Fluctuations in the Temperature Dependence of the Resistance of a One-Dimensional System

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New effects are predicted to occur in the resistance of a one-dimensional system when the inelastic scattering time becomes longer than the transit time of a carrier through the system. While the mean behavior of $R(T)$ is the same as in the multiple-hopping regime, the fluctuations in $R(T)$ change dramatically, depending logarithmically on the length. The behavior of $R(T)$ can give a detailed reconstruction of the microscopic eigenstates of the system.

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It is now well established that the resistance R of a one-dimensional system at $T=0$ displays large fluctuations even between macroscopically identical samples.[1,2] $R(T=0)$, a macroscopic observable, is strongly influenced by microscopic fluctuations in the random potential of the resistor. Mathematically this unusual behavior is typically manifested by the relation[1]

$$2\langle \ln R\rangle = \ln\langle R\rangle . \tag{1}$$

(Angular brackets denote the average over an ensemble of samples.) An ordinary random variable Q with small fluctuations obeys quite a different relation:

$$\langle \ln Q\rangle = \ln\langle Q\rangle . \tag{2}$$

It may be expected that for $T\neq 0$, thermal averaging effects, which are implicit in multiple-hopping descriptions of conductance, cause the more severe fluctuations in R to be removed, so that the random variable $R(T)$ would satisfy Eq. (2) rather than Eq. (1).[3] This expectation is likely to be true if the inelastic scattering time is much shorter than the elastic transit time of the carrier through the one-dimensional conductor: $\tau_{in} \ll L/v$, where v is the carrier velocity and L is the length of the sample.

In this Letter we explore the properties of $R(T)$ under the opposite condition, $\tau_{in} \gg L/v$. While it appears that no experiment is presently capable of reaching this regime,[4] it seems likely that it will be explored in the future. This condition should be satisfied at very low temperatures: $T \ll T_0/[\ln(L/a)]^2 \equiv \tilde{T}$,[5] where a is the dimension of a trap and T_0 is the characteristic temperature in the Mott law[6]:

$$R(T) \propto \exp[(T_0/T)^{1/2}] . \tag{3}$$

We find that the mean behavior of $R(T)$ at these low temperatures continues to obey Eq. (3). However, the two regimes differ in the physical origin of this behavior and in the magnitude of fluctuations which $R(T)$ displays. In the elastic conductance regime, we find $\varphi \equiv (L/L_0)^2(T/T_0)[(L_0/L)\ln(R)-1]$ to exhibit very unusual *logarithmic* (in L) relative fluctuations:

$$\delta \ln\varphi \sim \{2\ln[(L/L_0)(T/T_0)^{1/2}]\}^{-1} .$$

(L_0 is the localization length.) If $T \gg \tilde{T}$, these fluctuations are more conventional: $\delta \ln\varphi \sim v\tau_{in}/L$. We will show that $R(T)$ for $T \ll \tilde{T}$ continues to obey an equation similar to (1), and that these macroscopic fluctuations in the measurable quantity $R(T)$ can provide detailed information about the microscopic, quantum mechanical properties of the resistor.

It has been shown that at $T=0$, the conductance of a disordered one-dimensional system $G=1/R$, or equivalently (through Landauer's formula[7]) the transmission coefficient t contains huge resonances which occur at those energies coinciding with an eigenvalue ϵ_n of the random system.[8,9] The strength of these resonances is determined by the real-space distance Λ_n between the position of this (localized) eigenstate and the midpoint of the resistor. In particular, the width of these transmission resonances is $\delta\epsilon_n \propto \exp(-2\times|\tfrac{1}{2}L-\Lambda_n|/L_0)$ and the resonance transmission $t_{max}(=G_{max}) \cong \exp(-4\Lambda_n/L_0)$: away from the resonance $t \cong \exp(-2L/L_0)$, exponentially smaller than t_{max}.

If $\tau_{in} \gg L/v$, then the conductance for $T\neq 0$ may be obtained immediately from $t(T=0)$:

$$G(T\neq 0) = \int (-\partial n_F/\partial\epsilon)t(\epsilon)d\epsilon . \tag{4}$$

Here n_F is the Fermi distribution. The physical restriction implied by Eq. (4) is that the current carrier must pass through the entire sample in a single, quantum mechanical tunneling event. The temperature dependence of $G(T)$ is thus due solely to the Fermi distribution which is established in the metallic conductor attached to the ends of the resistor. Therefore the tunneling, which is restricted to occur at the Fermi level ϵ_F at $T=0$, is permitted in a range of energies $\sim T$ around ϵ_F for $T\neq 0$.

