Photoinduced Macroscopic Quantum Tunneling in Superconducting Interference Devices

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It is shown that a time-varying magnetic field through the superconducting loop in a rf SQUID can cause transitions between the fluxoid states. Experimental observation of these transitions should constitute evidence for the quantum mechanical character of the macroscopic variable, the total flux through the loop. In the case that dissipation is characterized by linear damping in the quasiclassical limit, the photon absorption process depends critically on the ratio of the resistance of the weak link to the fundamental unit of resistance h/e^2.

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It has recently been argued that quantum tunneling between the metastable fluxoid states in a superconducting quantum interference device (SQUID) may constitute evidence that a collective variable which describes the macroscopic state of a complex system, in this case the total flux through the superconducting ring, obeys the superposition principle of quantum mechanics.[1] In addition, since the stability of supercurrent and the associated phenomenon of flux quantization are such striking features of a superconductor, it is of some interest to ask when and how the stability of the supercurrent-carrying state breaks down, especially at very low temperatures where the only available fluctuations are quantum. Since for a bulk superconductor the decay rate due to quantum fluctuations is cosmologically long, it is indeed remarkable that in going from a superconducting ring to one interrupted by a weak link (a rf SQUID) the rate of decay of current-carrying states due to quantum fluctuations can be made experimentally accessible.

As important feature of macroscopic tunneling is the role of dissipation, as represented by the coupling between the macroscopic variable and the environment. This coupling can potentially take many forms, but of particular interest is the case where the quasiclassical equation of motion (where the total flux plays the role of a particle coordinate) contains a dissipative term linear in the time derivative of the coordinate. The applicability of this form of dissipation rests on the semiquantitative success of the phenomenological equation commonly known as the resistively shunted junction (RSJ) equation.[2] Recent calculations have shown that, while dissipation can be expected to result in a strong suppression of the tunneling rate, the rate should still, at least in principle, be observable at low temperatures with present-day experimental techniques.[3] Indeed such a tunneling rate may already have been observed in the related problem of a current-biased Josephson junction.[4] More recently one of us[5] examined the case in which the applied external flux (i.e., the flux through the loop due to the applied external magnetic field alone) in a SQUID is $(2n+1)\varphi_0/2$, where n is an arbitrary integer and φ_0, the flux quantum, is equal to $h/2e$. In this case, if $\beta_L = 2\pi L I_c/\varphi_0$ is greater than unity, the potential energy for the total flux consists of two degenerate wells. Here L is the inductance of the loop and I_c the critical current of the weak link. It was shown[5] that linear dissipation of the sort discussed above can result in a spectacular reduction of the tunneling rate, leading to a vanishing mean rate at zero temperature if the resistance of the weak link is less than a critical resistance.

In this paper we consider the effect of the coupling of the macroscopic variable, the total flux through the SQUID, to the electromagnetic field, and explore the possibility of photoassisted tunneling between the fluxoid states. Such a photoinduced transition, if observed, would be striking because it involves a transition between two macroscopically distinguishable states caused by the absorption of a single photon. Furthermore, the observation of the transition process would be a direct verification of the macroscopic quantum behavior since h appears explicitly in the relation

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between two measurable quantities, the energy difference of the two states between which the transition takes place and the threshold for the induced transition, as well as implicitly in the magnitude of the transition rate. Also striking is the fact that in the presence of linear dissipation the photon absorption rate depends critically on the ratio of the normal resistance to the same critical resistance mentioned above.

The Lagrangian that we use to describe the complete system is the one employed by Caldeira and Leggett[3]:

$$\mathcal{L}=\tfrac{1}{2}C\dot{\varphi}^2-U(\varphi)+\sum_\alpha(\tfrac{1}{2}m_\alpha\dot{x}_\alpha^2-\tfrac{1}{2}m_\alpha\omega_\alpha^2x_\alpha^2)-\varphi\sum_\alpha f_\alpha x_\alpha$$

Here C is the capacitance and the set of variables $\{x_\alpha,\dot{x}_\alpha\}$ represents the degrees of freedom of the environment whose spectral density $J(\omega)$ is

$$J(\omega)=\tfrac{1}{2}\pi\sum_\alpha(f_\alpha^2/m_\alpha\omega_\alpha)\delta(\omega-\omega_\alpha).$$

