Structure of Matter below Nuclear Saturation Density

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It is found that just below nuclear saturation density more stable forms of dense matter exist than the near-spherical nuclei or bubbles customarily assumed. Because of the large effect of the Coulomb lattice energy, cylindrical and planar geometries can occur, both as nuclei and as bubbles. It is suggested that in order to approximate more complicated kinds of short-range order, the dimensionality should be regarded as a continuous variable ranging from $d=3$ (spheres) to $d=1$ (planes). The dependence of d on density is illustrated, and its dependence on nuclear models discussed.

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In the process of stellar collapse, star material is compressed through many orders of magnitude in density at rather low entropy.[1,2] Modification in the physical structure of the matter during part of this process may affect significantly the ability of this mechanism to explain the origin of supernovae.[3] Within the density range from $\sim n_s/10$ to n_s (n_s being the nuclear saturation density) it is believed that dense matter may exist in other phases besides the customary one of nuclei immersed in a nucleon vapor. The other phase discussed in some detail has been the bubble phase, in which the vapor fills spherical holes in dense nuclear matter.[4-9] The purpose of this Letter is to enlarge the subject by discussing other configurations of matter. We find that over a wide density range, the energetically favored phases have geometries of quite different character.

The use of spherical geometry in discussions of dense matter derives from the knowledge that ordinary nuclei are more or less spherical. While not disputing this fact, we observe that in the density range we have cited, where the fraction of space filled by dense matter, u, ranges from 0.1 to 1, the contribution to the Coulomb energy of the system coming from neighboring nuclei (the so-called Coulomb lattice energy) rivals in importance the nuclear Coulomb self-energy.[9] Since it is the predominance at low densities of nuclear self-energies (surface and Coulomb) which produces sphericity (or almost sphericity), it is therefore not absurd to explore other geometries for dense matter.

The lattice contribution to the Coulomb energy is included in the Wigner-Seitz approach by considering a unit cell of matter which has zero net charge. We describe this unit cell in terms of the compressible-liquid-drop model, which we believe contains the essential physics. We shall further simplify that model by assuming that the density of the vapor phase is negligibly small, and that the nuclear surface tension σ depends only on the proton fraction x of the dense matter. We shall ignore all specifically temperature-dependent effects. In the nuclei phase the unit cell, of radius r_c, thus contains a nucleus of radius r, density n', and charge density xn'. In the bubble phase there is a void of radius r surrounded by dense matter out to radius r_c. While until now the geometry has been assumed to be three dimensional, so that the boundaries are spheres, it is possible to consider also two-dimensional geometry, corresponding to cyclindrical boundaries, and one-dimensional geometry, with planes (where the radius becomes the half width). In the last-mentioned case the nuclei and bubble phases are identical.

It is straightforward to obtain the surface and Coulomb energies in the cyclindrical and planar geometries, to supplement the $d=3$ result.[10] If the dimensionalities are denoted by $d=3$, 2, and 1 the results for all of these geometries may be represented by single expressions. Thus the surface energy *per unit volume of the matter* may be expressed as

$$E_s = u\sigma d/r, \tag{1}$$

and the Coulomb energy per unit volume as

$$E_C = 2\pi n'^2 x^2 e^2 r^2 u f_d(u), \tag{2a}$$

where

$$f_d(u) = \{2/(d-2)(1-\tfrac{1}{2}du^{1-2/d}) + u\}/(d+2). \tag{2b}$$

(In the case $d=2$ the limit yields the correct expression containing a logarithm.) The filling factor u, which is directly related to the average density n and the dense-phase density n' by $u=n/n'$, depends on the radii r and r_c according to $u=(r/r_c)^d$. The toal energy per unit volume is

$$E_{\text{tot}} = E_e + u\tilde{E}(n') + E_s + E_C, \tag{3}$$

where E_e is the electron energy and $\tilde{E}$ is the bulk energy of the dense phase. The latter will for simplicity be approximated when necessary by

$$\tilde{E}(n') = n'\{\tilde{E}_0 + (K_s/18)(1-n'/n_s)^2\}, \tag{4}$$

where $\tilde{E}_0$, the binding energy per baryon, K_s, the compressibility parameter, and n_s, the saturation density, have values appropriate to the proton fraction x.[11]

Thermodynamics requires that E_{tot} be stationary with respect to variations in n' and r at a given density n. The variation with respect to r produces immediately

$$E_s = 2E_C, \tag{5}$$

a result familiar for $d=3$,[10] but now seen to be independent of dimensionality. At the minimum, the radius of the nucleus is given by

$$r^{-3} = (4\pi n'^2 x^2 e^2/\sigma d) f_d(u),$$

so that the sum of surface and Coulomb energies per unit volume is for nuclei

$$E_{s+C} = n_s \zeta_s (n'/n_s)^{2/3} g_d(u), \tag{6}$$

where $\zeta_s = 3[\frac{9}{5}\pi x^2 e^2 \sigma^2/n_s]^{1/3}$ and $g_d(u) = u[5d^2 f_d(u)/18]^{1/3}$. The variation with respect to n' produces the pressure-equilibrium condition which, if we assume (4) for simplicity, becomes

$$\frac{n'}{n_s} = 1 - \frac{9\zeta_s}{K_s}\left(-g_d'(u) + \frac{2g_d(u)}{3u}\right)\left(\frac{n'}{n_s}\right)^{-4/3} \tag{7}$$

for nuclei. [The results for bubbles corresponding to Eqs. (6) and (7) are similar but not identical to these expressions.]

