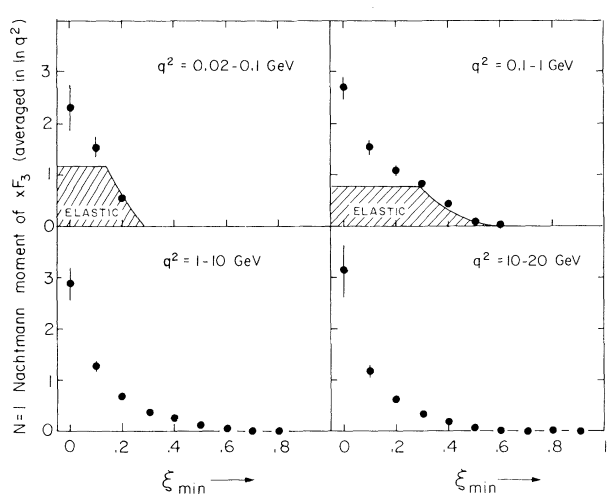

FIG. 2. The $N=1$ Nachtmann moment of F_3 plotted vs $\xi_{\min}$ showing the convergence of the Gross–Llewellyn Smith sum rule for various q^2 intervals.

the structure functions. The contribution to the integral coming from the first bin ($\xi = 0-0.1$) has been evaluated under the assumption that xF_3 behaves like $x^{0.5}$ within the bin (an approximate $x^{1/2}$ dependence at small x, independent of q^2, is expected on theoretical grounds[11]). Figure 2 shows the data for the integral Eq. (4) evaluated between limits $\xi_{\min}$ and $\xi_{\max}$ treating $\xi_{\min}$ as a variable for three different ranges of q^2. The convergence of the sum rule is clearly very slow particularly at high q^2. For $q^2 = 10-20$ GeV^2 almost two thirds of the integral comes from the first bin. In order to demonstrate the sensitivity of our results to the assumed form for xF_3 at small x we have varied this form within the limits $x^{0.5\pm0.1}$. This yields a "model" error which is comparable in magnitude to the statistical error and the flux error as summarized in Table I. Note that in Table I and in Fig. 2 the data have been averaged in $\ln q^2$ as recommended by Schrempp and Schrempp.[12]

In conclusion, in view of the several problems discussed above, very little attention should be paid to the precise placing of the high-q^2 points with respect to the asymptotic predictions (Fig. 1), except insofar as the data are consistent with these. However, the more remarkable conclusion, that the data show very little evidence for any q^2 dependence of the integral even at very low q^2, still stands.

We have performed fits to the data of Fig. 1 using Eq. (3) and taking into account both statistical and flux errors. Good fits are obtained with very small values for Λ and κ (χ^2 per degree of freedom $\to 11.3/11$ as $\Lambda, \kappa \to 0$). Figure 3 shows the 90%-confidence contours versus Λ and κ (for both positive and negative κ^2) as a function of the lower q^2 cut applied to the data. The contours are represented by broken curves for those re-

TABLE I. Results for the complete integral [Eq. (4)] showing statistical and flux errors together with an estimate for the uncertainty arising from model-dependent corrections.

q^2 (GeV^2)	Complete integral [Eq. (4)] ± statistical error	Flux error	"Model" error
0.02–0.10	2.31 ± 0.44	± 0.12	± 0.10
0.10–1.0	2.70 ± 0.21	± 0.19	± 0.16
1–10	2.89 ± 0.33	± 0.20	± 0.23
10–20	3.13 ± 0.48	± 0.15	± 0.28