All information concerning the effect of the environment on the flux dynamics is contained in $J(\omega)$. It can be shown that if $J(\omega)\to\omega/R$ as $\omega\to 0$, then in the classical limit φ obeys the equation of motion given by

$$C\ddot{\varphi}+\dot{\varphi}/R=-dU(\varphi)/d\varphi,$$

which is precisely the widely used phenomenological RSJ equation. We shall consider a model in which $J(\omega)=\omega/R$ up to a high-frequency cutoff ω_c. The potential energy $U(\varphi)$ is, in the case of a SQUID,

$$U(\varphi)=\frac{(\varphi-\varphi_{\rm ext})^2}{2L}-\left(\frac{I_c\varphi_0}{2\pi}\right)\cos\left(\frac{2\pi\varphi}{\varphi_0}\right).$$

When β_L is greater than unity, $U(\varphi)$ consists, in general, of more than one minimum (see Fig. 1). φ plays the role of a macroscopic "particle" coordinate. We consider the situation in which U has at least two minima so that the particle can make a transition from the lowest minimum to a metastable minimum an energy 2ϵ above it via the absorption of a photon with energy $\hbar\omega\geq 2\epsilon$.

Now, imagine introducing a small-amplitude time-dependent magnetic field through the superconducting loop which produces a time-dependent external flux, $\varphi_{\rm ext}=\varphi_{\rm ext}^0+\delta\varphi_{\rm ext}\cos(\omega t)$. We assume that we are in the linear response regime, or in other words that $\delta\varphi_{\rm ext}$ is small. We are interested in calculating the transition rate of the particle between the lowest states of adjacent metastable wells. The calculation will be performed in a truncated basis. We expect this approximation to be valid, at least so long as $\hbar\omega_0\gg 2\epsilon$, where $\hbar\omega_0$ is the spacing between levels in a single well (see Fig. 1). We are left with an effective two-level system (corresponding to the two wells) coupled to the environment described by the Hamiltonian

$$H=-\epsilon\sigma_z-\Delta\sigma_x+\sum_\alpha\hbar\omega_\alpha(b_\alpha^\dagger b_\alpha+\tfrac{1}{2})+\varphi_m\sigma_z\sum_\alpha f_\alpha(\hbar/2m_\alpha\omega_\alpha)^{1/2}(b_\alpha^\dagger+b_\alpha),$$

where $\pm\varphi_m$ are the locations of the minima, the σ's are the Pauli matrices, and the b's are phonon creation operators. The mapping to the effective two-level system requires two further adjustments. Δ is an effective tunneling matrix element renormalized by the dissipation and the cutoff ω_c is proportional to the inverse tunneling time. For the case of linear dissipation these can be obtained in the manner described in Ref. 5. One begins with the exact partition function of the symmetric double well and evaluates it by tunneling paths which move between the two wells. The resulting partition function is identical to that of the two-level system, *term by term*, provided the tunneling matrix element is renormalized to $\Delta=\Delta_0\exp[-\tfrac{1}{2}(2\varphi_m)^2/hR]$, where Δ_0 is the matrix element in the absence of dissipation, and the cutoff ω_c of the environment is given by

$$2\pi\omega_c=[1/(RC)^2+2\pi^2\overline{V}/\varphi_m^2C]^{1/2}-1/RC,$$

where

$$\overline{V}=(1/\varphi_m)\int_0^{\varphi_m}d\varphi[U(\varphi)-U(\varphi_m)].$$

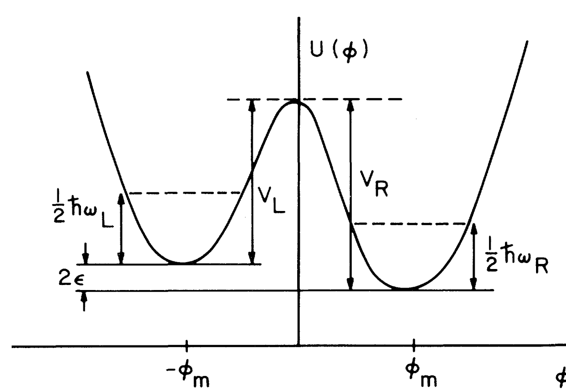

FIG. 1. Asymmetric double-well potential. For a typical set of parameters refer to the text.

