New Look at Magnetic Moments and Beta Decays of Mirror Nuclei

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A new and model-independent way of analyzing the data on magnetic moments and β decays of mirror nuclei gives firm evidence for a quenching of the axial-vector coupling constant in nuclei. Specifically, $R = |C_A/C_V| = 1.00 \pm 0.02$ is found, compared with the bare nucleon value $R = 1.25 \pm 0.01$. The same analysis yields effective orbital g factors $\tilde{g}_p = 0.94 \pm 0.05$ and $\tilde{g}_n = 0.10 \pm 0.04$ for proton and neutron, values which deviate from the bare values $g_p = 1$ and $g_n = 0$ in a direction opposite to that generally accepted.

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Sum-rule analyses[1] of nucleon transfer data strongly suggest that, for even-odd nuclei, *all odd-rank multipole operators in the variables of the nucleons with even number have vanishing expectation values in the nuclear ground state.* Here we shall not enquire further into underlying reasons for this principle,[2] but simply apply it to derive some remarkable correlations between magnetic moments and beta decays of mirror nuclei.

A nuclear magnetic moment is defined in the state $|J, M=J\rangle$ by $\mu = \langle JJ|\mu_z|JJ\rangle$, where μ_z is the z component of the magnetic dipole operator. With a similar notation we write J, L, and S for the stretched matrix elements of the z components of the total, orbital, and spin angular momentum operators, respectively. All the above operators are, of course, rank-one multipoles. It is conventional to discuss the magnetic moments of mirror nuclei in the isospin formalism, but we shall instead, for a given nucleus, distinguish between nucleons with odd number and even number, whether they happen to be protons or neutrons.

Assuming charge symmetry of nuclear forces and writing μ_P and μ_N for the magnetic moments of the odd-proton and odd-neutron members of a mirror pair, we have

$$\mu_P = g_p L_o + G_p S_o + g_n L_e + G_n S_e,$$
$$\mu_N = g_n L_o + G_n S_o + g_p L_e + G_p S_e,$$

where $g_p = 1$, $G_p = 5.586$, $g_n = 0$, and $G_n = -3.826$, all in nuclear magnetons, are the orbital and spin g factors for proton and neutron, while the odd- and even-nucleon contributions to the angular momenta are labeled by corresponding subscripts. For both nuclei the total J is equal to $L_o + S_o + L_e + S_e$ and, with $J_e = L_e + S_e$, μ_P and μ_N may for later convenience be rewritten in the form

$$\begin{aligned} \mu_P &= g_p J + (G_p - g_p)(S_o - S_e) \\ &\quad - (g_p - g_n)J_e + (G_p - g_p + G_n - g_n)S_e, \\ \mu_N &= g_n J + (G_n - g_n)(S_o - S_e) \\ &\quad + (g_p - g_n)J_e + (G_p - g_p + G_n - g_n)S_e. \end{aligned} \tag{1}$$

According to the principle given above only the odd-nucleon expectation values contribute appreciably and we obtain

$$\begin{aligned} \mu_P &\approx g_p J + (G_p - g_p)S_o, \\ \mu_N &\approx g_n J + (G_n - g_n)S_o. \end{aligned} \tag{2}$$

These relations are well known in the context of the extreme single-particle model[3] which assumes further that only the *last* odd nucleon is active. With $S_o = \frac{1}{2}$ for $J = L_o + \frac{1}{2}$ and $S_o = -J/(2J+2)$ for $J = L_o - \frac{1}{2}$ they lead to the Schmidt estimates for magnetic moments. The strong deviations of actual nuclear moments from these estimates can be understood if we assume instead that *all* the odd nucleons are potentially active so that S_o does not necessarily take on the single-particle values. This odd-group model[4] can be tested independently of the values of S_o by eliminating S_o from Eqs. (2). This implies that

$$\mu_P \approx \alpha\mu_N + \beta J, \tag{3}$$

where $\alpha = (G_p - g_p)/(G_n - g_n)$ and $\beta = g_p - \alpha g_n$. Substitution of the bare nucleon g factors gives $\alpha = -1.199$ and $\beta = 1$. Earlier investigations[5] led to the conclusion that Eq. (3) was not a particularly accurate relation.