Since $t(\epsilon)$ is exponentially larger at eigenstate resonances than elsewhere, the only significant contributions to the integral in Eq. (4) will come from the eigenvalues ϵ_n. The contribution of the nth level from the Fermi level will be

$$\left(-\frac{\partial n_F}{\partial \epsilon}\right)\delta\epsilon_n\, t_{max}{}^n \propto \exp\left(-\frac{L}{L_0}\right)\exp\left[\left(-\frac{|\epsilon_n-\epsilon_F|}{T}-\frac{2\Lambda_n}{L_0}\right)\right]. \tag{5}$$

The exponent in Eq. (5) is a competition between the Fermi function factor and the strength of the transmission. For one particular ϵ_n this expression will be greatest; because it appears as an exponent, this maximum value will completely dominate Eq. (4), and a very good approximation for $G(T)$ is[9]

$$G(T)=\exp\left(\frac{-L}{L_0}\right)\exp\left[\max_n\left(-\frac{|\epsilon_n-\epsilon_F|}{T}-\frac{2\Lambda_n}{L_0}\right)\right]. \tag{6}$$

A convenient approximation which simplifies the analysis below is the replacement of $|\epsilon_n-\epsilon_F|$, the distance of the nth farthest level from the Fermi level, by $n\,\Delta\tilde{\epsilon}$, where the average distance between eigenvalues $\Delta\tilde{\epsilon}\approx 1/L\rho_s(\epsilon)$, with $\rho_s(\epsilon)$ the electronic density of states. This approximation is quite good for $n\gg 1$, which is the regime of interest. We can also put the second part of Eq. (6) in a convenient dimensionless form. The statistical homogeneity of the random system guarantees that the distances Λ_n are uniformly distributed between 0 and $L/2$. (We have confirmed this property of Λ_n by detailed numerical study.[10]) This suggests defining a new scaled variable $\kappa_n\cong(2/L)\Lambda_n$, so that κ_n is a uniformly distributed random variable between 0 and 1. With these Eq. (6) becomes

$$G(T)=\exp\left(-\frac{L}{L_0}\right)\exp\left[\max_n\left(\frac{-n}{\rho_s LT}-\frac{L}{L_0}\kappa_n\right)\right]. \tag{7}$$

This may be put in a compact form:

$$G(T)=e^{-C}\exp[-(C/M)\varphi(M)], \tag{8a}$$

$$\varphi(M)\equiv\min_n(n+M\kappa_n). \tag{8b}$$

Here $C\equiv L/L_0$ and $M\cong\rho_s L^2T/L_0=(L^2/L_0{}^2)T/T_0$.

Equation (7) provides an accurate approximation for Eq. (4) so long as $C^2\gg M\gg C$. This equation is quite amenable to both accurate numerical and analytical study of the statistical properties of $G(T)$, or equivalently of $\varphi(M)$. As a first step, the mean value of $\varphi(M)$ may be estimated by quite a simple argument: Among the first n samples of κ_n, it is likely that the smallest value of κ_n will be $\kappa_n\sim 1/n$. Equation (8b) may be rewritten $\langle\varphi(M)\rangle\sim\min_n(n+M/n)$, which gives $n_{min}\sim\sqrt{M}$, $\langle\varphi(M)\rangle\sim\sqrt{M}$. Without loss of generality, $\varphi(M)$ can be written as $\varphi(M)=\alpha(M)M^{1/2}$, where α is of order unity. While α depends on M for any particular resistor, an approximate statistical analysis of Eq. (8b) indicates that in an ensemble of samples, α is statistically independent of M if $M\gg 1$. This analysis gives the probability density function of α to be

$$A(\alpha)=\alpha\exp(-\alpha^2/2). \tag{9}$$

Thus $\langle\varphi(M)\rangle=\langle\alpha\rangle M^{1/2}=(\pi/2)^{1/2}M^{1/2}$. Combining (9) with (8a) gives us information about the mean conductance:

$$\langle\ln G(T)\rangle=-C-\frac{C}{M}\langle\varphi(M)\rangle$$

$$=\frac{-L}{L_0}\left(\frac{1}{\rho_s L_0}\right)^{1/2}\langle\alpha\rangle T^{-1/2}. \tag{10}$$

This temperature dependence is the same as in Mott's variable-range hopping[6]; the behavior of the mean of G is incapable of distinguishing the two models, despite their radically different physical origins.