In the weak damping limit ω_c is equal to the small oscillation frequency ω_0 in the well and in the strong damping limit it is proportional to $\omega_0^2 RC$. For small asymmetry 2ϵ (which is a tunable parameter) these adjustments are completely sufficient. For other models of the dissipation, the effective parameters can be obtained in an analogous fashion. The additional time-dependent term in the Hamiltonian is given by

$$\Delta H = (\varphi_m/L)\delta\varphi_{\mathrm{ext}}\cos(\omega t)\sigma_z.$$

We now calculate the transition rate using Fermi's golden rule and evaluate the expression by expanding the wave functions in perturbation theory in powers of $\Delta/2\epsilon$. The calculation proceeds by exactly diagonalizing H with Δ set equal to zero, and then calculating the energy shifts and wave functions in perturbation theory in Δ. Finally the thermally averaged transition rate is calculated by averaging over the initial states of the environment and summing over the final states subject to an energy-conserving (for the complete system) δ function. The transition rate can be expressed as

$$\nu(\omega) = \frac{2\pi}{\hbar}\left(\frac{\varphi_m\delta\varphi_{\mathrm{ext}}}{L}\right)^2\left(\frac{2\Delta}{\hbar\omega}\right)^2 g(\omega) + O\left(\frac{\Delta}{\hbar\omega}\right)^4.$$

The expression for $g(\omega)$ is given by

$$g(\omega) = \int_{-\infty}^{\infty}\frac{dt}{2\pi\hbar}\exp[it(\omega - 2\epsilon/\hbar) - X(t),$$

where $X(t)$ is given by

$$X(t) = \frac{(2\varphi_m)^2}{h}\int_0^\infty d\omega\frac{J(\omega)}{\omega^2}\{(1-e^{-i\omega t})[1+N(\omega)] + N(\omega)(1-e^{i\omega t})\}.$$

The boson occupation factor $N(\omega) = (e^{\beta\hbar\omega}-1)^{-1}$. So far the results are quite general and no assumptions regarding $J(\omega)$ have been made. We shall now present the results for the case $J(\omega)\to\omega/R$ as $\omega\to 0$ and then discuss the results for other possible forms of the spectral density.

At $T=0$, $g(\omega)$ is given by the expression

$$g(\omega) = [\hbar\omega_c\Gamma(\alpha)]^{-1}\left(\frac{\hbar\omega_c}{\hbar\omega - 2\epsilon}\right)^{1-\alpha}\exp\left(\frac{2\epsilon - \hbar\omega}{\hbar\omega_c}\right)\Theta(\hbar\omega - 2\epsilon),$$

where $\alpha = (2\varphi_m)^2/hR$, Θ is the step function, and $\Gamma(\alpha)$ is the gamma function. Thus the transition rate at the threshold diverges as a power law if $\alpha < 1$ or vanishes as a power law if $\alpha > 1$. The dimensionless parameter α is precisely the one which determines whether the ground state exhibits broken symmetry or not in Ref. 5. If we believe that the model of the harmonic-oscillator heat bath is reasonable for the purpose of calculating the transition rate at finite temperatures (or at least for low temperatures), it is not difficult to calculate an expression for $g(\omega)$ at finite temperatures as well:

$$g(\omega) = \frac{1}{\pi\hbar}\,\mathrm{Re}\int_0^\infty dt\, e^{it(\omega - 2\epsilon/\hbar)}\left(\frac{\pi t/\beta\hbar}{(1+i\omega_c t)\sinh(\pi t/\beta\hbar)}\right)^\alpha.$$

To obtain the threshold behavior at finite temperatures, the integrand can be approximated by its large-t limit and one obtains

$$g\left(\frac{2\epsilon}{\hbar}\right) = \frac{2}{\alpha\pi\hbar\omega_c}\left(\frac{\hbar\omega_c}{2\pi k_B T}\right)^{1-\alpha}\cos\left(\frac{\alpha\pi}{2}\right).$$

For values of $\omega \sim 2\epsilon$, the behavior of $g(\omega)$ is insensitive to any details of the high-frequency behavior of $J(\omega)$; it depends only on the magnitude of the cutoff frequency, ω_c. It is always symptomatic of a problem involving an infrared divergence that all scales up to a high-frequency cutoff contribute. At present, it is not entirely clear that in all physically interesting cases the low-frequency behavior of $J(\omega)$ is well represented by $J(\omega)\sim\omega/R$. In fact, it probably is not a good representation of an oxide-layer junction.[6] Although in this case the spectral dependence of $J(\omega)$ can be quite different, the basic phenomena of photoinduced transitions are unaffected. For example, so long as $J(\omega)$ vanishes for small ω faster than ω^ν with $\nu > 1$, $g(\omega)$ will consist of a zero-phonon line at $\omega = 2\epsilon/h$ and a multiphonon background.

