Electrons on Films of Helium: A Quantum Mechanical Two-Dimensional Fermion System

F. M. Peeters[a] and P. M. Platzman
Bell Laboratories, Murray Hill, New Jersey 07974

(Received 31 March 1983)

The phase diagram of electrons on films of helium is investigated with use of simple qualitative ideas. It is shown that several new very interesting experimentally accessible regimes are possible. Quantum effects can be important and a new fluidlike state ($n \to 0$, $T \to 0$) appears.

PACS numbers: 67.50.-b, 67.20.+k, 05.30.Fk

Electrons on the surface of liquid helium make a particularly interesting almost ideal two-dimensional (2D) fermion system[1,2] For bulk helium (the only system investigated experimentally), a finite concentration of them ($10^5 < n < 10^9$ cm^{-2}) are held onto the free surface by a combination of an external field and an image potential which traps the electrons in their lowest quantum state for motion perpendicular to the surface. Motion parallel to the surface is essentially unrestricted although coupling to the ripplons (capillary waves) of the surface can play an important role.[3]

For bulk helium such a system is very accurately characterized by a 2D gas with a pairwise potential $V_\infty(r) = e^2/r$ immersed in a uniform background of charge.[1] At realistically achievable densities of $n \cong 10^9$ cm^{-2} the mean potential energy of the electrons $\langle V\rangle_\infty = e^2\pi^{1/2}n^{1/2}$ is much greater than the mean classical kinetic energy $\langle K\rangle = k_B T$, i.e., $e^2\pi^{1/2}n^{1/2}/k_B T \cong 10^2$ for $n = 10^9$ cm^{-2}, $T = 1$ K. In this regime we expect solidification as one raises the density or lowers the temperature for a fixed density. Such a classical freezing transition has been observed[4] when $e^2\pi^{1/2}n^{1/2}/k_B T_{KT} = \Gamma_{KT} \cong 137$. By now it is generally agreed that the mechanism for melting of the classical solid is the unbinding of pairs of defects as first suggested by Kosterlitz and Thouless (KT)[5] and elaborated on by Halperin and Nelson.[5] This theory predicts[6] a $\Gamma_{KT} = 125$ for a Coulombic potential and tells us that

$$\Gamma_{KT} \sim \frac{1}{c_t^2}\left[\frac{1}{1 - c_t^2/c_l^2}\right] \qquad (1)$$

for more general solids. Here c_t and c_l are the transverse and longitudinal sound velocities.

There is, however another region of the phase diagram, i.e., the low-temperature region, where the Fermi energy $\langle K\rangle = E_F = h^2\pi n/m$ dominates the kinetics of the electron gas. In this regime ($T \cong 0$) one expects a melting transition (Wigner transition) to occur as one *increases* the density, i.e., when $r_0/a_B = me^2/h^2\pi^{1/2}n^{1/2} \equiv r_s < r_s^0$. Very little is known about this transition. Physical intuition might argue that an $r_s^0 \cong 100$ is expected. However, this region is not normally accessible to current experiments on bulk He. Lowering the temperature to 10^{-2} K at a density of 10^9 cm^{-2} does nothing but cool down the classical solid ($r_s \cong 3 \times 10^3$ for $n \cong 10^9$ cm^{-2}).

In a typical experimental situation the He layer sits on top of an insulating substrate of dielectric constant ϵ which in turn is adjacent to a metal electrode. This electrode provides electrical contact to an external voltage source permitting a holding field to be applied between the electron layer and the metal. A sketch of the geometric arrangement of the various substrates is shown in Fig. 1. When the bulk He substrate is thinned down (thickness d) so that d becomes comparable to the interparticle spacing, i.e., $d \cong 100$ Å; then several very amusing and physically interesting changes occur.

(1) The maximum density to which the surface can be charged increases drastically.[7] If the underlying substrate is an insulator and has a dielectric constant ϵ then for films $d < 1000$ Å the maximum density to which the film may be charged is $n_c = (3\alpha\tau)^{1/4}/(2\pi e^2\epsilon)^{1/2}d$, with α the van der Waals constant for the substrate and τ the surface tension of the helium. For $n = n_c$ the

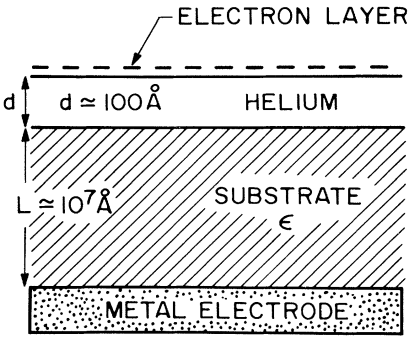

FIG. 1. The geometry.

surface is unstable at a wave vector $k_c=(3\alpha/\tau)^{1/2}/d^2$. Typically $n_c \cong 10^{11}$ cm^{-2} for $d=100$ Å, $\epsilon=10$. When the substrate is a metal, $\epsilon=\infty$ and the surface is unstable near $k_c=0$ for an $n_c=(2\alpha/4\pi e^2 d^3)^{1/2} \cong 10^{11}$ cm^{-2} for $d=100$ Å.

