Inelastic p-Air Cross Section at Energies between 10^{16} and 10^{18} eV Estimated from Air-Shower Experiments

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The inelastic cross section of p-air collisions [$\sigma_{\rm in}(p\text{-air})$] between 10^{16} and 10^{18} eV is estimated by the observation of extensive air showers at Akeno. The flux of air showers at different zenith angles is analyzed with both a fixed muon number and a fixed electron number. $\sigma_{\rm in}(p\text{-air})$ increases with energy as $290\times E_{\rm lab}^{0.06\pm0.01}$ mb ($E_{\rm lab}$ in teraelectronvolts) up to 10^{18} eV with the assumptions of Feynman scaling in the fragmentation region and at least 10% of the primary particles being protons.

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Recent results from the CERN super proton synchrotron (SPS) collider give the total cross section of p-p collisions, (66 ± 7) mb, at 150 TeV in the laboratory system.[1] This fits the extrapolation from lower energies which was derived by Yodh, Pal, and Trefil[2] from the cross section for p-air collisions based on measurements of unaccompanied hadron spectra at a high altitude.[3] In the energy region higher than the SPS collider, several attempts have been made to infer the values of the cross section in cosmic-ray experiments.[4,5] In this paper we derive the collisional mean free path for p-air interactions in the energy range between 10^{16} and 10^{18} eV.

The total number of muons (N_μ) in an extensive air shower (EAS) remains almost constant after reaching its maximum and hence it is a good measure of the primary energy at any observation level beyond that point. On the other hand, the total number of electrons (N_e) is attenuated after its maximum development, so that its size and lateral distribution reflect the stage of development. With a fixed number of muons we can select showers of the same primary energy, and with the further condition of a fixed number of electrons we can choose showers of the same depth between the first interaction point and the observation level, under the assumption of no fluctuations in the successive interactions. By measuring the attenuation of frequencies of showers with fixed N_e and N_μ at different zenith angles, we can estimate the collisional mean free path of the primary particles in the atmosphere as was done by Fukui *et al.*[6]

The Akeno EAS array[7] is at 900 m above sea level. The detectors relevant to the present investigation are as follows: 150 scintillation detectors each of 1 m^2 area, 6 of 2 m^2, and 9 muon detectors each of 25 m^2 are distributed over an area of 1 km^2. The threshold energy of muons is $1.0\sec\theta$ GeV, where θ is the incident zenith angle. Arrival directions of showers are determined by the fast timing method with an accuracy of 4.0°, using 53 scintillation detectors. The standard deviations in the determination of N_e and N_μ are 0.05 and 0.15, respectively, in logarithmic scale. Details of the experiment and the method of analysis are discussed by Tan, Nagano, and Kawaguchi[8] and Hara *et al.*[9]

The total observation time to date is 15 288 h and more than 500 000 showers have been recorded. About 60 000 of these showers whose cores hit inside an area where the detection efficiency is 100% are used for the present analysis.

In order to determine the cross section for p-air collisions, a certain amount of protons must exist in the primary cosmic-ray beam. We assume that at least 10% of the primaries are protons, which is compatible with the recent observations up to 10^{16} eV.[4,10,11] Above 10^{16} eV many EAS experiments can be explained by this assumption.[12-14]

Air showers initiated by heavier primaries have generally smaller N_e at the Akeno level, when fixed-N_μ showers are selected. In order to bias our analysis towards showers which have a high probability of having been produced by a proton we make a cut on our data as follows. At fixed N_μ, we have selected the showers with N_e larger than a certain threshold value of N_e independent of zenith angle. The threshold value of N_e is determined so that selected showers are 10% of the vertical showers with fixed N_μ. The observed attenuation length (λ_{obs}) is determined from the intensity variation with zenith angle. With a fixed primary energy, however, N_μ changes slightly with zenith angle θ, since both the threshold energy and the decay probability of muons increases with zenith angle. This is taken into consideration in the analysis by the equal-intensity method. The zenith-angle dependence of the flux is plotted in Fig. 1.

The values λ_{obs} (in grams per square centimeter) have been calculated from a weighted least-squares fitting of the fluxes by the equation

$$f(\theta) = f(0)\exp[-x_0(\sec\theta - 1)/\lambda_{obs}],$$

where $f(\theta)$ is the flux at a zenith angle θ and x_0 is the atmospheric depth at Akeno (920 g/cm^2).

