Velocity-Dependent Factors for the Rubakov Process for Slowly Moving Magnetic Monopoles in Matter

J. Arafune
Institute for Cosmic Ray Research, University of Tokyo, Tanashi, Tokyo 188, Japan

and

M. Fukugita [(a)]
National Laboratory for High Energy Physics, Tsukuba, Ibaraki 305, Japan
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Cross sections for monopole-catalyzed nucleon decays (Rubakov process) receive a velocity-dependent correction factor in matter due to an extra angular momentum carried by the monopole–electric-charge system. This leads to a strong suppression factor for slowly moving monopoles in detectors using heavy elements.

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It has been pointed out recently[1] that magnetic monopoles catalyze the nucleon decay with a typical cross section of strong interactions (we call it the Rubakov process). Then one naturally expects that, if this is the case, this effect is readily observable in current underground experiments with massive detectors provided that the monopole flux is large enough.[2] In practical estimates of the cross section one may usually assume that it takes the form $\sigma \sim \bar{\sigma}/\beta$.[3,4]

We shall point out in this paper that such a naive expectation should be modified by a long-range force which originates from an extra angular momentum carried by a monopole–electric-charge system: $q=(eg/4\pi)Z = Z\times(\pm\frac{1}{2}, \pm 1, \dots)$ with Z the charge of the nucleus.

In the case of a spinless nucleus, for instance, this gives a suppression factor for the monopole catalysis of nucleon decay $\sim(\beta/\beta_0)^{2\nu}$ with $\nu = -\frac{1}{2} + (\frac{1}{4} + |q|)^{1/2}$, β being the monopole velocity in units of light velocity, and β_0 a constant which depends on the nucleus. Thus the cross section for the Rubakov process in iron with $Z=26$ receives a strong suppression factor of order 10^{-6} for monopole velocity $\beta \sim 10^{-4}$. Such a suppression factor, however, disappears when the monopole velocity increases to $\beta \gtrsim 10^{-3}$. For oxygen the suppression factor is 10^{-5} for $\beta \sim 10^{-4}$ and a suppression of order 10^{-2} persists even for $\beta \sim 10^{-3}$.

Let us take the Hamiltonian for the monopole-nucleus system as

$$H = (2m_A)^{-1}(\vec{p} - Ze\vec{A})^2 - \vec{\mu}\cdot(g\hat{r}/4\pi), \tag{1}$$

with $\vec{\mu} = (eZ/2m_A)2\vec{s}(1+\kappa)$ the magnetic moment, κ the anomalous moment, $\vec{s}$ the spin, and $m_A = Am_N$ the mass of the nucleus.

From this we obtain[5]

$$H = \frac{1}{2m_A}\left(-\frac{d^2}{dr^2} - \frac{2}{r}\frac{r}{dr} + \frac{L^2}{r^2}\right), \tag{2}$$

where L^2 is the operator commutable with r and H,

$$L^2 = [\vec{r}\times(\vec{p} - Ze\vec{A})]^2 - 2q\vec{s}\cdot\hat{r}(1+\kappa), \tag{3}$$

with

$$q = (eg/4\pi)Z. \tag{4}$$

We may rewrite L^2 with the conserved total angular momentum $\vec{J}$,

$$L^2 = \vec{J}^2 + \vec{s}^2 - q^2 - 2\vec{s}[\vec{J} + q\hat{r}(1+\kappa)], \quad \vec{J} = \vec{r}\times(\vec{p} - Ze\vec{A}) - q\hat{r} + \vec{s}. \tag{5}$$

For the lowest angular momentum state $J = |q| - s$ (assuming $|q| \geq s$), we find that L^2 takes the minimum value

$$L^2 = |q|\{1 - 2(1+\kappa)s\}. \tag{6}$$

In particular,

$$L^2 = |q| \text{ for } s=0, \quad L^2 = -|q|\kappa \text{ for } s=\tfrac{1}{2}. \tag{7}$$