However, the new evidence from sum-rule analyses[1,2] and the recent accumulation of magnetic data for mirror nuclei (displayed in Table I) suggest that it may be worthwhile to look again at this simple correlation between nuclear moments. With the new idea that agreement with experiment may be substantially improved by using fitted values of α and β, this is most efficiently done by defining the gyromagnetic ratios $\gamma_{P,N} = \mu_{P,N}/J$ so that Eq. (3) becomes

$$\gamma_P \approx \alpha\gamma_N + \beta. \tag{4}$$

A plot of γ_P against γ_N using values from Table I is shown in Fig. 1 together with a statistically fitted straight line.[6] The Schmidt estimates for γ_P and γ_N must, of course, lie close to this line, but the interesting feature now emerges that the points representing measured moments deviate from those estimates simply by sliding along the same line. With a correlation coefficient of $r = -0.999$ the fit is obviously not accidental and we must now examine the significance of the ~5% deviations of the extracted parameters $\alpha = -1.145 \pm 0.012$, $\beta = 1.056 \pm 0.021$ from the "bare" values $\alpha = -1.199$, $\beta = 1$ given above.

To this end we return to Eqs. (1), which contain the possible small contributions from even-nucleon expectation values, and consider also the effects of meson currents in the nucleus. It is generally believed[7] that the dominant one-pion exchange mechanism does not modify the isoscalar moment $\mu_0 = \mu_P + \mu_N$; hence the main effect of these currents is to introduce additional terms $-\mu_M$ and $+\mu_M$ into the expressions for μ_P and μ_N, respectively. Some allowance should also be made for the small Jensen-Mayer correction to μ_P, which we shall denote by μ_x. This can be regarded as arising from the restoration of gauge invariance in spin-orbit forces[3,8] or, more directly,[7] as an effect of heavy-vector-meson exchange currents. Model calculations[9] suggest that μ_x is well represented by $\mu_x = -x(S_o - S_e)$. Equations (1) thus become

$$\begin{aligned} \mu_P &= g_p J + (G_p - g_p - x)(S_o - S_e) \\ &\quad - (g_p - g_n)J_e - \mu_M + (G_p - g_p + G_n - g_n)S_e, \\ \mu_N &= g_n J + (G_n - g_n)(S_o - S_e) \\ &\quad + (g_p - g_n)J_e + \mu_M + (G_p - g_p + G_n - g_n)S_e. \end{aligned} \tag{5}$$

The quantity $S_o - S_e$ is closely related to the β-decay ft value for a mirror pair transition. Up to an overall sign, which is determined unambig-

TABLE I. Mass numbers A, angular momenta J, and ratios γ_P, γ_N, and γ_β as described in text. Starred mass numbers indicate that these nuclei have not been included in the statistical fits of Figs. 1–3.