As a result of the dispersion in zenith-angle determination, the fluxes at deeper atmospheric depths are overestimated and hence λ_{obs} is also. Thus corrected $\lambda_{obs}{}^c$ are listed together with λ_{obs} in Table I. The cross section σ_{obs} has been calculated from (p-air)(mb) = $2.41\times10^4\ \lambda_{obs}{}^c$(g/cm^2). The λ_{obs} are insensitive to the threshold cut (N^{th}) on N_e within experimental errors up to $R = (10-20)\%$, where R is the ratio of showers larger than N^{th} to total showers for fixed N_μ.

In order to confirm that we are selecting showers at the same stage of longitudinal development, the shower ages for the different zenith angles have been compared with each other. The average age of the selected showers is indeed independent of depth for fixed N_μ and N_e, suggesting an equal degree of development.

If the longitudinal development of each shower would not fluctuate after the first interaction, $\lambda_{obs}{}^c$ could give the true value of the mean free path. However, the fluctuations from shower to shower give rise to an error in the estimation of the mean free path. We simulated proton showers to find out the effect of the fluctuation on λ_{obs}. Calculations were made for the energy-dependent cross sections for p-air in the cases of both an $E^{0.03}$ and an $E^{0.06}$ dependence, with the assumption of Feynman scaling in the fragmentation region ($x \geq 0.05$). A multiplicity increasing as

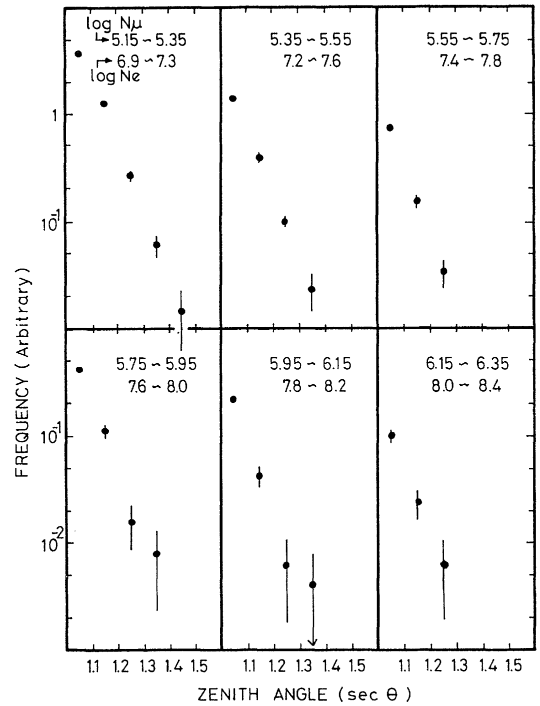

FIG. 1. Zenith-angle dependence of the flux for six muon-number ($\log_{10}N_\mu$) ranges. The logarithm of the electron numbers used for each N_μ are also shown.

TABLE I. Measured inelastic p-air cross sections $\sigma_{in}(p\text{-air})$ in each muon-number and electron-number bin listed together with their estimated energies. λ_{obs}, observed attenuation length; $\lambda_{obs}{}^c$, corrected for uncertainty in zenith-angle determination; σ_{obs}, cross section before correction due to the fluctuation of shower development.

$\log_{10}N_\mu$	$\log_{10}N_e$	Estimated energy $\log_{10}E$ (eV)	λ_{obs} (g/cm^2)	$\lambda_{obs}{}^c$ (g/cm^2)	σ_{obs} (mb)	$\sigma_{in}(p\text{-air})$ (mb)
5.15–5.35	6.9–7.3	16.25–16.5	73.1 ± 6.9	67.1 ± 6.9	359 ± 37	539 ± 56
5.35–5.55	7.2–7.6	16.55–16.8	70.2 ± 1.4	64.2 ± 1.4	375 ± 8	562 ± 12
5.55–5.75	7.4–7.8	16.85–17.1	60.2 ± 3.7	54.2 ± 3.7	445 ± 30	667 ± 45
5.75–5.95	7.6–8.0	17.15–17.4	66.0 ± 7.3	60.0 ± 7.3	401 ± 49	601 ± 74
5.95–6.15	7.8–8.2	17.45–17.7	59.7 ± 7.1	53.7 ± 7.1	449 ± 59	674 ± 89
6.15–6.35	8.0–8.4	17.75–18.0	65.9 ± 5.9	59.9 ± 5.9	402 ± 40	603 ± 60

TABLE II. Calculated attenuation lengths λ_{sim} from simulated showers compared with the input values λ_{input} for two energy-dependent cross sections which increase as $E^{0.03}$ and $E^{0.06}$ at 10^{16} and 10^{17} eV, respectively. The experimental values $\lambda_{obs}{}^c$ are also shown for comparison.

