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Scaling Treatment of Critical and "Chaotic" Dynamics of the Dilute Heisenberg Chain

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A lattice rescaling method is applied to the equations of motion of the dilute Heisenberg chain and leads via a probabilistic integral equation to an iterative map for the characteristic frequency β and concentration p. Dilution induces a crossover in the β scaling from "chaotic" (ergodic and mixing) behavior, corresponding to the sampling of the pure band, to periodic orbits corresponding to isolated cluster response. A dynamic scaling form is obtained for the critical dynamics by fixed-point analysis.

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Much recent progress has been made in understanding static critical properties of disordered systems by the use of lattice rescaling methods.[1] Up to now, however, there has been no satisfactory development of such methods for the dynamic properties, partly because of some constructional and interpretive difficulties in lattice rescaling methods for the dynamics of the pure case[2] and partly because of insufficient care with the disorder. These difficulties are overcome in this Letter. I will treat the particular case of the diluted Heisenberg chain at absolute zero.

I give here the first lattice-rescaling treatment transforming the distribution function of the dynamic variable. I extract an equation for the scaling of the characteristic frequency which is able to deal with the critical dynamics and to treat both band (extended) and localized (cluster) response. The scaling equation is an iterative map[3-5] with control variable ranging, as the concentration variable scales, from values yielding "chaotic" behavior associated with band response to values yielding a hierarchy of bifurcated stable orbits corresponding to isolated cluster dynamics.

If one is not interested in the full crossover, the low-frequency (critical) dynamics can be obtained simply by linearizing about the doubly unstable zero-frequency percolation fixed point of

the recursion map, in the usual procedure of renormalization-group theory.[6] This yields a dynamic scaling form,[7] involving the percolation correlation length ξ_p with dynamic exponent $z=2$.

I start from the equation of motion[8] for the transverse spin components in a zero-temperature disordered Heisenberg chain, having exchange J_l on the lth bond. The equation for the $2n$th spin involves the spins at neighboring sites $(2n\pm1)$. These latter spins can be eliminated by using their equations of motion, which involve spins at sites $2n, 2(n\pm1)$. This elimination of every other site of the original chain ("decimation"[9]) leads exactly to an equation of identical form to the initial one, but with rescaled value β_l' of the original random "frequency" variable $\beta_l \equiv \omega/J_l$ for the lth bond:

$$\beta_l' = 2\beta_{2l} + \beta_{2l-1} - \beta_{2l-1}\beta_{2l} + \beta_{2l-3}(\beta_{2l-1} + \beta_{2l} - \beta_{2l-1}\beta_{2l})/(\beta_{2l-3} + \beta_{2l-2} - \beta_{2l-3}\beta_{2l-2})$$
$$= g(\beta_{2l-3}, \beta_{2l-2}, \beta_{2l-1}\beta_{2l}). \tag{1}$$

The probability distribution $Q(\beta_l)$ for the independent random variables β_l thus transforms[10] according to the recursive integral equation $Q \to Q'$, where

$$Q'(\beta_l') = \int d\beta_1 \ldots d\beta_4\, Q(\beta_1)\ldots Q(\beta_4)\delta(\beta_l' - g(\beta_1,\beta_2,\beta_3,\beta_4)). \tag{2}$$

In the dilute case, $Q(\beta_l)$ is initially two δ functions, of respective weights p and $1-p$ at ω/J and ∞, where p is the bond concentration. The scaled distribution $Q'(\beta_l')$ is (after a single scaling) a δ function at ∞ (whose weight is denoted by $1-p'$, so that p' is the scaled value of p) and several other δ functions at finite values of β_l', whose overall weight is p' and whose center of gravity is denoted by β'. β' can be regarded as the scaled value of the original characteristic frequency $\beta \equiv \omega/J$. The resulting scalings of concentration variable p and characteristic frequency $\beta \equiv \omega/J$ are

$$p' = p^2, \tag{3}$$

$$\beta' = (1-p)^2(3\beta - \beta^2) + p(1-p)\beta(8 - 9\beta + 2\beta^2)/(1-\beta) + p^2(4\beta - \beta^2). \tag{4}$$

Equation (3) is exact, but (4) is approximate since it is based on the same sort of binary approximation and neglect of induced correlations often used to reduce the scaling of the distribution of the random variable in static disordered critical problems.[1,11] Equations (3) and (4) give the complete static and dynamic renormalization-group transformation for the system.