We now turn to experimental implications of our results for the transition rate. In order that the truncation to a two-level problem be justified it is necessary that the barrier heights $V_{L,R}$ (L standing for the left well and R for the right) be considerably larger than $h\omega_{L,R}/2$, where $h\omega_{L,R}$ is the spacing between the energy levels in the respective wells, and that $h\omega_{L,R}$ be large compared to 2ϵ. For the perturbation theory in Δ to be valid we must also have $2\epsilon \gg \Delta$. Although the calculations are difficult (though not unmanageable) to perform for a situation in which the truncation to two levels is not justified, the physical effect, at least in a qualitative sense, surely exists regardless. In any case the parameter space is sufficiently large so that the above conditions can be satisfied in a number of experimental situations. Finally, the transition rate at threshold is nonzero only for $\alpha < 1$. For a typical value of $\varphi_m = 0.3\varphi_0$, $\alpha < 1$ when R is greater than $R_c \sim 2323\ \Omega$.

To be more specific, let us calculate the transition rate for a possible set of parameters: $\alpha = 1$, $\beta_L = 2.0$, $C = 10^{-14}$ F, $I_c = 10^{-6}$ A, $L = 6.6 \times 10^{-10}$ H, and $\varphi_{ext}^{(0)} = (2n+1)\varphi_0/2 + 0.025\varphi_0/2\pi$. (Of course, $\varphi_{ext}^{(0)}$ can be varied to obtain the desired value of 2ϵ.) For these parameters we estimate the following: $h\omega_{L,R} \sim 3.8$ K, $V_L = 9.5$ K, $V_R = 10.6$ K, $2\epsilon = 1.1$ K, $\Delta \sim 0.4 \times 10^{-4}$ K, and $\varphi_m \sim 0.3\varphi_0$. If we assume that $\varphi_m \delta\varphi_x \sim 10^{-3}\varphi_0^2$, such that the amplitude of the oscillating magnetic field is indeed very small, it is easy to see that the transition rate is of the order of 10^{+2} Hz. An important point to note is that it is not necessary that the experimental temperature, T, be small compared to Δ, which is tiny, but only that $T \ll 2\epsilon$.

Finally, we should emphasize that this set of parameters was chosen to demonstrate experimental feasibility. However, the parameter space is large and no attempt was made to find experimentally optimal parameters.

Unlike the case of atomic spectroscopy, where one can take advantage of a large ensemble of atoms, the power absorbed from the ac field by a single SQUID is very small. The signature of the transition is probably to be found in the actual change of the total flux through the loop. Since this is a macroscopic quantity, it is measurable. Finally, we note that if the system is prepared in the upper well, it is possible for it to decay to the lower well via the emission of a photon. This transition can either be induced, by an ac field with frequency $\omega < 2\epsilon/h$, or proceed via spontaneous emission of a photon. The spontaneous emission rate can be derived from an Einstein relation. At temperature very small compared to 2ϵ,

$$\nu_s = \nu_0 \int_0^{2\epsilon/\hbar} d\omega \left(\frac{2\epsilon/\hbar - \omega}{\omega_c}\right) g(\omega + 2\epsilon/\hbar)$$
$$\approx \frac{\nu_0}{\Gamma(\alpha+2)} \left(\frac{2\epsilon}{\hbar\omega_c}\right)^{\alpha+1},$$

where $\nu_0 = \frac{1}{3}(8\varphi_m S_0 \Delta/Lhc)^2(\omega_c/hc)$, S_0 is the area of the SQUID, and in the second line we have evaluated the integral for the RSJ model. Using the same set of parameters discussed previously, with $S_0 \sim 10^{-4}$ cm^2, we find $\nu_0 \sim 75$ Hz.

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