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Nuclear Physics and the Quark Model: A Study of Six Quarks with Chromodynamics

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The six-quark system has been studied in a nonrelativistic quark model which incorporates some of the features expected from chromodynamics. With use of a large basis space which includes color, spin, and orbital excitations, and a Hamiltonian completely determined by previous studies of baryon structure, a number of the features of low-energy nuclear physics are derived. When supplemented with a reasonable model for one-pion exchange, the residual color interactions give a good account of the properties of the deuteron.

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While progress is constantly being made in rigorously understanding the consequences of QCD, the theory is so complex that the only sufficiently versatile tools presently available for unraveling the phenomena of the confinement regime are some relatively crude QCD-like models. Nevertheless, since this sector of the theory seems likely to remain unsolved for some time, except perhaps in the simplest circumstances, we believe it is worthwhile to study some central issues of the strong-interaction problem in the context of these models.

The derivation of nuclear physics from QCD is one such central issue, both naturally and historically. We present here the results of a study[1] of the six-quark system in a QCD-like nonrelativistic quark model[2] which has had some success in describing hadronic structure, especially in the baryonic sector most relevant to the present calculation. By specializing to the case of three u and three d quarks and using variational methods, we have been able to work in a much larger basis space for the two-nucleon system than has heretofore been possible. The associated increases in accuracy and reliability of our direct six-quark calculation, together with its ease of interpretation, are its main advantages over previous calculations (which have all used what are essentially cluster scattering methods); its basic physics input is quite similar to most other work. Within this new context we have derived, using only parameters previously extracted from fits to baryon spectroscopy,[2] a number of features of low-energy nuclear physics.

The starting point for our calculations is the model Hamiltonian[2,3]

$$H=\sum_{i=1}^{6}(m_i+p_i^2/2m_i)+\sum_{i<j}(H_{\rm conf}{}^{ij}+H_{\rm hyp}{}^{ij}), \quad (1)$$

where, with $\vec{r}_{ij}=\vec{r}_i-\vec{r}_j$ and

$$S_{ij}=3\vec{S}_i\cdot\vec{r}_{ij}\,\vec{S}_j\cdot\vec{r}_{ij}/r_{ij}^2-\vec{S}_i\cdot\vec{S}_j,$$

$$H_{\rm conf}{}^{ij}=-[e_0+\tfrac{1}{2}kr_{ij}^2+U(r_{ij})]\tfrac{1}{2}\vec{\lambda}_i\cdot\tfrac{1}{2}\vec{\lambda}_j \quad (2)$$

is a two-body color confining potential and

$$H_{\rm hyp}{}^{ij}=-(\alpha_s/m_im_j)[\tfrac{8}{3}\pi\vec{S}_i\cdot\vec{S}_j\,\delta^3(\vec{r}_{ij})+r_{ij}^{-3}S_{ij}]\tfrac{1}{2}\vec{\lambda}_i\cdot\tfrac{1}{2}\vec{\lambda}_j \quad (3)$$

is the color hyperfine interaction expected from one-gluon exchange. $U(r_{ij})$ in (2) is an anharmonic term which represents the expected strong short-range attractive Coulomb-like interaction of QCD and also other departures from the harmonic limit.[4] We believe that this foundation for our calculation is fairly secure. For example, inaccuracies associated with the absence of spin-orbit terms in (1) and with relativistic corrections (relative to two separated clusters) are probably small.[1] We also believe that, while one must be cautious (as indeed we are; see below) with the presence of spurious long-range van der Waals-like forces in the calculation,[5] (1) is otherwise an acceptable phenomenological representation of confinement.[1]

Given the complicated structure of the six-quark system it is important to choose spatial coordinates which make a physical interpretation of the states obtained as simple as possible. In this regard we have been guided by the success of standard nuclear physics (especially in light of the results of Ref. 6) and have chosen as natural relative coordinates the internal coordinates of two three-quark clusters and the corresponding intercluster coordinate. Of course, if the dynamics of the calculation are such that three-quark clustering does not dominate the resulting state, this will be reflected in large exchange overlaps and an inability to interpret the intercluster wave functions in a simple manner.