Primary energy of proton (eV)	Energy dependence of cross section (E in TeV)	λ_{input} (g/cm^2)	λ_{sim} (g/cm^2)	$\lambda_{obs}{}^c$ (g/cm^2)
10^{16}	$E^{0.03}$	63.0	93 ± 4	70 ± 8
10^{16}	$E^{0.06}$	47.8	74 ± 8	
10^{17}	$E^{0.03}$	58.8	91 ± 5	57 ± 7
10^{17}	$E^{0.06}$	41.7	61 ± 5	

$(\ln s)^2$, corresponding to the rise of the plateau in the central region, has also been included, but this does not have a significant effect on the structure function in the fragmentation region. Details of the simulation are discussed in Kasahara, Torii, and Yuda.[15] We simulated 1000 showers for each mean free path (λ_{input}) at 10^{16} and 10^{17} eV, respectively, to get λ_{sim} in the same way as in the experiment. The results are tabulated in Table II.

Since $\lambda_{obs}{}^c$ agrees well with λ_{sim} for $E^{0.06}$, we corrected the experimental $\lambda_{obs}{}^c$ at each energy by the ratio $\lambda_{sim}/\lambda_{input} \sim 1.5$. As is seen in the table, the ratio $\lambda_{sim}/\lambda_{input}$ is almost independent of cross section. Such a large value of $\lambda_{sim}/\lambda_{input}$ is discussed by Ellsworth *et al.*[16] The inelastic cross sections $\sigma_{in}(p\text{-air})$ thus obtained are plotted by squares in Fig. 2. Horizontal bars show the ambiguity in the primary energy and the vertical bars the standard deviations in fitting to the experimental points.

The total cross section of p-$\bar{p}$ as measured at the SPS collider is converted to a corresponding p-air inelastic cross section by the formula[17] $\lambda(p\text{-air}) = 760\ [\sigma_{in}(p\text{-}p)]^{-0.63}$, where $\sigma_{in}(p\text{-}p) = 0.83\sigma_{total}(p\text{-}p)$. The value calculated from the SPS data is also plotted in Fig. 2. $\sigma_{in}(p\text{-air})$ estimated from the gamma-ray family flux at Mt. Fuji is shown by the shaded region.[4] The preliminary result of the "fly's eye" experiment[5] is shown by a large circle.

The contamination by particles other than protons makes λ_{obs} smaller than the attenuation length of protons. This effect is examined by the simulation of 1000 showers from alpha particles at 10^{17} eV. Here $\lambda_{sim}(\alpha\text{-air}) = 43.5(E/4)^{-0.06}$ g/cm^2 (E in teraelectronvolts) is assumed. It is found that even if alpha particles are of the same flux as protons in the primary beam, the cross section decreases by only about a few percent. The contribution of other particles heavier than alpha particles is negligible. So long as the fraction of protons is higher than 10% and the flux of alphas is the same as or less than that of protons, our conclusion is not affected, under the assumption of similar energy dependence of $\lambda(\alpha\text{-air})$ to that of $\lambda(p\text{-air})$.

It is worth noting that we are selecting showers near their maximum development. Since fluctuation of the depth of the shower maximum is smaller in the case of larger multiplicity in the hadronic interaction, the correction factor ($\lambda_{sim}/\lambda_{input}$) for a model with a high-multiplicity law should be smaller than in the case of scaling. If scaling

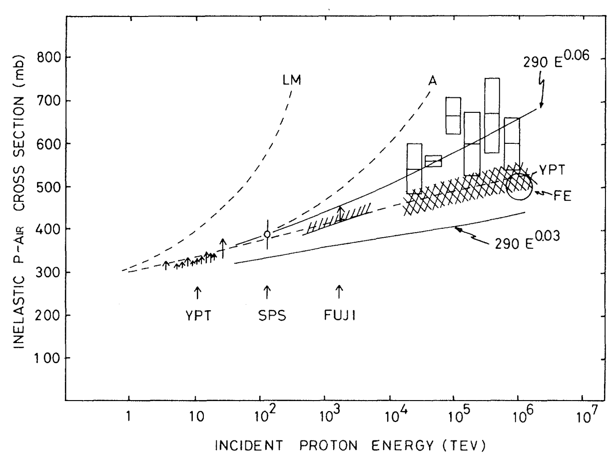

FIG. 2. Energy dependence of the inelastic p-air cross section. Other experimental data are as follows: YPT from unaccompanied hadron spectra (Yodh, Pal, and Trefil, Ref. 2); FUJI from the gamma-ray family spectrum (Ref. 4); SPS from the SPS collider (Ref. 1); and FE from the "fly's eye" experiment (Ref. 5). Solid lines are the ones assumed for the present simulation. Three types of proposed energy dependences are shown by dotted lines: LM and YPT are calculated on the basis of equations in Ref. 22; A is taken from the figure of Ref. 16.

in the fragmentation region breaks weakly as proposed by Wdowczyk and Wolfendale,[18] $\lambda_{sim}/\lambda_{input} \sim 1.3$, which is derived for the shower maximum by Stanev and Gaisser,[19] may be applied in our case and the result corrected with this value is shown by the hatched region in Fig. 2. From these considerations, the squares in Fig. 2 represent the upper bound.