A	J^π	$\gamma_P = \mu_P/J$	$\gamma_N = \mu_N/J$	$\gamma_\beta = R(S_o - S_e)/J$
3	$1/2^+$	+5.9578 (2)	−4.2552 (2)	+1.2055 (17)
11	$3/2^-$	+1.7923 (1)	−0.6600 (200)	+0.1911 (13)
13	$1/2^-$	−0.6442 (7)	+1.4048 (2)	−0.3245 (12)
15	$1/2^-$	−0.5662 (2)	+1.4378 (16)	−0.3649 (17)
17	$5/2^+$	+1.8890 (5)	−0.7575 (−)	+0.2172 (3)
19	$1/2^+$	+5.2576 (2)	−3.7708 (2)	+0.9226 (29)
21	$3/2^+$	+1.5907 (1)	−0.4412 (−)	+0.1823 (15)
23*	$3/2^+$	+1.4783 (−)		+0.1389 (15)
25	$5/2^+$	+1.4582 (5)	−0.3422 (−)	+0.1347 (5)
27	$5/2^+$	+1.4565 (1)	−0.3422 (2)	+0.1160 (7)
29	$1/2^+$	+2.4698 (6)	−1.1105 (1)	+0.3000 (46)
31	$1/2^+$	+2.2634 (2)	−0.9759 (2)	+0.2968 (52)
33*	$3/2^+$		+0.4292 (−)	−0.0762 (15)
35	$3/2^+$	+0.5479 (−)	+0.4213 (13)	−0.0720 (15)
37*	$3/2^+$	+0.1355 (−)		−0.1508 (21)
39	$3/2^+$	+0.2610 (−)	+0.6810 (2)	−0.1671 (23)
41	$7/2^-$	+1.5514 (57)	−0.4556 (−)	+0.1328 (4)
43*	$7/2^-$	+1.3200 (114)		+0.1120 (16)

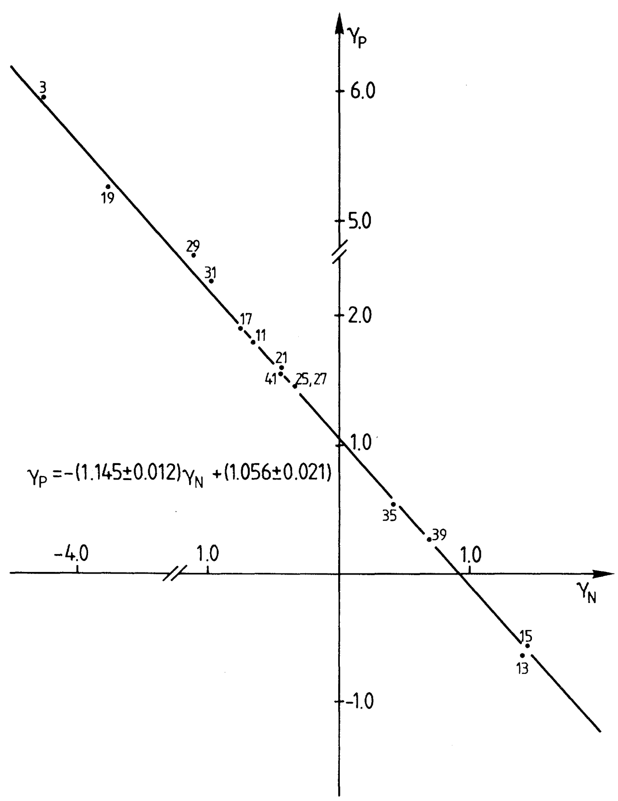

FIG. 1. Plot of γ_P vs γ_N with gyromagnetic ratios γ_P and γ_N taken from Table I.

uously from the systematics, we can write, for the $T=\frac{1}{2}$ nuclei considered here,

$$|R\langle\sum_i s_{iz}\tau_{iz}\rangle| = |R(S_o - S_e)| = \left[\left(\frac{B}{ft}-1\right)\frac{J}{J+1}\right]^{1/2}, \qquad (6)$$

where s_{iz} and τ_{iz} are the z components of spin and isospin for the ith nucleon, $R=|C_A/C_V|$ is the magnitude of the ratio of axial-vector to vector coupling constants, and $B=6170$ s is a constant determined from superallowed $0^+\to 0^+$ transitions. To simplify Eqs. (5) we define $\gamma_\beta = R(S_o - S_e)/J$ and in Table I list relevant values of this quantity taken from a compilation[9] which includes various small corrections to Eq. (6).