Equation (3) is the usual percolation scaling[12] and has two fixed points, $p^* = 1, 0$, corresponding to pure and empty chains, respectively. At these two values of the concentration, (4) reduces to

$$\beta' = \alpha(p)\beta - \beta^2, \tag{5}$$

with $\alpha(1)=4$, $\alpha(0)=3$. Equation (5) has the form of the common one-dimensional (Myrberg) iterative map.[3-5] It has very rich iterative properties of which the simplest (for small control variable α) are monotonic or periodic flows into a stable fixed point. As α is increased through 3 a bifurcation into a stable orbit of period 2 takes place and then further period-doubling bifurcations occur until a limit point of bifurcations is reached at $\alpha \sim 3.57$, at which the system is ergodic but not mixing. For α between 3.57 and 4 ergodic mixing behavior occurs in which the iterated variable samples a series of bands in frequency space. Then for $\alpha = 4$ the ergodic mixing behavior samples a single complete band. This last situation, occurring in the present system at the pure limit $p=1$, can easily be seen to be the simple band behavior of the pure system, since $\beta = 2(1 - \cos k)$ (with $k \to k' = 2k$ under length scaling) solves the frequency recursion equation for this case and gives the full band sampling of the iteration, with the pure density of states $\rho(\beta) \propto [\beta(16 - \beta^2)^{1/2}]^{-1}$. The opposite (low-concentration) limit of our development gives $\alpha = 3$, which is the marginally stable situation just about to bifurcate for the first time. It corresponds to the response due to single bonds, which have overwhelming weight as $p \to 0$. This can be seen from the density of states, which can be calculated by iterating many times, or by analytic methods to be described elsewhere. This has two dominant contributions, one at $\beta = 2$ and one at $\beta = 0$: These peaks are at the two energies for a pair of spins joined by a bond, and are indicative of nonmixing behavior.

Any initial (physical) value of the concentration not equal to 1 or 0 scales through a series of iterated values towards the $p=0$ fixed point. This takes $\alpha(p)$ from the ergodic mixing regime corresponding to band behavior into the orbit cascade regime (α below 3.57) which corresponds to the

response from isolated finite clusters of smaller and smaller size. Thus any initial $p < 1$ picks up appropriately weighted dynamic response from finite clusters of all sizes and, in general, dilution induces a crossover from "extended" to localized behavior. The full details of this crossover will be given elsewhere.

The low-frequency critical dynamics is determined only by the scaling in the neighborhood of the zero-frequency percolation fixed point $(\beta, p) = (0, 1)$ [since the percolation threshold at which the percolation correlation length ξ_p diverges is $p_c = 1$ (Refs. 12 and 13)]. Here linearized forms of the recursion equations (3) and (4) are adequate, and the full discussion of the points of the preceding paragraphs is not required. From the eigenvalues of the linearized recurrence equations we obtain scaling fields $1-p$, $\beta^{1/2}$ which scale like the dilatation factor. The other (obvious) scaling fields are the wave vector k and $1/\xi_p$. Usual length-scaling homogeneity then implies the following dynamic scaling form[7] for the characteristic frequency:

$$\beta = k^z F(k\xi_p) \quad (\beta, k, 1/\xi_p \to 0), \tag{6}$$

where $z = 2$ and $\xi_p \propto (1-p)^{-1}$. Standard asymptotic arguments show that $F(x)$ goes to a constant (the spin-wave stiffness) for x large and like $1/x^2$ for x small. These again correspond to extended and localized results, but are special cases of the general description since we are here only concerned with $k, 1/\xi_p \to 0$. A discussion of the critical dynamics and of asymptotic forms for this case can be obtained by other methods[14] based on direct cluster statistics.

The main features discussed here will persist in higher dimensionality.[15] However, infinite cluster response now occurs for a range $(p_c < p < 1)$ of values of p, and so the crossover is still more complicated[16] and involves a percolative dynamics (at p_c) different from the pure dynamics, and crossover from this to both pure and finite cluster dynamics. Also, in this case, the function g occurring in (2) is expected to exhibit, in the appropriate limit, a relationship to the dilute resistor network scaling function[11] because of a result of Last[17] and Kirkpatrick.[18] This relationship does not occur in the one-dimensional case because of the absence of the finite cluster for $p < 1$.

Finally, the close connection between the initial (configurationally dependent) equation of motion used here and that occurring in disordered tight-binding models and, in particular, quantum percolation[19] and the Anderson model[20] suggest that the present method should have interesting consequences and useful applications in those problems.

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