Observation of Muon Inner Bremsstrahlung in Deep-Inelastic Neutrino Scattering

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In a neutrino bubble-chamber experiment with a mean event energy of 90 GeV, clear evidence is observed for muon inner bremsstrahlung. The corrected rate, for gamma rays with $E_\gamma > 1$ GeV and within 32 mrad of the muon direction, is 0.027 ± 0.005 per charged-current event. Comparison with QED calculations is discussed.

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In charged-current (CC) neutrino interactions, rapidly moving muons are suddenly created. This process can be viewed as a violent acceleration of the charge of the muon, and hence one expects an accompanying emission of electromagnetic radiation, called inner bremsstrahlung (IB); cf. Fig. 1. The number of photons emitted per unit energy and per unit solid angle is expected to vary approximately with the reciprocal of the photon energy and to peak very close to the muon direction.[1] Analogous radiative processes have been observed in nuclear beta decay,[2] in K capture,[1] and in $K^+ \to \mu^+$ decay.[3] We know of no previous observation of real photons from IB in neutrino interactions.[4]

IB is a major component of the radiative corrections to CC interactions, which have received considerable attention recently.[5-7] A measurement of the IB process should allow a partial check of these calculations. IB is also important in studies of the hadron system produced in CC interactions. The usual assumption that all observed photons come from the hadron system is not exactly correct; the IB photons could lead to asymmetries which may falsely be attributed to the hadron system, and to incorrect neutrino energy estimates, for example.

This experiment[8] used the Fermilab 15-ft bubble chamber with a two-plane external muon identifier (EMI). The chamber was filled with a neon-hydrogen mix (47% atomic neon, radiation length 53 cm, pion absorption length 193 cm) and was exposed to the quadrupole triplet neutrino beam produced by 400-GeV incident protons. 8485 CC events were fully measured; each event has a μ^- with energy $E_\mu > 4$ GeV and a good two-plane match in the EMI, and has a primary vertex at least 70 cm from the downstream chamber wall. The measurement included all primary charged tracks, all visible decays or interactions of neu-

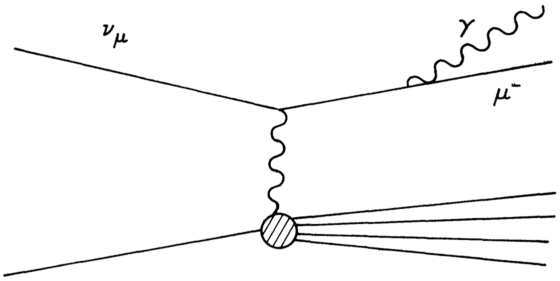

FIG. 1. Muon inner-bremsstrahlung diagram.

tral hadrons, and all possibly primary gamma rays that converted into e^+e^- pairs within 110 cm of the primary vertex. After measurement, a gamma ray was selected as primary if it (a) pointed to the primary vertex, within errors, and (b) was not consistent with being bremsstrahlung from an e^+ or e^-.

In all that follows, we require $E_\gamma > 1.0$ GeV and $L_\gamma > 10$ cm, where L_γ is the distance from the primary vertex to the gamma-ray conversion point. The E_γ cut is made to reduce backgrounds to the IB process. The L_γ cut ensures that gamma-ray directions are well measured. The error in $\theta_{\mu\gamma}$, the angle in space between the directions of the muon and a nearby gamma ray, is approximately 9 mrad for $E_\gamma = 1$ GeV and $L_\gamma = 10$ cm, and is ~2 mrad on average over our sample. On average 1.1 primary gamma rays per event with energy $E_\gamma > 1.0$ GeV were observed.