Inserting numerical "bare" values for the g factors in Eqs. (5), writing $\tilde{J}_e = J_e + \mu_M$, dividing through by J, and using the definitions of γ_P, γ_N, and γ_β, we find

$$\gamma_P - 1 = \frac{4.586 - x}{R}\gamma_\beta + \left(0.760\frac{S_e}{J} - \frac{\tilde{J}_e}{J}\right) \approx M_P\gamma_\beta + A_P,$$

$$\gamma_N = -\frac{3.826}{R}\gamma_\beta + \left(0.760\frac{S_e}{J} + \frac{\tilde{J}_e}{J}\right) \approx M_N\gamma_\beta + A_N. \qquad (7)$$

Plots of $\gamma_p - 1$ and γ_N against γ_β, with values taken from Table I, are shown in Figs. 2 and 3. Excellent linear fits are obtained with correlation coefficients $r=\pm 0.997$ in the two cases. Extracted values of the slopes and intercepts are

$$M_P = +4.38 \pm 0.10, \quad A_P = -0.061 \pm 0.046,$$

$$M_N = -3.82 \pm 0.10, \quad A_N = +0.101 \pm 0.043.$$

The following conclusions are strongly suggested by the data:

(i) The ratio S_e/J exhibits small fluctuations about a small average value, in accord with other determinations.[9-12] The most accurate value for the average of S_e/J is found by plotting $\gamma_P + \gamma_N - 1$ against γ_β, thereby eliminating $\tilde{J}_e/J$. This yields $(S_e/J)_{\mathrm{av}} = 0.027 \pm 0.013$.

(ii) The quantity $\tilde{J}_e/J$ shows small fluctuations about a somewhat larger average value which, from the data alone, we cannot separate into its components J_e/J and μ_M/J. The average value of $\tilde{J}_e/J$ can be deduced from the results for A_P, A_N, and $(S_e/J)_{\mathrm{av}}$ given above or, more accurately, by plotting $\gamma_P - \gamma_N - 1$ against γ_β, which eliminates S_e/J. Hence we find $(\tilde{J}_e/J)_{\mathrm{av}} = 0.081 \pm 0.044$.

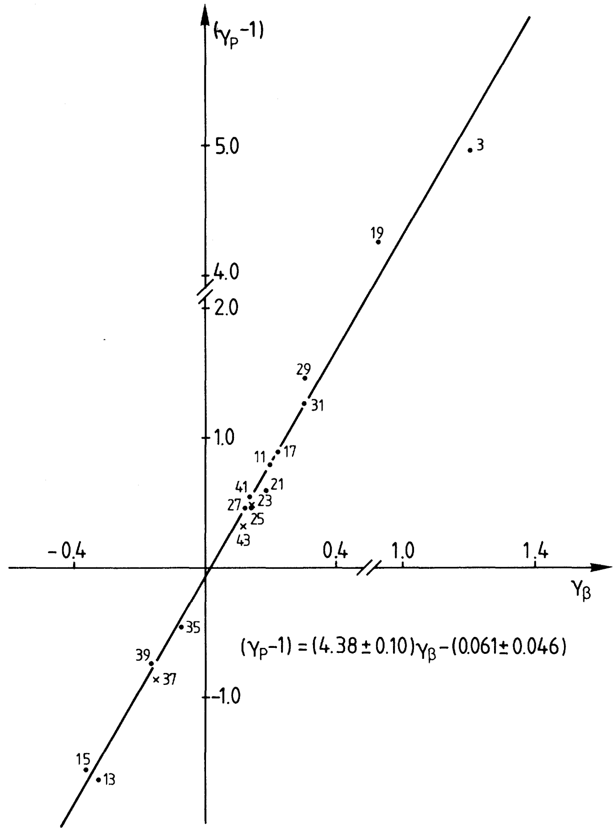

FIG. 2. Plot of $\gamma_P - 1$ vs γ_β with $\gamma_P - 1$ and γ_β from Table I. Crosses indicate cases not included in the fitting procedure.