(2) The law of force between the electrons is changed because of the presence of the image charge when the thickness of the film is small compared to the interparticle spacing. For the case of a film lying on a substrate with dielectric constant ϵ,

$$V_d(r)=e^2\left[\frac{1}{r}-\frac{\delta}{(r^2+4d^2)^{1/2}}\right] \tag{2}$$

with $\delta=(\epsilon-1)/(\epsilon+1)$.

In this Letter we will make the simplest estimate of the phase diagram of such a 2D fermion gas supported on a film of He typically 100 Å thick. This estimate will be based on KT theory and on the law of force given in Eq. (2). This estimate which is basically a dimensional argument is expected to be quantitatively correct at high temperatures $k_B T > E_F$ and at best qualitative at low temperatures. The picture which emerges from such qualitative considerations is one of experimental interest, i.e., a 2D fermion system with an easily accessible quantum as well as classical solid-to-liquid phase transition.[8] In addition there will be a region which will remain a normal fluid as $n \to 0$, $T \to 0$. We neglect any ripplon coupling. When ripplon coupling is taken into account the normal fluid will surely become a superfluid. In addition, all of these properties are variable by simply changing film thickness and surface charge density.

Since we physically expect any liquid to crystallize when the potential energy is large we can get a qualitative picture of the shape and nature of the phase diagram by calculating $\langle V\rangle_d/\langle K\rangle$ in the *gas* phase and setting it equal to a quantity Γ which we determine in the classical regime from KT, i.e.,

$$\langle V\rangle_d/\langle K\rangle=\Gamma_{KT}(d). \tag{3}$$

For the case under consideration,

$$\langle V\rangle_d=e^2\left[\frac{1}{r_0}-\frac{\delta}{(r_0^2+4d^2)^{1/2}}\right], \tag{4}$$

with $\pi\tau_0^2=n^{-1}$, and

$$\langle K\rangle=\frac{2}{n}\int\frac{d^2p}{(2\pi)^2}\,\frac{\epsilon_p}{\exp[\beta(\epsilon_p-\mu)]+1}, \tag{5}$$

with $\epsilon_p=p^2/2m$ and with

$$n=2\int\frac{d^2p}{(2\pi)^2}\,\frac{1}{\exp[\beta(\epsilon_p-\mu)]+1}, \tag{6}$$

KT tells us that,

$$\Gamma_{KT}(d)=\Gamma_{KT}(\infty)\,\frac{c_t^2(\infty)}{c_t^2(d)}\left[\frac{1}{1-c_t^2(d)/c_l^2(d)}\right]. \tag{7}$$

We take $\Gamma_{KT}(\infty)=137$ and use zero-temperature values of the sound velocities (10%–20% accuracy). The sound velocities were calculated[9] for a harmonic solid with a pairwise potential of the form given in Eq. (2) by methods identical to those used in Ref. 10. The results are shown in Fig. 2.

Figure 3 shows the essential results of the cal-

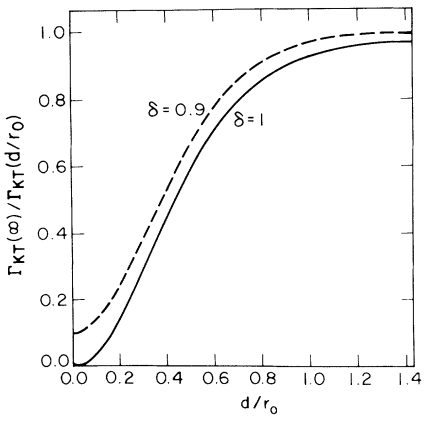

FIG. 2. A plot of $\Gamma(d)$, Eq. (7), for $\delta=0.9$ and 1.

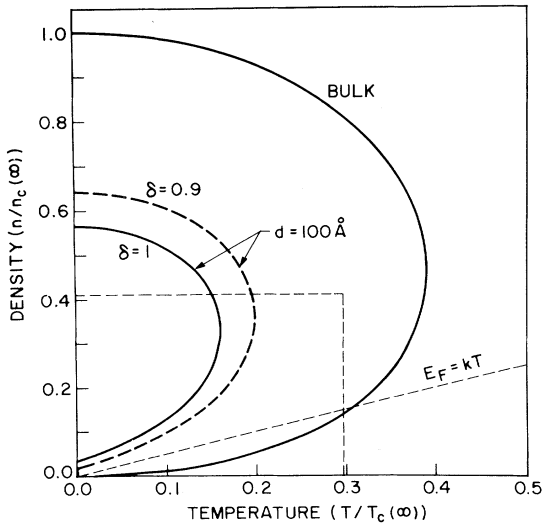

FIG. 3. The phase diagram [with use of Eq. (3)] for bulk helium and for a $d=100$-Å film lying on two different substrates (sapphire $\delta=0.9$ and metallic $\delta=1$). $n_c=2.4\times10^{12}$ cm^{-2} and $T_c=33$ K.