Our combined scanning, measuring, reconstructing, and pointing efficiency for gamma rays with $E_\gamma > 1$ GeV is 0.90. This estimate follows from a model which assumes that all gamma rays come from π^0 decay and that the number of π^0 equals one-half the number of π^+ plus π^- everywhere, the latter taken from the observed charged hadrons. Uncertainties in the kaon and baryon components of the charged hadrons cause a 3% uncertainty in the efficiency estimate. The gamma-ray momentum and angle distributions predicted by this model are in reasonable agreement with the observed distributions.

The $\cos\theta_{\mu\gamma}$ distribution for primary gamma rays with $E_\gamma > 1$ GeV and $\cos\theta_{\mu\gamma} > 0.998$, Fig. 2, shows a strong peak near 1.0 over a slowly varying background. Most primary gamma rays come from the hadron vertex; over a limited solid angle near the muon direction, one expects that the hadron-associated gamma rays will have an approximately flat $\cos\theta_{\mu\gamma}$ distribution. (This expectation is confirmed by the observed flatness of the distribution near the direction of the reflection of the muon direction through the visible hadron momentum vector's projection onto the lepton plane.) Therefore we attribute this peak to muon-associated gamma rays.

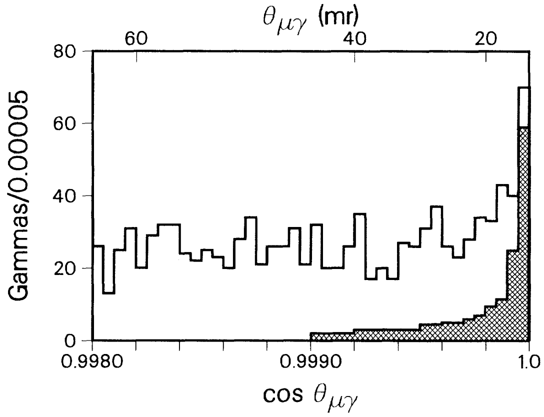

FIG. 2. $\cos\theta_{\mu\gamma}$ for gamma rays with $E_\gamma > 1$ GeV. The shaded histogram is the prediction from Ref. 5, for $\cos\theta_{\mu\gamma} > 0.9990$, smeared by our angular resolution.

Muon-associated gamma rays are produced by three processes as the muon passes through the bubble-chamber liquid: (a) outer bremsstrahlung, (b) bremsstrahlung from delta rays, and (c) tridents ($\mu Z \rightarrow \mu Z e^+e^-$) where the e^+e^- is taken to be a gamma-ray conversion. Taking the pointing criterion into account, the expected[9-11] numbers of gamma rays in our sample from these three sources are 1, 0, and 5, respectively, all with $\cos\theta_{\mu\gamma} > 0.9999$. We have examined all $\cos\theta_{\mu\gamma} > 0.9999$ events on the scanning table, and found 0 consistent with a delta-ray origin and 5 consistent with being tridents with $L_\gamma > 20$ cm, in agreement with the calculation. (For $L_\gamma < 20$ cm where we expect 0.5 trident, it becomes difficult to distinguish tridents from nontridents.) In all that follows, these 5 observed tridents are removed, we neglect the remaining 1.5 events from sources (a)–(c), and we attribute the muon-associated gamma rays to IB. As a check that the muon-associated gamma rays originate at the primary vertex, we have verified that there is no dependence of the L_γ distribution on $\cos\theta_{\mu\gamma}$.

We determine a net signal of IB gamma rays, for $\cos\theta_{\mu\gamma} > 0.9995$, by subtracting the background of hadron-associated gamma rays. To estimate the latter, we assume that all gamma rays with $\cos\theta_{\mu\gamma} = 0.9990$–$0.9995$ are hadron associated,[12] and that the distribution of hadron-associated gamma rays with $\cos\theta_{\mu\gamma} > 0.9990$ is either (1) flat, or (2) given by the π^0 model discussed above (also approximately flat). In Tables I and II and Fig. 3 we use assumption (2) [with the assumption-(1) results given in parentheses in Table I]; the difference is less than the statistical error for all subsamples considered. The rates are corrected for our estimated efficiency of 0.90 and for an average conversion probability (including cuts) of 0.64.[13] These rates are restricted to $E_\mu > 4$ GeV, $E_\gamma > 1$ GeV, and $\cos\theta_{\mu\gamma} > 0.9995$, and are appropriate averages over our neutrino energy spectrum. The total is (2.7 ± 0.5)% muon IB gamma rays per CC event.