Expressing the eigenvalue of L^2 as $\nu(\nu+1)$ and correctly normalizing the radial wave function at $r=\infty$, we find that the wave function should behave as $(kr)^\nu$ or $(kr)^{-\nu-1}$ for small r. When we apply our argument to the Rubakov process, we take r as the order of magnitude of the nuclear radius $r \sim r_0 A^{1/3}$ ($r_0 = 1.2$ fm). Then we have a suppression factor for the monopole catalysis due to the distortion of the wave function as

$$F(\beta) = (kr)^{2\nu} \sim (\beta/\beta_0)^{2\nu} \tag{8}$$

TABLE I. Suppression factors $F(\beta) = (\beta/\beta_0)^{2\,\mathrm{Re}\,\nu}$ for slowly moving monopoles in matter.

nucleus	Z	2Reν	β_0	F(β)		
				$\beta=10^{-3}$	$\beta=5\times10^{-4}$	$\beta=10^{-4}$
^{4}He	2	1.236	0.0275	0.017	0.0071	9.7×10^{-4}
^{12}C	6	2.605	0.00636	0.0081	0.0013	2.0×10^{-4}
^{16}O	8	3.123	0.00434	0.0098	0.0012	7.7×10^{-6}
^{28}Si	14	4.385	0.00206	0.042	0.0020	1.7×10^{-6}
^{40}A	18	5.082	0.00128	-----	0.0084	2.4×10^{-6}
^{40}Ca	20	5.403	0.00128	-----	0.0062	1.0×10^{-6}
^{56}Fe	26	6.280	0.00082	-----	0.046	1.9×10^{-6}
^{208}Pb	82	11.84	0.00014	-----	-----	0.0157

with

$$\beta_0 = 1/(r_0 A^{1/3} m_A) = 1/(r_0 m_N A^{4/3}). \qquad (9)$$

We here note that when the nucleus has a positive anomalous magnetic moment $\kappa > 0$, the minimum value of L^2 is always negative [see Eq. (6)], giving a complex value for ν. In this case $\mathrm{Re}\,\nu = -\frac{1}{2}$, and we obtain an "enhancement" factor rather than the suppression factor. For example, we expect an enhancement of a factor 1.7×10^2 for monopoles with $\beta \sim 10^{-3}$ in hydrogen.[6]

The long-range force due to the angular momentum barrier thus modifies the cross section of monopole catalysis $\sigma_0 \sim 1/\beta$ (Refs. 3 and 4) to be

$$\sigma \sim \sigma_0(\beta/\beta_0)^{2\,\mathrm{Re}\,\nu} \sim \beta^{2\,\mathrm{Re}\,\nu - 1}.$$

We present some examples of the suppression factor $F(\beta)$ for $\beta = 10^{-3}$, 5×10^{-4}, and 10^{-4} in Table I. We see that in the heavy targets like ^{12}C, ^{16}O, ^{56}Fe, or ^{58}Fe we encounter a significant suppression of the Rubokov effect, and hence the naive argument would overestimate the probability of the process by a factor $F = 10^{-2}$–10^{-6} in the low-β region. Therefore the constraints on the monopole flux obtained from the Rubakov process in the underground experiments[2,3,7] should be modified by this factor F.

Finally we comment on a recent argument for the limit on the local abundance of monopoles from the observed subterranian heat of the Earth.[8] Monopoles with the velocity $\beta \lesssim 3\times10^{-5}$ may be trapped by the Earth. The Rubakov process caused by such slowly moving monopoles, however, is largely suppressed by two facts: The dominant components of the Earth are even-even nuclei and they contribute very little in the Rubakov process because of the suppression discussed above. Furthermore in any heavy elements more important suppression arises from a strong repulsive force against a slowly moving monopole because of the Zeeman effect and the diamagnetic effect on the atomic electrons.[5] Even in the case of light atoms such as helium the repulsive potential is about 16 eV (the energy difference between the ground state of the free helium atom and the helium atom united with the monopole),[9] and the monopole with $\beta < 10^{-4}$ can hardly touch the helium nucleus. For heavier atoms such a repulsive force greatly suppresses the Rubakov process, and it is unlikely to obtain the strong constraints as in Ref. 8 on the monopole abundance in the Earth or on the local monopole flux near the Earth.

[a]Present address: Research Institute for Fundamental Physics, Yukawa Hall, Kyoto University, Kyoto, Japan.

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