The calculation then proceeds in three phases. In the first we restrict ourselves to states which are the appropriately antisymmetrized versions of two three-quark s-wave clusters in a relative s wave, but with all possible spin and color excitations. In the isospin basis these states are $N\frac{1}{2}N\frac{1}{2}$, $\Delta\frac{3}{2}\Delta\frac{3}{2}$, $N_c\frac{3}{2}N_c\frac{3}{2}$, $N_c\frac{1}{2}N_c\frac{1}{2}$, $N_c\frac{3}{2}N_c\frac{1}{2}$, and $\Delta_c\frac{1}{2}\Delta_c\frac{1}{2}$ in the $I=0$, $J=1$ channel and $N\frac{1}{2}N\frac{1}{2}$, $\Delta\frac{3}{2}\Delta\frac{3}{2}$, $N_c\frac{3}{2}N_c\frac{3}{2}$, $N_c\frac{1}{2}N_c\frac{1}{2}$, $\Delta_c\frac{1}{2}\Delta_c\frac{1}{2}$, and $\Delta_c\frac{1}{2}N_c\frac{1}{2}$ in the $I=1$, $J=0$ channel (our notation gives the isospin and spin of the clusters as $I_1S_1I_2S_2$; the subscript "c" denotes a cluster in a color octet state). The spatial wave functions of the three-quark clusters are taken to be those of the nucleon ground state,[2] while the intercluster wave function is taken to be of the form

$$\Psi(r) \sim \sum_{i=1}^{i_{\max}} \xi_i \exp(-\tfrac{1}{2}\beta_i^2 r^2) \tag{4}$$

in order to facilitate the calculations of the spatial matrix elements. The spin and color states in the six-quark color singlet sector are readily obtainable for any value of total spin. The physical states of a given sector are then obtained by antisymmetrizing in the three u quarks and three d quarks separately, allowing isospin to emerge dynamically in the space of available $I_3=0$ states. This procedure considerably reduces the calculational effort required and allows us to handle many more states than has previously been possible. In each sector of the problem the Hamiltonian matrix was constructed and diagonalized for various choices of β_i and ξ_i in (4) until a variational minimum was found. [In the event that no bound state was found the Hamiltonian (1) was supplemented by an artificial weak harmonic attraction which allowed one to extract information on the short-range behavior of the system.] In both the $J=0$ and $J=1$ sectors the resulting intercluster wave function was strongly suppressed at $r=0$ and the system showed an almost complete dynamical clustering into the neutron plus proton configuration (as measured by small admixtures of other color, spin, and isospin components and by small exchange integrals even within the np component). The "phase-I" bound-state effective np potential[7]

$$V_{\text{eff}} \equiv E + \frac{\nabla^2\Psi(r)}{M_N\Psi(r)} \tag{5}$$

was characterized by a large repulsive core and weak (~5-MeV) intermediate-range attraction, with the $^1S_0$ potential more repulsive for $r \lesssim 1$ fm and less attractive for $r \gtrsim 1$ fm than the $^3S_1$ potential.

In the second phase of the calculation we expanded the basis space of our system to include the most important states with up to two units of orbital excitation: states consisting of two spatially excited P-wave color octet clusters coupled to the phase-I np ground state by the spin singlet color-dependent potentials, and those (such as the np relative D-wave state) coupled by the tensor interaction. Given the (dynamically enforced) dominance of the np component and the smallness of the intercluster overlap integrals, we considered only matrix elements connecting these new states directly to the np system, their effect on the system being treated perturbatively and to leading order in the small exchange overlap integrals.[1] The ground-state energy was then reminimized in the old β_i, ξ_i as well as with respect to independent variations of the intercluster wave functions of the new states. While the tensor interaction proved to have an almost negligible effect, admitting spatially excited clusters significantly increased both the depth of the intermediate-range attraction and the $^3S_1$-$^1S_0$ splitting, while producing only small changes in the character of the repulsive core. The resulting "phase-II" effective nucleon-nucleon potentials associated purely with residual quark forces are shown in Fig. 1. They are quite similar to potentials com-

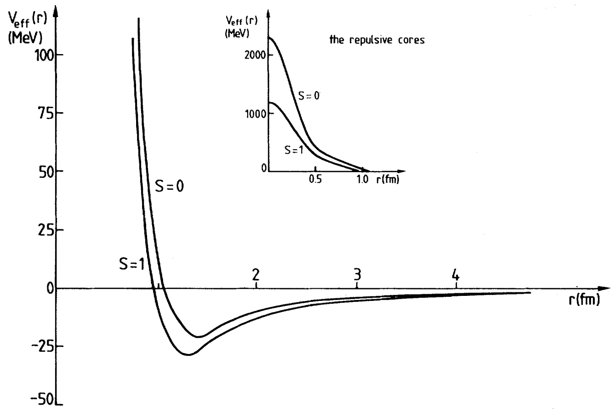

FIG. 1. The effective nucleon-nucleon potential from residual quark forces in the $^3S_1$ and $^1S_0$ channels.