Table I gives the net number of IB gamma rays,

TABLE I. IB angle dependence. $N_{\rm obs}$ is the observed number of gamma rays, $N_{\rm net}$ is the net signal after background subtraction (values in parentheses result from a different background subtraction—see text), and $r = (1/N_{e\nu})(dN/d\cos\theta_{\mu\gamma})$, with $N_{e\nu}$ the total number of events (8485) and N the corrected number of IB gamma rays. The theoretical calculations labeled r_1^s and r_5^s are based on Eq. (1) and Ref. 5, respectively, and have been smeared by our experimental resolution. The r_5 is the prediction of Ref. 5 before smearing.

$\cos\theta_{\mu\gamma}$	$N_{\rm obs}$	$N_{\rm net}$	r	r_1^s	r_5^s	r_5
0.9995–0.9999	255	75(79) ± 20	38 ± 10	23	27	26
0.999 90–0.999 95	40	16(18) ± 7	64 ± 27	80	98	82
0.999 95–1.0	65	41(43) ± 8	167 ± 34	209	230	262

and the corrected rate, r, per neutrino CC event and per $\cos\theta_{\mu\gamma}$ for three angular intervals. [The rate is corrected for detection efficiency, but not for angular resolution. The effect of our angular resolution on the Kiskis angular distribution (discussed below) is shown in the bottom two lines of Table I.] Table II gives the corrected IB rate, R, per event for all $E_\gamma > 1$ GeV, and in three E_γ intervals. Figure 3 shows the dependence of the corrected IB rate per event on the event variables E_ν, x, and y. The neutrino energy E_ν for each event was estimated with use of an average correction factor[8]: $E_\nu = P_X^{\rm lepton} + 1.16 P_X^{\rm hadron} + 3.3$ GeV, where P indicates momentum, lepton means muon plus IB gamma ray (if present), and the incident neutrino direction is along the X axis; we expect an average 15% uncertainty in this E_ν estimate. The quantities x and y are the usual scaling variables, evaluated by taking only the observed muon as the lepton. The errors given in the tables and Fig. 3 are statistical errors, which are much larger than the estimated systematic errors.

We compare our results with theoretical calculations, which are restricted to $E_\gamma > 1$ GeV and $E_\mu > 4$ GeV, use our known E_ν spectrum, and are insensitive to the input structure functions.

The first is a semiclassical calculation[1,14] of the ratio, R, of the rates for inner bremsstrahlung due to the diagram of Fig. 1 to that for neutrino CC interactions:

$$R = \frac{\sigma_{\rm IB}}{\sigma_{\rm CC}} \approx \frac{\alpha}{2\pi}\int \frac{dE_\gamma}{E_\gamma}\int \frac{\beta_\mu^2 \sin^2\theta_{\mu\gamma}\, d(\cos\theta_{\mu\gamma})}{(1-\beta_\mu\cos\theta_{\mu\gamma})^2}. \tag{1}$$

TABLE II. IB rate, R (in percent), $R = \int (1/N_{e\nu}) \times (dN/d\cos\theta_{\mu\gamma})\, d\cos\theta_{\mu\gamma}$, with limits of integration 0.9995 and 1.0000 and E_γ dependence. Our calculations of the predictions of Ref. 5 (R_5) and of Eq. (1) (R_1) are also given.