We determine approximately which of the phases is most stable at a given density by looking for the lowest E_{tot}. (For reasons given later, it is not necessary to examine the phase transitions in more detail.) The nuclear parts of the energies of the five phases, after subtraction of a common term, are illustrated in Fig. 1(a). It is seen that although the $3N$ phase (three-dimensional nuclei) is the most stable configuration at low densities, as we certainly expect, at a quite low density the $2N$ phase appears, followed by the $1NB$. This phase, which is nuclei and bubbles at the same time, then gives way to $2B$ and to $3B$ as the density further increases. There is finally a phase change to uniform matter at $n/n_s \sim 0.85$. Thus the transition from $3N$ to $3B$ has been made in a series of smaller transitions, going through

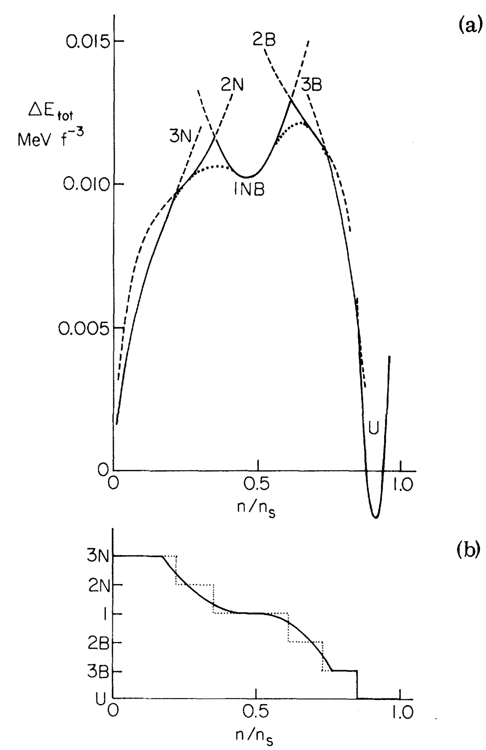

FIG. 1. (a) A plot of E_{tot} vs n/n_s for the five phases $3N$, $2N$, $1NB$, $2B$, and $3B$, and the uniform matter phase. Each is shown as a dashed curve except for the region in which it is the most stable phase, where it is shown as a full curve. The dotted curves show E_{tot} for the continuous dimensionality phase. For illustrative purposes a common background function of n/n_s has been subtracted. (b) The continuous dimensionality d vs n/n_s (full curve). The dotted lines correspond to the energy crossings of the discrete-d phases.

quite different geometries, and covering a considerably larger density range.

It will be interesting to explore the consequences of these spaghettilike and lasagnalike phases of dense matter. Their physical properties will have to reflect the great departure from isotropy that these phases possess. Neutrino scattering processes, an important ingredient in the stellar collapse process, may well exhibit the pronounced diffractive effects characteristic of these geometries. (Neutrino wave numbers are of order 0.1 to 0.5 fm^{-1}, and the widths of the one-dimensional slabs are of order 10 fm.)

The planar, cylindrical, and spherical geometries we have considered are clearly oversimplifications of the real situation. We have in mind more complicated shapes for the regions occupied by nuclear matter. For example, in considering the $3N$ phase one can well imagine the generation, by the close proximity of neighbors in the lattice, of quadrupole deformations which turn the spherical nuclei into prolate spheroids. As the density increases, these spheroids will touch and fuse into tubes of dense matter of varying radii. Similarly, the tubes may deform, touch, and become nonuniform plates. We propose to generalize our procedure to try to accommodate these more general shapes. The results contained in Eqs. (1) and (2) were not guessed as portmanteau formulas for the three special cases $d=3$, 2, and 1, but were derived for a geometry of general dimensionality d. We believe that it is useful to regard d as a continuous variable. All of the results we have quoted are still mathematically valid, and the expressions vary smoothly with d. They provide an energy expression which may represent the continuum of deformed configurations intermediate between the geometrically perfect cases discussed earlier.