monly used in low-energy nuclear physics.[8]

Given that mesons and nucleons have radii (r_M and r_N, respectively) which characterize the spatial extent of their quark substructure, it is difficult to imagine—in the most naive geometrical terms—that meson exchange can be very relevant in the two-nucleon system for distances $r \lesssim 2r_N + 2r_M$. For pions r is $\gtrsim 2$ fm; for excited mesons, it is even larger. Thus, in our view, it is probable that only residual quark forces contribute to the nucleon-nucleon potential at distances $r \lesssim 2$ fm. Conversely, at distances much greater than this, one expects simple quark dynamics to be inadequate: $q\bar{q}$ pair creation in this region will both cut off the spurious long-range part of the van der Waals potential generated by (1) and create meson-exchange contributions to the nucleon-nucleon potential. With the assumption of naive quark-pair-creation values for the couplings, however, all meson contributions except that of the pion are negligible at these distances, because of suppression by the Yukawa factor e^{-mr}/r. We therefore advocate a picture in which the long-range part of the nucleon-nucleon potential is dominated by pion exchange and the short-range part by residual quark forces.

In the third and final phase of the calculation we have implemented this hybrid picture in a semiquantitative manner by adding to our phase-II effective potential a modified one-pion exchange potential obtained by suppressing the pion field (and thereby the resulting potential) within the nucleons. The resulting potential produces negligible changes in the diagonal $^3S_1$ and $^1S_0$ potentials of Fig. 1, but leads to a significant $^3S_1$-$^3D_1$ mixing through its tensor part. Bearing in mind the qualitative nature of the hybrid model, we have made no attempt to "fine tune" the pion potential: We have only insisted that the suppression be operative below roughly 2 fm. With hybrid potentials having this character we always obtain a good qualitative description of the deuteron. Our results are given in Table I. The theoretical errors that we quote are rough estimates based mainly on difficulties associated with the spurious long-range part of the van der Waals force; far more reliable is the difference in energy between the deuteron and $^1S_0$ channels, for which we obtain -2.3 ± 0.3 MeV, in good agreement with experiment. While the existence of the core is unambiguously demonstrated by the strong suppression of the wave function Ψ as $r \to 0$, the magnitude of the repulsion at $r=0$ is less sensitively determined than is the attractive region and could, we estimate, still be in error by as much as 20%.

Two comments are in order regarding the origins of the major qualitative features of our effective potentials. First, as we have seen, most of the intermediate-range attraction traditionally associated with either two-pion or scalar meson exchange[8] is produced in the present model by the excitation of colored P-wave clusters. We expect this conclusion to remain valid independent of problems with the long-range part of this interaction since the features of (1) have been tested in the relevant intermediate region by spectroscopic studies of excited mesons and baryons. Second, our results on the antibinding effect of the spin-spin piece of the hyperfine interaction[1] are compatible with conclusions of other authors regarding this interaction as the source of the repulsive core.[9]

TABLE I. Some properties of the six-quark ground states.

	E_d (MeV)	$(r_E^2)_d^{1/2}$ (fm)	Q_d (mb)	μ_d [a]	$E(^1S_0)$ (MeV)
Theory	$-2.9^{+0.8}_{-0.3}$	4.5 ± 1.1	$+2.1 \pm 0.5$	$(+0.859 \pm 0.003)\mu_N$	$-0.4^{+0.4}_{-0.1}$
Experiment	-2.23	3.9	$+2.86$	$+0.857\mu_N$	Unbound

[a] μ_d is calculated under the assumption that the departure from $\mu_p + \mu_n$ is due only to our 3.6% D-wave mixing.