E_γ (GeV)	R	R_1	R_5
1–2	0.5 ± 0.3	0.64	0.72
2–4	1.0 ± 0.3	0.59	0.69
> 4	1.3 ± 0.3	1.16	1.38
> 1	2.7 ± 0.5	2.39	2.79

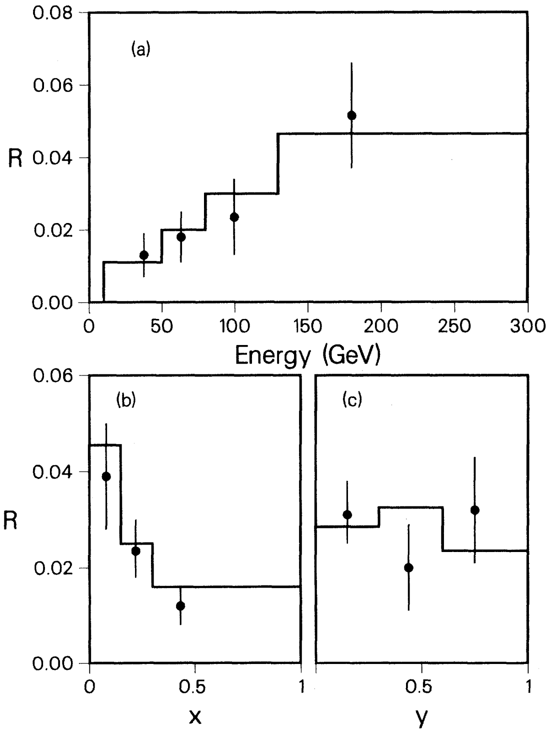

FIG. 3. Variation of IB rate R with (a) neutrino energy, (b) x, and (c) y. R is defined as in Table II but with $N_{e\nu}$ the number of events in a given bin. The histograms are predictions of Ref. 5 (see text).

Equation (1) predicts a peak in $dR/d\cos\theta_{\mu\gamma}$ at $\cos\theta_{\mu\gamma}=\beta_\mu$, i.e., at $\theta_{\mu\gamma}=m_\mu/E_\mu$ if $E_\mu \gg m_\mu$ (as usual for QED processes). For typical E_μ of 50 GeV the peak is near 2 mrad, an angle of the order of our resolution. We multiply the right-hand side of Eq. (1) by a lepton phase-space factor of $E_\mu/(E_\mu+E_\gamma)$, require E_γ to be less than the kinematic limit, and obtain the predictions given in Tables I and II. The total rate for $\cos\theta_{\mu\gamma} > 0.9995$ is 2.4%, and for all $\theta_{\mu\gamma}$ is 5.3% (2.9% and 6.7%, respectively, if the phase-space factor is omitted). Interference with diagrams where the γ comes from the target or recoil, and recoil kinematics, are neglected here.

The second calculation uses the theory of Kiskis.[5,14] This theory uses the quark-parton model and includes amplitudes for radiation by the incident and outgoing quarks. We set the quark mass equal to zero, and the incident and outgoing quark charges to zero and one, respectively. The results are given in Tables I and II and Fig. 3. For small $\theta_{\mu\gamma}$ the resulting angular distribution is similar to that of Eq. (1). The total > 0.9995 is 2.8%. The prediction of Kiskis, integrated over all angles, is 5.1% after removal of the gamma rays in the peak in the outgoing quark direction. This Kiskis estimate for the muon IB plus its interference with the IB from the recoil charge can be compared also with a calculation[14] based on formula (3.1) of De Rújula, Petronzio, and Savoy-Navarro.[6] This formula includes a term due to the diagram in Fig. 1. Calculation of this term, with integration over appropriate variables, yields 5.2%.

In conclusion, we have observed a clear signal for muon inner bremsstrahlung in neutrino CC interactions. The corrected rate per event for $E_\gamma > 1$ GeV and $\cos\theta_{\mu\gamma} > 0.9995$ is 0.027 ± 0.005. The data agree with our calculation (0.028) of the theory of Kiskis[5,14] and with a semiclassical approximation[1,14] (0.024).

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