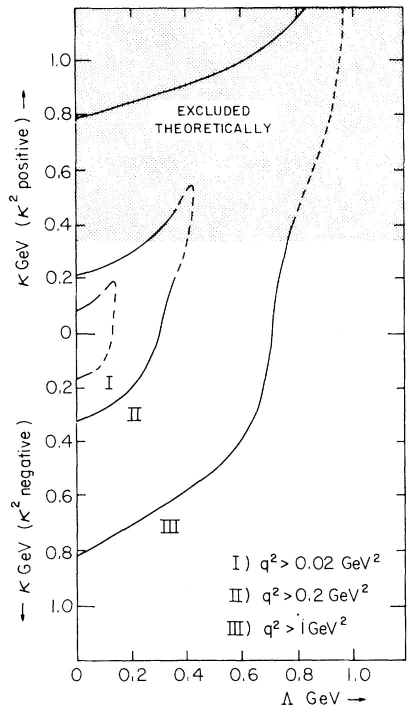

FIG. 3. 90%-confidence contours for Λ and κ (see text) as a function of the lower q^2 cut applied to the data. The contours are shown by broken curves when α_s/π (or $|\kappa^2|/q^2) > 1$. The shaded region represents the theoretical bound due to Ellis, Furmanski, and Petronzio (Ref. 13).

gions of Λ and κ for which α_s/π (or $|\kappa^2|/q^2) > 1$. In this case Eq. (3) is unlikely to be a sensible theoretical approximation and the corresponding bounds on Λ and κ are not considered to be physically relevant. A theoretical bound on the twist-4 correction has been given by Ellis, Furmanski, and Petronzio.[13] The twist-4 correction is bounded such that $\kappa^2 < \frac{4}{3}m^2 M_3/M_1$, where M_3/M_1 is the ratio of $N=3$ to $N=1$ Nachtmann moments of xF_3. In this experiment we measure M_3/M_1 to be in the range 0.03–0.08 depending on the value of q^2 (Ref. 6). In Fig. 3 the region excluded by the theoretical bound is represented by the shaded area. On the basis of the data shown in Fig. 3 we conclude that Λ, $|\kappa| < 350$ MeV, allowing for a partial cancellation of the higher-order and higher-twist corrections. In view of the near saturation of the sum rule by elastic and quasielastic events at low q^2 the small value obtained for the twist-4 parameter κ should perhaps be considered surprising. We emphasize that the above bounds are independent of theoretical assumptions regarding the x dependence of the higher-twist contribution.

Finally, we note that acceptable fits are also obtained using Eq. (3) but with no higher-twist correction (i.e., $\kappa = 0$). The best fit with all the data ($q^2 > 0.02$ GeV^2) gives $\Lambda = 92^{+20}_{-36}$ MeV (χ^2 per degree of freedom $= 8.1/12$).

We thank R. K. Ellis and R. L. Jaffe for informative discussions on higher twist.

---

[a]Now at Fermilab, Batavia, Ill. 60510.

[b]Now at the Rutherford-Appleton Laboratory, Chilton, Didcot, Oxfordshire OX11 0QX, United Kingdom.

[1]D. J. Gross and C. H. Llewellyn Smith, Nucl. Phys. B14, 337 (1969). Below the threshold for charm-particle production this prediction is modified by a factor $(1+\cos^2\theta)/2$ (for an isoscalar target) where θ is the Cabbibo angle, a correction of ~2.8%. In the above and subsequent discussion the effect of the nonzero Cabbibo angle is neglected.

[2]M. Calvo, Phys. Rev. D 15, 730 (1977).

[3]W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, Phys. Rev. D 18, 3998 (1978).

[4]E. V. Shuryak and A. I. Vainshtein, Phys. Lett. 105B, 65 (1981).

[5]R. L. Jaffe and M. Soldate, Phys. Lett. 105B, 467 (1981).

[6]P. C. Bosetti *et al.*, Nucl. Phys. B142, 1 (1978), and B203, 362–384 (1982).

[7]S. Wandzura, Nucl. Phys. B112, 412 (1977).

[8]J. G. H. de Groot *et al.*, Z. Phys. C 1, 143 (1979).

[9]P. Bosetti *et al.*, Phys. Lett. 110B, 167 (1982).

[10]T. Eichten *et al.*, Phys. Lett. 46B, 281 (1973).

[11]See, for example, C. Lopez and F. J. Yndurain, Nucl. Phys. B171, 231 (1980).

[12]B. Schrempp and F. Schremmp, Z. Phys. 6, 7–15 (1980).

[13]R. K. Ellis, W. Furmanski, and R. Petronzio, CERN Theory Report, Ref. TH. 3254-CERN and TH. 3301-CERN (to be published).