We have confirmed numerically the above approximate analytical results for the present model. The top panel of Fig. 1 shows $\ln\langle\varphi(M)\rangle$ vs $\ln M$ computed by generating uniform random numbers for κ_n in Eq. (8b), with averaging performed over 2000 different samples. For comparison, the statistical prediction of $\ln\langle\varphi\rangle$ above is plotted as a dotted line on Fig. 1. The agreement between direct numerical simulation and our approximate analytical theory is clearly ex-

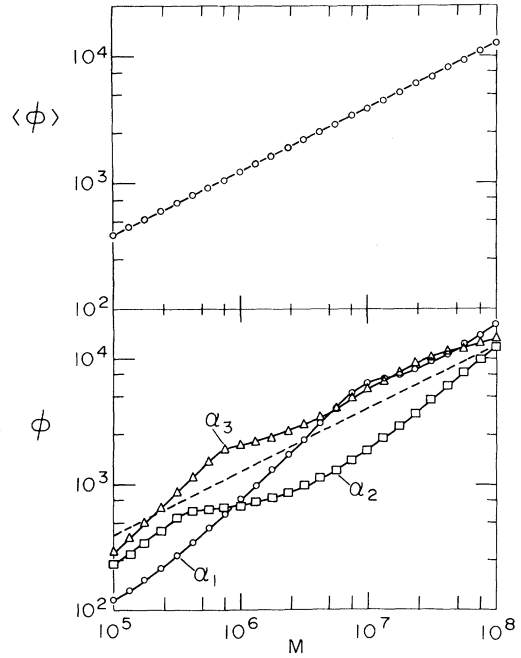

FIG. 1. Top panel: Open circles show $\langle\varphi\rangle$ vs M on a logarithmic scale. Average is computed over 2000 samples. Dotted line is the prediction of an approximate statistical theory for $\langle\varphi\rangle$. Bottom panel: φ vs M for three different samples α_1 (circles), α_2 (squares), and α_3 (triangles). Note the substantial fluctuations about the mean behavior $\langle\varphi\rangle$ (dotted line). A study of the full 2000-sample ensemble confirms that these fluctuations are of order of unity and independent of M. This directly leads to the logarithmic dependence of the fluctuations on L (see text).

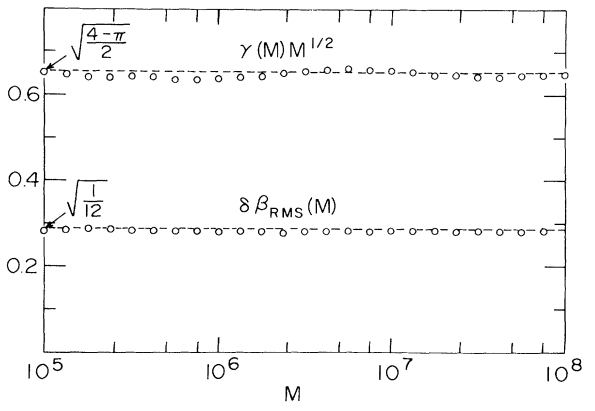

FIG. 2. Upper curve: $\gamma(M)M^{1/2}$ vs M. γ is the rms fluctuation of φ. Circles are from a numerical average over 2000 samples; the dotted line at $[(4-\pi)/2]^{1/2}$ is the result of a statistical theory. Lower curve: $\delta\beta_{\rm rms}$, the rms fluctuations of β, the "local power of T," vs M, computed over 2000 samples. The circles approach the dotted curve at $1/(12)^{1/2}$, which is the exact result if β is uniformly distributed.

cellent, and the above analysis of the mean behavior of $\varphi(M)$ is confirmed.

