Laser Absorption and Heat Transport by Non-Maxwell-Boltzmann Electron Distributions

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A reduced Fokker-Planck theory of Coulomb collisional electrons in high-Z laser-fusion plasmas is found to give self-consistent results of lower than classical absorption and heat transport in numerical calculations characterized by laboratory parameters. These results are determined by non-Maxwell-Boltzmann distributions which are established when laser absorption is balanced by heat transport.

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The failure of classical absorption and heat-transport theory[1,2] to explain laboratory experience with high-Z laser-fusion plasmas is well known.[3] The theory overestimates both laser-beam absorption and electron heat transport into the irradiated target. *Ad hoc* normalization of the classical theory implemented in computer simulation codes is required to replicate experimental data. In particular, heat transport is reduced by employing the flux-limited quantity $Q_{\rm fl} = Q/[1+|Q|/F_{\rm fl}n(T/m)^{1/2}T]$ in place of Q, the classical heat flux. When Q is unsuitably large heat is transported according to $Q_{\rm fl} \sim F_{\rm fl} \times n(T/m)^{1/2}T$. The phenomenological flux limit parameter, $F_{\rm fl}$, often $\sim \frac{1}{20}$, assumes considerable significance as it represents the rate and thereby the efficiency with which the energy to do the work of driving the target may be transported.

Naturally, there has been reconsideration[4] of the classical theory and improvements[5-7] which relax its limitations. Indeed, classical heat-transport theory ought not be expected to describe laser-fusion plasmas because it fails to self-consistently govern the higher-energy electrons of the assumed Maxwell-Boltzmann distribution which transport the heat. The same statement is made of classical absorption theory and the relatively lower-energy electrons which absorb the energy from the laser beam.

Here I show that although electron collision times are short compared with the time scales of interest their mean free paths are comparable with the scale lengths. Collisions with ions drive the distribution to near isotropy over a short distance while thermalization in energy due to self-collisions occurs over a significant distance. I present a new kinetic, reduced Fokker-Planck theory of electron transport which provides this first improvement to the classical theory. The present theory aims to be the counterpart in laser fusion of the neoclassical transport theory in magnetic fusion in that we seek a solution of the Coulomb collisional transport equation for systems of programmatic interest.[8] I have obtained self-consistent results of lower than classical absorption and heat transport in numerical calculations characterized by laboratory parameters. These results are determined by non-Maxwell-Boltzmann distributions which are established when laser absorption is balanced by heat transport.

In one spatial dimension the Fokker-Planck equation is solved by ordering electron-ion collisions dominant so that the distribution is nearly isotropic:

$$f = f_0(\epsilon, x, t) + \mu f_1(\epsilon, x, t) + P_2(\mu) f_2(\epsilon, x, t) + \cdots.$$

Here $\epsilon = mv^2/2 - e\varphi$, where $\varphi(x,t)$ is the electrostatic potential and $\mu = \hat{x}\cdot\hat{v}$ is also the first Legendre polynomial. We have[2]

$$\frac{\partial f}{\partial t} - \left(\frac{\partial e\varphi}{\partial t}\right)\frac{\partial f}{\partial \epsilon} + \mu v \frac{\partial f}{\partial x} + \left(\frac{\partial e\varphi}{\partial x}\right)\frac{(1-\mu^2)}{mv}\frac{\partial f}{\partial \mu} = \left(\frac{v}{\lambda_{ei}}\right)\left(\frac{\partial}{\partial \mu}\right)\left((1-\mu^2)\frac{\partial f}{\partial \mu}\right) + \frac{df}{dt_{ee}}.$$

Here $v = [2(\epsilon + e\varphi)/m]^{1/2}$ and $\lambda_{ei} = m^2v^4/2\pi e^4 Z^2 N \times \ln\Lambda_{ei}$. Terms associated with ion motion are neglected as small in U/v. Fluxes are calculated from f_1 which obeys[2]

$$v\,\partial f_0/\partial x = -(2v/\lambda_{ei}) f_1. \tag{1}$$

Several terms have been dropped here. Electron-electron terms are small by $1/Z$ compared with the electron-ion term retained. The term in f_2 is neglected in comparison with the term in f_0. Finally, the rates of temporal variation of f_1 and $e\varphi$ are assumed slow compared with the electron-ion collision rate.