(iii) The intercepts A_P and A_N may be interpreted as the deviations of effective orbital g factors $\tilde{g}_P = 1 + A_P$ and $\tilde{g}_n = 0 + A_N$ of nucleons in a nucleus from the bare values 1 and 0. Thus our results imply $\tilde{g}_p = 0.939 \pm 0.046$, $\tilde{g}_n = 0.101 \pm 0.043$. These deviations are of a similar magnitude but *opposite in sign* to currently accepted values.[13]

(iv) The slope M_N yields immediately a well-determined result for the weak-interaction ratio $R=|C_A/C_V|$ appropriate to nucleons in a nucleus. We obtain

$$R = 1.00 \pm 0.02,$$

a value which is strongly quenched compared with the ratio $R = 1.249 \pm 0.006$ measured in neutron decay experiments.[14] A renormalization of R in nuclei has long been suspected.[11] All previous attempts to determine R have been based on consideration of the isoscalar moment $\mu_0 = \mu_P + \mu_N$. However, the intrinsic smallness of μ_0 makes such attempts extremely sensitive to model-dependent values of the (albeit small)

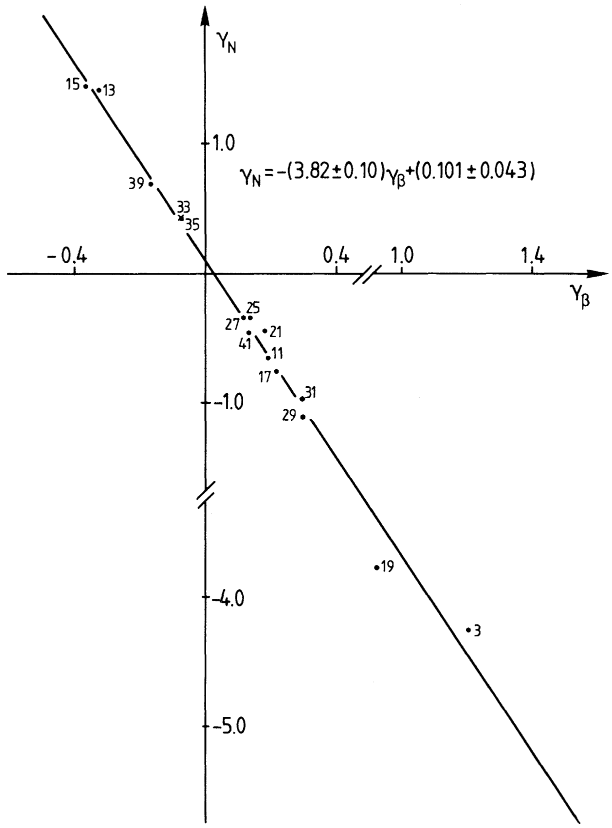

FIG. 3. Plot of γ_N vs γ_β with γ_N and γ_β from Table I. Cross indicates case not included in the fitting procedure.

quantities S_e and μ_x, and the extracted values[9-12] lie in the range $1.0 \leq R \leq 1.3$.

(v) With our deduced value of R, the slope M_P gives a value for the Jensen-Mayer correction of $x = 0.21 \pm 0.10$. A better result can be found from the fitted line $\gamma_P = \alpha\gamma_N + \beta$ of Fig. 1, where $\alpha = -1.145 \pm 0.012$ now represents the quantity $(G_p - 1 - x)/G_n$; this yields

$$x = 0.203 \pm 0.046.$$

We note also that $\beta = 1.056 \pm 0.021$ of Fig. 1 now signifies the combination $\tilde{g}_p - \alpha\tilde{g}_n$ of effective orbital g factors; so this combination is much better determined than $\tilde{g}_p$ and $\tilde{g}_n$ separately, though our values above are consistent with the fitted β.

In conclusion, we believe that the accuracy with which the effective g factors, correction terms, and beta-decay coupling constants appear to have been determined could have far-reaching consequences for nuclear-structure studies.[15,16] We stress that *these results are model independent,* except for the *forms* assumed for small quantities, forms which seem amply justified by their internal consistency and by model calculations. The three diagrams demonstrate very striking correlations between some of the most accurately known quantities in nuclear physics.[17] Any alternative interpretation of the data would have to take these into account.

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