The lower panel of Fig. 1 shows $\ln\varphi$ vs $\ln M$ for three particular samples among the 2000 studied. These curves clearly display the distinction between the present model and Mott's variable-range hopping, for instead of converging to the mean for large T (large M) as it would in the Mott theory, $\ln\varphi$ displays fluctuations around the mean which are of order of unity and are independent of M. This directly implies the very unusual logarithmic behavior of the fluctuations mentioned above: $\Delta\ln\varphi/\ln\langle\varphi\rangle \sim 1/\ln M$. We have confirmed this observation quantitatively by explicitly computing the rms fluctuations of φ, $\gamma \equiv [\langle(\varphi-\langle\varphi\rangle)^2\rangle]^{1/2}$, over the same 2000 samples. Using Eq. (9) we expect $\gamma = [(4-\pi)/2)]^{1/2} M^{1/2}$; our numerical simulations confirm this result quite precisely, as Fig. 2 shows. Thus we demonstrate that even $\varphi \propto \ln G$ is not a typical well-behaved random variable with power-law decay of fluctuations with L, but rather has a much slower logarithmic decay. Accordingly, $R(T\neq 0)$ displays the same sort of large fluctuations as are generally believed to occur in $R(T=0)$. In fact, by using Eq. (9) we can show that $R(T\neq 0)$ satisfies an equation similar to (1): $(1+C/M)\langle\ln R\rangle = \ln\langle R\rangle$. That is, for very low T, i.e., $M \sim C$, $R(T)$ displays the same sort of fluctuations as $R(T=0)$. As T (or M) is increased, the fluctuations in R gradually decrease and this equation approaches Eq. (2).

We wish to demonstrate one explicit way in which a real experiment should be capable of distinguishing between the present model and the multiple-hopping picture of conduction, despite the fact that the two give identical predictions for the average behavior of $G(T)$. Consider the quantity

$$\beta \equiv \frac{\Delta\ln\varphi(T)}{\Delta\ln T} \equiv \frac{\ln\varphi(T+\Delta T)-\ln\varphi(T)}{\ln(T+\Delta T)-\ln T}\,.$$

β, the logarithmic derivative of $\ln\varphi$ computed by a finite difference, measures the "local power of T" in the law $G(T) \propto \exp[-(T_0/T)^{\beta}]$. In the usual variable-range hopping model this derivative will give the mean value of the exponent, $\frac{1}{2}$, with very little uncertainty. We predict that as $\Delta T/T \to 0$, the distribution of $\beta(T)$, rather than being peaked around $\frac{1}{2}$, is equally likely to be any value between 0 and 1! Thus the rms fluctuations of β, $\delta\beta_{\rm rms} = [\langle(\beta-\langle\beta\rangle)^2\rangle]^{1/2}$, rather than being near 0, should approach $1/(12)^{1/2}$. We

have computed β numerically for 2000 samples using $\Delta T/T \cong 30\%$. Even for this rather large relative temperature difference which is easily attainable experimentally, the fluctuations in β are found to be almost precisely $1/(12)^{1/2}$ (see Fig. 2), confirming the above analytical result. A well-resolved experiment should be capable of measuring $\beta(T)$ and observing these fluctuations.

These variations in $\varphi(T)$ imply that there are strong differences between $G(T)$ measured for different samples. Consequently, the *macroscopic* measurement of $G(T)$ contains information about the *microscopic* details of the resistor being studied. This is brought out by Fig. 1 in $\ln\varphi$ vs $\ln M$ for several samples. Observe that in the range of $M \sim 10^7$, the slope of the curve marked α_2 is larger than the average, that is, $\beta(M) \approx 1$. From Eq. (7) we can see that this implies that there is an extraordinarily low concentration of strong eigenstate resonances in the region $n \sim (10^7)^{1/2}$ around ϵ_F. Conversely, $\beta(M) \approx 0$, as occurs for $M \sim 10^6$ on sample α_2, implies strong resonances at the energies corresponding to $n \sim (10^6)^{1/2}$. Generally speaking, the measurement of $\beta(M)$ over a large range of M should provide rather detailed information about the zero-temperature eigenstate spectrum.

Finally, it is interesting to reconsider the case $\tau_{in} \ll L/v$, in which the existing multiple-hopping model may be thought to be correct. In the language of the present model, the "hops" are still identified with eigenstate tunneling; the distance of the tunneling, instead of being over the whole length of the sample L, is over the length $v\tau_{in}$. This is somewhat different from Mott's variable-range hopping, in which the hop distance depends on L_0, the localization length. Since the carrier will perform a random walk of hops through the sample, the number of hops needed to travel the entire sample in the present model will be $N = (L/v\tau_{in})^2$. Since each hop is independent, the conductance is determined by a summing over N independent expressions of the form of Eq. (6) with L replaced by $v\tau_{in}$. This implies that the mean results for $G(T)$ given above for $\tau_{in} \gg L/v$ will remain true for $\tau_{in} \ll L/v$, but that all fluctuations will be reduced by the factor $1/\sqrt{N} = v\tau_{in}/L$. So as expected, when inelastic scattering is important, almost all of the zero-temperature fluctuations are suppressed.

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