Classical transport[1,2] is obtained by use of the

Maxwell-Boltzmann distribution, $n/(2\pi T/m)^{-3/2} \times\exp[-(\epsilon+e\varphi)/T]$, for f_0 in Eq. (1) to evaluate the heat flux due to a temperature gradient. The potential is determined by quasineutrality through the condition that the total electron-number flux vanishes:

$$\Gamma=(4\pi/3)\int_{-e\varphi}^{\infty} d\epsilon(v/m)v[f_1=-(\lambda_{ei}/2)\partial f_0/\partial x]=0;$$
$$\partial e\varphi/\partial x=(\tfrac{5}{2})\partial T/\partial x+T\,\partial \ln N/\partial x$$

so that

$$f_1=\frac{2\pi^{1/2}}{32}\,\frac{Q}{n(T/m)^{1/2}T}\left(\frac{mv^2}{2T}\right)^2\left(\frac{mv^2}{2T}-4\right)f_0 .$$

The heat flux carried by electrons of energy less than ϵ^* is

$$Q^*=(4\pi/3)\int_{-e\varphi}^{\epsilon^*} d\epsilon(v/m)v(mv^2/2)f_1$$

and the classical heat flux

$$Q=\lim(\epsilon^*\to\infty)Q^*$$
$$=-[64/(2\pi)^{1/2}]n(T/m)^{1/2}\lambda_{\rm mfp}\,\partial T/\partial x .$$

Here $\lambda_{\rm mfp}=(T/mv^2)^2\lambda_{ei}$. For the sake of definiteness, we adopt $|f_1(\epsilon^*)/3|<f_0(\epsilon^*)$, where $Q^*(\epsilon^*)=0.9Q$ as necessary for the self-consistent validity of heat-transport theory employing Eq. (1).[8] The inequality expresses the physical requirement of finite flux in $d\epsilon^*$. For the classical theory we find $\epsilon^*=11.4T$ and $|f_1^*/3|=25[|Q|/n(T/m)^{1/2}Tf_0^*<f_0^*$ for $|Q|/n(T/m)^{1/2}T<0.04$ or $\lambda_{\rm mfp}|\partial\ln T/\partial x|<0.002$. This inequality is ubiquitously violated in laser-fusion plasmas. For the parameters $Q=10^{14}$ W/cm^2, $n=10^{21}$/cm^3, and $T=10^3$ eV, $|f_1^*/3|\sim 13f_0^*$!

Previous efforts to improve heat-transport theory essentially involve improving Eq. (1). Greater μ structure has been included in studying idealized systems wherein f_0 is a Maxwell-Boltzmann distribution in a hot region from which heat flows into an adjacent cold region.[6]

It turns out that for modeling laser-fusion plasmas Eq. (1) is adequate to describe f_1; our new theory involves the calculation of the isotropic but not necessarily Maxwell-Boltzmann distribution, f_0, which obeys[2]

$$\frac{\partial}{\partial t}(vf_0)-\left(\frac{\partial}{\partial t}e\varphi\right)\frac{\partial}{\partial\epsilon}(vf_0)+\frac{\partial}{\partial x}\left(\frac{v^2f_1}{3}=-\frac{v^2\lambda_{ei}}{6}\frac{\partial f_0}{\partial x}\right)$$
$$=Ym\ln\Lambda_{ee}\frac{\partial}{\partial\epsilon}\left(f_0 4\pi\int_{-e\varphi}^{\epsilon}\frac{d\epsilon'v'}{m}f_0'+\frac{\partial f_0}{\partial\epsilon}4\pi\int_{-e\varphi}^{\epsilon}\frac{d\epsilon'v'}{m}\int_{\epsilon'}^{\infty}d\epsilon''f_0''\right)+v_{\rm laser}^2\frac{m^2}{6}YZ^2N\ln\Lambda_{ei}\frac{\partial^2f_0}{\partial\epsilon^2}. \qquad (2)$$

First on the right-hand side of Eq. (2) is the self-thermalization term which fails to be dominant in laser-fusion plasmas so that f_0 is non-Maxwell-Boltzmann and has finite flux in $d\epsilon^*$ according to Eq. (1). Here $Y=4\pi e^4/m^2$. Second is the laser-heating term[1,2] recently investigated by Langdon.[7]

Equation (2) is coupled to a WKB equation for the electron oscillating velocity, $v_\pm$, in the incident and returning laser beam of intensity $I_\pm=n_c(mv_\pm^2/2)v_g$:

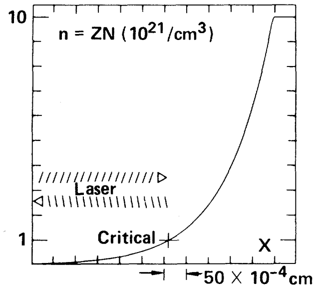
$$\pm\frac{\partial}{\partial x}(v_g v_\pm^2)=-\tfrac{1}{3}Y\frac{Z^2N}{n_c}\ln\Lambda_{ei}f_0(\epsilon=-e\varphi)v_\pm^2 . \qquad (3)$$