If this procedure is followed, then at a given density the appropriate "phase" is to be obtained by minimizing the energy with respect to the dimensionality d. We have not found a simple way to express the minimum in closed form, but the numerical minimization is trivial, and the results are shown in Fig. 1(b). The dimensionality varies continuously from $3N$ through $d=1$ and then on to $3B$. The surface-plus-Coulomb energy has a symmetry between u for nuclei and $q=1-u$ for bubbles, so that in the approximation that the dense matter is incompressible ($n'=n_s$) the curve in Fig. 1(b) will become symmetrical about $n/n_s = \frac{1}{2}$. [The subtraction used to exhibit the small energy differences of Fig. 1(a) does not possess this symmetry.]

There remains a first-order phase transition between $3B$ and the uniform-matter phase, for reasons given elsewhere.[9] But over most of the density range the transitions are smooth, reflecting what we believe to be the actual physical progression of configuration changes. The incorporation of such effects into the equation of state used in stellar-collapse calculations would produce a natural smoothing of the present phase changes from three-dimensional nuclei through three-dimensional bubbles, thereby making less violent the resulting "ripple."

A correction to the Wigner-Seitz approximation comes from the calculation of the long-range part of the Coulomb energy for a lattice, rather than for a cell of idealized geometry. The correction is small, and slightly enhances the energy differences in favor of the phases we propose. For $d=3$ the body-centered cubic lattice, the most stable, has a lattice Coulomb energy 0.45% higher than that of the Wigner-Seitz cell.[10] For $d=1$ there is no correction. (We are working on the $d=2$ correction, but it is not expected to be out of line with the $d=3$ and 1 results.) Our calculation also neglected screening of the Coulomb interaction by the electrons. We calculate that the corrections due to screening are, at $u=0.1$, -3.3% of the Coulomb energy for $d=3N$ and -7.3% for $d=1$. They are thus not negligible, and they favor the lower values of d.

Without going into details we observe that the results presented are model dependent to the extent that we have used a particular nuclear Hamiltonian.[11] The dependence of the physics on $9\zeta_s/K_s$, i.e., on the ratio of surface-plus-Coulomb energy to the compressibility parameter,[9] implies that with a softer interaction, as used, e.g., in Ref. 5, the transition at the upper end of the density range could well be from uniform matter to bubbles with $d<3$, i.e., to very deformed bubbles. An effect going in the other direction, i.e., making $3B$ more stable, is the inclusion of surface curvature.[12] Curvature will also push the onset of the initial departure of d from $3N$ to lower densities.

Some temperature-dependent corrections which are not known exactly even for $d=3$ (Coulomb and translation energies) presumably will be modified for $d<3$, and need study. In the $3N$ and $3B$ geometries the $T=0$ behavior is found to be insensitive to T up to temperatures which are a reasonable fraction of the critical temperature (~18 MeV at $x=0.3$). We see no reason why the new geometries should not show similar temperature properties. After all, the cooking of spaghetti, while it spoils the perfect straightness of the strands, does not destroy the characteristic short-range order.

It is clear that the numerically more elaborate Hartree-Fock[5] and Thomas-Fermi[6] calculations can be done for our generalized geometry. Although it is not clear what new significance they will have for d not an integer, they will be extremely useful for $d < 3$ as confirmation of our liquid-drop model predictions. The possibility of $d < 3$ phases, or of smaller-scale effects like them, occurring during the fragmentation part of heavy-ion collisions also should not be overlooked.

In neutron-star matter the presence of an appreciable vapor of dripped neutrons (whose density we could neglect at low temperatures for $Y_e = 0.3$) shifts the lower end of the range of expected transition densities to $\sim 0.2(n' + n_{\text{vapor}})$, while the upper end remains somewhat below n_s. The span of densities is thus somewhat reduced. Also there may still exist regular phases with more complex nuclear/bubble arrangements than the 3N, etc., discussed initially. Transitions involving them would presumably be of first order, and involve density discontinuities, although on a reduced scale.

Consideration of some consequences of the lower dimensionality on the physical properties of the transition from uniform nuclear matter to quark matter at higher densities is in progress.

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[11]For consistency with the earlier work of some of us (see, e.g., Ref. 9) we use here the Skyrme 1′ interaction. In the customary notation its parameters are $t_0 = -1057.3$ MeV fm^{-3}, $t_1 = 235.9$ MeV fm^{-5}, $t_2 = -100$ MeV fm^{-5}, $t_3 = 14\,463.5$ MeV fm^{-6}, $x_0 = 0.2885$, $x_1 = x_2 = 0$, and $x_3 = 0.2557$. At a proton fraction of $x = 0.3$, a typical value in stellar collapse, $\tilde{E}_0 = -11.4$ MeV, $K_s = 291$ MeV, and $n_s = 0.147$ fm^{-3}. The corresponding surface tension is $\sigma = 0.74$ MeV fm^{-2}, so that the surface-plus-Coulomb energy parameter ζ_s is 4.18 MeV.

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