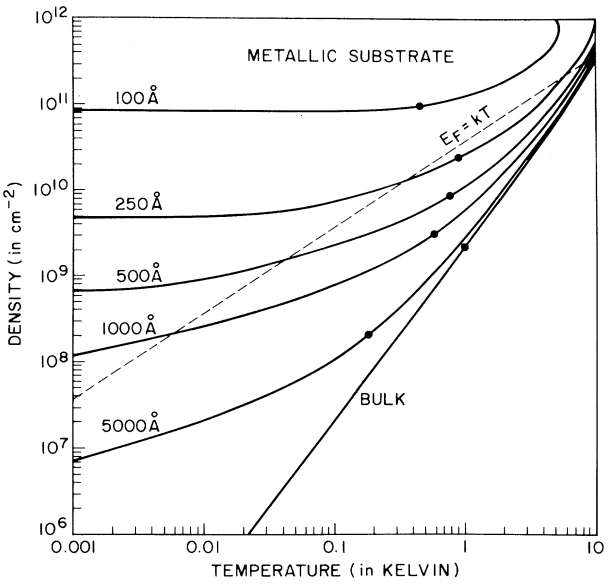

FIG. 4. The phase diagram near $T=0$ [with use of Eq. (3)] for a metallic substrate, and varying film thicknesses. The solid dots are the stability points for the film.

culation with $n_c = 2.4\times10^{12}$ cm^{-2} and $T_c = 33$ K. The curve labeled $\delta = 0.9$ is for a typically insulating substrate like sapphire $\epsilon \cong 10$. For the metallic substrate ($\delta = 1$) we show a blow up of the phase diagram, Fig. 4, in the quantum regime for a variety of film thickness. The solid dots on each curve indicate the stability limit for such films, i.e., lower densities are stable.

The drastic screening of the Coulomb interactions by the substrate is obvious. The curved quantum region of the diagram is pushed to lower densities and temperatures. This fact and the fact that the films can support higher densities makes this region experimentally accessible. In addition we see a fluidlike region remaining as $n \to 0$, $T \to 0$ for the metallic substrate. This new interesting feature arises because the force is strictly dipolar at large distances, $\langle V\rangle_d \sim n^{3/2}$ ($n \to 0$), so that the system must remain a liquid ($T = 0$) for some $n < n_{DP}$. At higher densities it will be solid and then melt again for $n > n_w$, the Wigner transition. The values of n_{DP} and n_w depend on the film thickness. In fact for $d = 70$ Å in our model $n_{DP} = n_w$ so that the solid region has shrunk to a point. For thinner films the system is always a normal liquid. Our estimates show that for a 100-Å film $n_{DP} \cong 10^{11}$ cm^{-2}. From the study made in Ref. 2 we know that typical phonon energies are of order 10^{-2} K so that we could legitimately expect a simple BCS superfluid transition of the 2D liquid.

For $\delta = 0.9$ there is similar behavior; however, the fluid does not persist for finite n to $T = 0$. The boundary curves in our estimate drop (vertical dotted line in Fig. 3) at very low temperatures ($T < 2\times10^{-3}$ K) approaching the classical parabola

$$e^2(1-\delta)^2 n^{1/2}\pi^{1/2}/kT = \Gamma_{KT}(\infty). \quad (8)$$

While our approximation, Eq. (3), is a naive qualitative estimate of the properties of this fascinating ideal system, and while experimental observation for a metallic substrate may be difficult (though not impossible with a thin film) our conclusion is that such a system can be an almost ideal lab for testing the properties of interacting 2D systems in the quantum regime.

We would like to thank E. Andrei, C. C. Grimes, Y. Iye, and S. A. Jackson for stimulating discussions. This work was supported in part by the Nationaal Fonds voor Wetenschappelyk Onderzoek (Belgium), FKFO (Belgium), and NATO.

[a] On leave from Department Natuurkunde, Universitaire Instelling Antwerpen, B-2610 Wilrijk, Belgium.

[1] C. C. Grimes, Surf. Sci. 73, 379 (1978); V. Shikin, Surf. Sci. 73, 396 (1978).

[2] Y. P. Monarkha and V. Shikin, Fiz. Nizk. Temp. 8, 563 (1982) [Sov. J. Low Temp. Phys. 8, 279 (1982)].

[3] S. A. Jackson and P. M. Platzman, Phys. Rev. B 24, 499 (1981).

[4] C. C. Grimes and G. Adams, Phys. Rev. Lett. 42, 795 (1979); D. S. Fisher, B. I. Halperin, and P. M. Platzman, Phys. Rev. Lett. 42, 798 (1979).

[5] M. Kosterlitz and D. Thouless, J. Phys. C 6, 1181 (1973); B. Halperin and D. Nelson, Phys. Rev. B 19, 2457 (1979).

[6] R. H. Morf, Phys. Rev. Lett. 43, 931 (1979).

[7] H. Ikezi and P. M. Platzman, Phys. Rev. B 23, 1145 (1981).

[8] Some aspects of the classical regime for electrons on helium films were considered in Ref. 2.

[9] F. Peeters, to be published.

[10] L. Bonsall and A. A. Maradudin, Phys. Rev. B 15, 1959 (1974).