The lines in Fig. 2 represent the energy dependence of σ assumed for the present simulation. The best fit is $290\times E^{0.06\pm0.01}$ up to 10^{18} eV (E in teraelectronvolts), or $290+(1.44\pm0.083)\,[\ln(s/100)]^2$ mb (s in gigaelectronvolts squared). In the figure, various types of energy dependence proposed by other authors[2,20,21] are also shown. According to the scaling-violation model discussed above, the energy dependence given by Yodh, Pal, and Trefil,[2] which is based on parametrization of Maor and Nussinov[22] may hold up to 10^{18} eV. Such a rapid increase of p-air cross section as shown by the line denoted by LM is inconsistent with the present experiment.

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[1]R. Battiston *et al.*, Phys. Lett. 117B, 126 (1982).

[2]G. B. Yodh, Yash Pal, and J. S. Trefil, Phys. Rev. Lett. 28, 1005 (1972).

[3]C. Aguirre *et al.*, Nuovo Cimento B 27, 263 (1975).

[4]M. Akashi *et al.*, Phys. Rev. D 24, 2353 (1981).

[5]G. L. Cassiday *et al.*, in Proceedings of the Workshop on Very High Energy Cosmic Ray Interactions, University of Pennsylvania, Philadelphia, 1982 (unpublished), p. 72.

[6]S. Fukui *et al.*, Prog. Theor. Phys., Suppl. 16, 1 (1960).

[7]T. Hara *et al.*, in *Proceedings of the Sixteenth International Conference on Cosmic Rays, Kyoto, Japan, 1979* (University of Tokyo, Tokyo, Japan, 1979), Vol. 8, p. 135.

[8]Y. H. Tan, M. Nagano, and S. Kawaguchi, Tokyo University Report No. ICR-99-82-2, 1982 (unpublished).

[9]T. Hara *et al.*, in *Proceedings of the Seventeenth International Cosmic Ray Conference, Pairs, 1981* (Centre d'Etudes Nucléaires de Saclay, Gif-sur-Yvette, France, 1981), Vol. 11, p. 227.

[10]B. S. Acharya *et al.*, in Ref. 9, Vol. 11, p. 385.

[11]S. I. Nikolsky *et al.*, in Ref. 9, Vol. 2, p. 129.

[12]Summary is found in A. M. Hillas, in Ref. 9, Vol. 13, p. 69.

[13]M. P. Chantler *et al.*, J. Phys. G 9, 127 (1983).

[14]M. Nagano and S. Kawaguchi, in Proceedings of the Workshop on Very High Energy Cosmic Ray Experiments, University of Utah, Salt Lake City, January 1983 (to be published).

[15]K. Kasahara, S. Torii, and Y. Yuda, in *Proceedings of the Sixteenth International Conference on Cosmic Rays, Kyoto, Japan, 1979* (University of Tokyo, Tokyo, Japan, 1979), Vol. 13, p. 70.

[16]R. W. Ellsworth *et al.*, Phys. Rev. D 26, 336 (1982).

[17]A. M. Hillas, in *Proceedings of the Sixteenth International Conference on Cosmic Rays, Kyoto, Japan, 1979* (University of Tokyo, Tokyo, Japan, 1979), Vol. 6, p. 13.

[18]J. Wdowczyk and A. W. Wolfendale, Nuovo Cimento A 54, 433 (1979).

[19]T. Stanev and T. K. Gaisser, in Proceedings of the Workshop on Very High Energy Cosmic Ray Interactions, University of Pennsylvania, Philadelphia, April 1982 (unpublished), p. 125.

[20]E. Leader and U. Maor, Phys. Lett. 43B, 505 (1973).

[21]Y. Afek *et al.*, Phys. Rev. Lett. 45, 85 (1980). The dotted line denoted by A in Fig. 2 is cited from the figure of Ref. 15.

[22]U. Maor and S. Nussinov, Phys. Lett. 46B, 99 (1973).