Here v_g is the group velocity of light and n_c is the laser critical density. In Eq. (2), $v_{\rm laser}^2=v_+^2+v_-^2$. Note that the laser opacity depends upon the distribution at zero velocity. The potential is determined by quasineutrality:

$$n=4\pi\int_{-e\varphi}^{\infty}d\epsilon(v/m)f_0=ZN. \qquad (4)$$

Figure 1 is an overview of three numerical calculations of laser-fusion plasmas according to Eqs. (2)–(4). The ionization state, Z, is a constant in each calculation in which a constant-intensity laser beam irradiates a fixed plasma until the heat transport out of the absorbing subcritical region into the colder supercritical region equals the (slowly varying) absorbed laser intensity. At this time the subcritical plasma has reached a quasisteady state which is taken to be the conclusion of the absorption and transport calculations. Beyond this state the hydrodynamic ablation should be included. However, the subcritical plasma may not be influenced by this except as it determines the density scale length. Thus the present calculations may reveal the long-time-scale subcritical plasma state.

Inspection of Fig. 1 shows that *classical* calculations are significantly in error in finding excessive absorption and heat transport. *Mixed* calculations in which the kinetically absorbed energy is the source in a classical heat-transport calculation also find excessive heat trans-



$\lambda_{laser} = 1.0 \times 10^{-4}$cm
$n_{critical} = 1.1 \times 10^{21}/\mathrm{cm}^3$
$\left(\frac{d}{dx}\ln n\right)^{-1} = 100 \times 10^{-4}$cm
$n_{min} = 5 \times 10^{19}/\mathrm{cm}^3$
$T_{initial} = 0.1$ keV
$v_{group,min} = c/\sqrt{2}$

	$Z = 10$ $I = 10^{14}$W/cm^2	$Z = 10$ $I = 10^{15}$W/cm^2	$Z = 40$ $I = 10^{14}$W/cm^2
	classical kinetic mixed	classical kinetic mixed	classical kinetic mixed
t_{data} (sec)	125×10^{-12}	41×10^{-12}	168×10^{-12}
$f_{absorbed}$	0.63 0.46	0.36 0.19	0.84 0.63
T_{corona} (keV)	1.28 1.33 1.16	2.20 2.28 1.76	2.05 2.20 1.85
$Q_{max}/n\sqrt{T/m}\,T$	0.19 0.14 0.16	0.48 0.26 0.36	0.13 0.09 0.11
f_1^*/f_0^* @ Q_{max}	14 1.4 12	35 1.4 27	9.4 0.70 8.3
$Q_{max}^{kin}/Q^{class}(T^{kin})$	0.34	0.17	0.31
$F_{flux\ limit}$ @ Q_{max}	0.21	0.32	0.13

FIG. 1. Common conditions and overview at the conclusion of three numerical calculations of laser-irradiated plasmas. In the expression for λ_{ei} and in Eqs. (2) and (3), Z^2N is replaced by $Z(Z+1)N$. The Coulomb logarithms are $\ln\Lambda_{ei} = 2.5$ and $\ln\Lambda_{ee} = 5$ throughout.

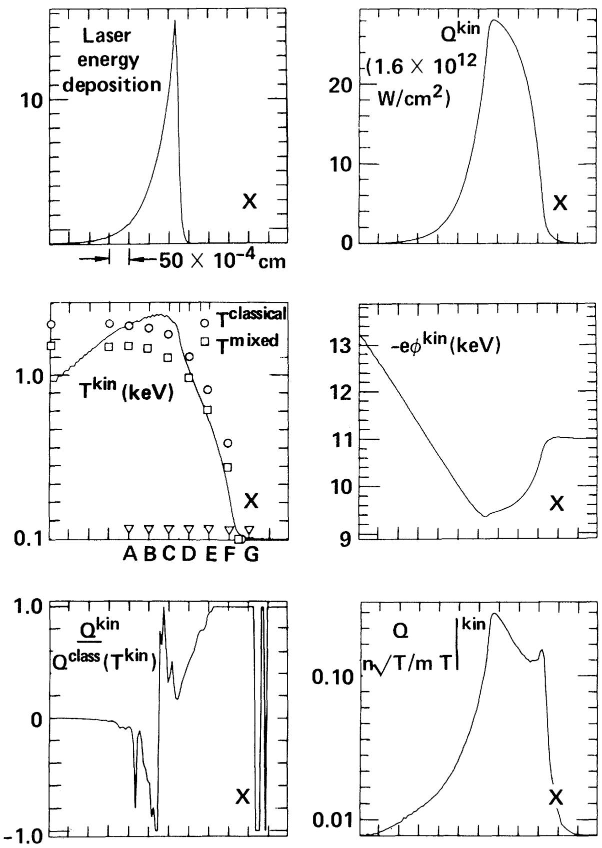


FIG. 2. Macroscopic plasma properties at the conclusion of the $Z = 10$, $I = 10^{14}$-W/cm^2 calculation.

port. The new *kinetic* calculations are a vast improvement over the classical ones and are fully self-consistent according to the condition defined after Eq. (1).