Many improvements could be made in this calculation, but unfortunately all those of which we are aware appear to require a considerable increase in effort. It would be especially interesting to perform a fuller variational calculation in which cluster sizes were no longer fixed beforehand but variationally determined (to do this would not only require a restudy of baryon spectroscopy, but also the calculation of much more complicated six-quark matrix elements since the simple forms[4] we have used for U and $H_{\rm hyp}$ would have to be modified) and in which intercluster wave functions were allowed to vary independently for distinct cluster configurations. This would, among other things, allow us to complete the proof of our conclusion (already strongly indicated[1]) that three-quark clustering completely dominates the deuteron, since the results of this more complete calculation would be independent of the initial cluster decomposition chosen. The present calculation can only draw conclusions on other clusterings (such as q^6) when the system as a whole has a size comparable to the fixed cluster size. We can now only offer the observation that our results are stable under reasonable variations of the cluster size about the value obtained from baryon spectroscopy.

To the reader unfamiliar with previous work in the area we should stress that attempts to understand the nucleon-nucleon force in the context of the quark model have a long history.[10] Our results appear to strongly confirm the emerging view that the repulsive core has a quark-model origin, and to point (for the first time we believe) to a similar origin for the intermediate-range attraction of the nuclear force.

In conclusion, although there are reasons for caution (as outlined above) one might optimistically interpret our results as evidence that even the present rather crude models for QCD are capable of explaining many of the basic features of low-energy nuclear physics.

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[1]This study is more fully described in the Ph.D. thesis [Department of Physics, University of Toronto, 1983 (unpublished)] of one of us (K.M.) and in Kim Maltman and Nathan Isgur, to be published.

[2]For a recent review see Nathan Isgur, in *Testing the Standard Model*, AIP Conference Proceedings No. 81, edited by C. Heusch and W. T. Kirk (American Institute of Physics, New York, 1982), p. 1.

[3]The $\vec{\lambda}_i \cdot \vec{\lambda}_j$ confinement potential has a long history, beginning with H. Nambu, in *Preludes in Theoretical Physics*, edited by A. de Shalit, H. Feshbach, and L. van Hove (North-Holland, Amsterdam, 1966), p. 133; H. J. Lipkin, Phys. Lett. 45B, 267 (1973). In its modern form, especially as applied to multiquark systems, it may be traced from H. J. Lipkin, in *Common Problems in Low- and Medium-Energy Nuclear Physics*, edited by B. Castel, B. Gouland, and F. C. Khanna (Plenum, New York, 1979), p. 173; R. S. Willey, Phys. Rev. D 18, 270 (1978); M. B. Gavela *et al.*, Phys. Lett. 79B, 459 (1979); N. Isgur, in *The New Aspects of Subnuclear Physics*, edited by A. Zichichi (Plenum, New York, 1980).

[4]The baryon spectrum up to $N=2$ is independent of the form of the perturbation U which we here chose to be a δ function to simplify the calculation. As with the hyperfine interaction, this δ function is smeared over our clusters, making it an allowed interaction and also reducing our sensitivity to this choice.

[5]For a recent pedagogical discussion, see O. W. Greenberg and H. J. Lipkin, Nucl. Phys. A370, 349 (1981), from which the literature can be traced.

[6]John Weinstein and Nathan Isgur, Phys. Rev. Lett. 48, 659 (1982), and Phys. Rev. D 27, 588 (1983). The calculations described in these papers are the meson-meson version of the present calculation.

[7]There are some ambiguities in such a definition associated with the existence of forbidden states; see Ref. 1.

[8]M. Lacombe *et al.*, Phys. Rev. C 21, 861 (1980); K. Holinde and R. Machleidt, Nucl. Phys. A256, 479 (1976); R. V. Reid, Ann. Phys. (N.Y.) 50, 411 (1968).

[9]See, for example, I. T. Obukhovsky *et al.*, Phys. Lett. 88B, 321 (1979); J. E. F. T. Ribiero, Z. Phys. C 5, 27 (1980).

[10]A discussion of this history along with an extensive set of references may be found in Ref. 1. The situation was recently reviewed by M. Harvey, in Proceedings of the Summer School on Progress in Nuclear Dynamics, Vancouver Island, 1982 (to be published). See also David A. Liberman, Phys. Rev. D 16, 1542 (1977); Carleton de Tar, Phys. Rev. D 17, 323 (1978); Obukhovsky *et al.*, Ref. 9; M. Harvey, Nucl. Phys. A352, 326 (1981); C. S. Warke and R. Shanker, Phys. Rev. C 21, 2643 (1980); Ribiero, Ref. 9; M. Oka and K. Yazaki, Phys. Lett. 90B, 41 (1980); M. Rosina *et al.*, Prog. Part. Nucl. Phys. 8, 417 (1982); Amand Faessler *et al.*, Phys. Lett. 112B, 201 (1982).