In the following I display and analyze the plasma state at the conclusion of the $Z = 10$, $I = 10^{14}$-W/cm^2 calculation. The greater penetration of the classical and mixed temperature fronts is evident in Fig. 2 and results in comparable or lower subcritical temperature for greater or equal absorbed energy. Near the peak of the heat flux the classical heat flux associated with the kinetic temperature profile is considerably greater than the kinetic heat flux. This is due to the fractionally higher temperature which enters as $T^{7/2}$ in the classical heat flux and also to the steeper gradient. We do not feel that Q_{fl} offers a hopeful transport treatment; nevertheless, we have evaluated F_{fl} which is required in Q_{fl} to match simultaneously the kinetic temperature and heat flux; near the peak of the heat flux $F_{fl} \sim \frac{1}{5}$.

In Fig. 3 the electron distribution at various positions is shown together with the local Maxwell-Boltzmann distribution. The subcritical plasma is characterized by the distribution at $x = B$. The flow in phase space there is upwards in energy as a result of laser heating and/or self-collisions and then into the colder denser plasma as a result of spatial transport. The electric field induces a return flow of lower-energy electrons from the transport-heated plasma to the laser-heated plasma. The quasi-steady distributions in the subcritical plasma result from the balance between laser heating at lower energy and spatial transport at higher energy with self-collisions mediating. Previous work on absorption involved similar but nonsteady distributions which evolved under heating and self-collisions only,[7] and so did not address this physics. Laser heating diffuses electrons away

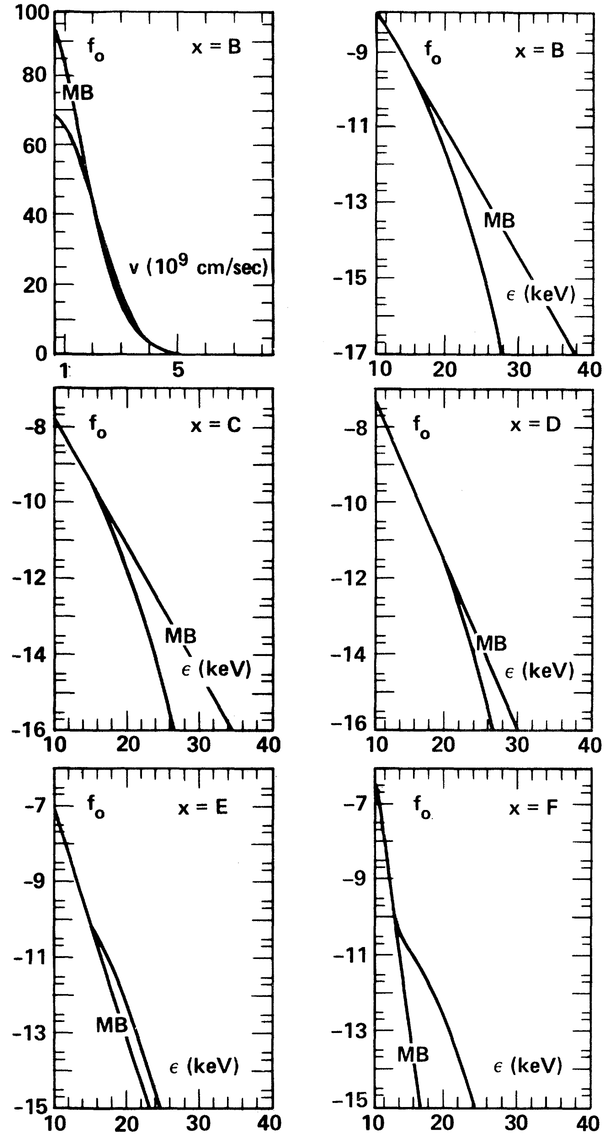


FIG. 3. Electron distribution functions and local Maxwell-Boltzmann (MB) distributions at various positions as labeled on the temperature profile in Fig. 2.

from low energy sufficiently rapidly that self-collisions cannot establish a Maxwell-Boltzmann distribution there. The reduced opacity to the laser beam results from the relative depopulation of low-energy electrons. In the laser-heated plasma the thermalization mean free path of the higher-energy electrons is long enough so that they transport spatially before self-collisions can establish a Maxwell-Boltzmann tail. It is intuitively satisfying that the heat flux associated with the resulting truncated distributions is less than that associated with local Maxwell-Boltzmann distributions. Above critical density the distribution relaxes to a Maxwell-Boltzmann distribution at low energy more rapidly than at high energy. The tail of the subcritical distribution penetrates as preheat as evidenced by the two-component distribution at $x = F$. Except in the preheat region, we find underfull tails on the distribution and not the overfull ones of previous work.[6]

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