Quasilinear Evolution of Current-Driven Ion-Acoustic Instability in a Magnetic Field

O. Ishihara and A. Hirose
Department of Physics, University of Saskatchewan, Saskatoon S7N 0W0, Canada
(Received 10 June 1982; revised manuscript received 1 March 1983)

Three-dimensional evolution of current-driven ion-acoustic instability in a magnetic field is studied numerically in the quasilinear framework. The electron-cyclotron resonance (anomalous Doppler effect) is shown to be responsible for the observed rapid cross-field heating of electrons. The formation of a high-energy ion tail, previously studied in a one-dimensional model, remains effective both parallel and perpendicular to the electron current and governs the saturation of the instability, subsequent decay, and anomalous k spectrum dominated by cross-field modes.

PACS numbers: 52.35.Dm, 52.35.Py

The simulation by Dum, Chodura, and Biskamp[1] has indicated that the evolution of current-driven ion-acoustic instabilities in a collisionless plasma is governed by quasilinear effects. Recent studies[2,3] indeed support this, although these studies have been limited to either one-dimensional[2] or unmagnetized plasma.[3] In practice, most experiments[4-6] relevant to the instabilities are done in a strong magnetic field which may greatly modify the evolution of the instabilities. In a strong magnetic field, turbulent electron heating is expected to occur only along the magnetic field since the cross-field electron motion is governed by $\vec{E}\times\vec{B}$ velocity, which is small for reasonable fluctuation level. Yet it has been widely observed that effective electron heating perpendicular to a magnetic field occurs in turbulent heating experiments. It will be shown that transfer from parallel electron energy to perpendicular energy is governed by the anomalous collision frequency, and thus extremely rapid.

Another surprising observation[6,7] in current-driven ion-acoustic instability in a magnetic field is that the $\vec{k}$ spectrum of the fluctuations is dominated by those modes propagating at large angles to the magnetic field, or the electron drift. (Linear analyses for Maxwellian electrons and ions predict the maximum growth along the magnetic field.) It will be shown that the anisotropic high-energy ion tail, which is formed quasilinearly, can explain the anomalous $\vec{k}$ spectrum.

We consider a plasma in which hot electrons are drifting with constant velocity relative to cold ions along a uniform external magnetic field. The magnetic field intensity is varied in a wide range, $\omega_{ce}/\omega_{pe}=0.2-5$, where $\omega_{ce}=eB/mc$ is the electron cyclotron frequency and $\omega_{pe}=(4\pi n_0 e^2/m)^{1/2}$ is the electron plasma frequency. Ion-acoustic instability is characterized by the frequency range $\omega \lesssim \omega_{pi}$ (ion plasma frequency), and we may assume the following ordering: $\omega_{ci} \ll \omega \lesssim \omega_{pi} \ll \omega_{ce}$ where $\omega_{ci}=eB/Mc$ is the ion cyclotron frequency. Ions are then assumed to be unmangetized, while electrons are magnetized and expected to be strongly influenced by the magnetic field. The distribution functions of ions and electrons evolve according to the quasilinear equation

$$\frac{\partial f_j}{\partial t}=\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}v_\perp\left(D_{\perp\perp}\frac{\partial f_j}{\partial v_\perp}+D_{\perp\parallel}\frac{\partial f_j}{\partial v_\parallel}\right)+\frac{\partial}{\partial v_\parallel}\left(D_{\perp\parallel}\frac{\partial f_j}{\partial v_\perp}+D_{\parallel\parallel}\frac{\partial f_j}{\partial v_\parallel}\right)-\frac{e_j}{m_j}E_0(t)\frac{\partial f_j}{\partial v_\parallel}, \tag{1}$$

where $f_j=f_i$, $e_j=e$, $m_j=M$ for ions and $f_j=f_e$, $e_j=-e$, $m_j=m$ for electrons. The external electric field $E_0(t)$ is imposed to maintain constant current. The ion diffusion tensor contains both resonant and nonresonant contributions and is formulated by Ishihara and Hirose.[8] The problem of "negative diffusion coefficient" for nonresonant particles has been resolved by Bodner[9] and Kaufman.[10] For damped waves, nonresonant particles obviously cannot diffuse as argued by Bodner, and we put γ_k (growth or damping rate) in $D_{\rm NR}$ (the nonresonant part of the diffusion coefficient) equal to zero whenever γ_k becomes negative. This procedure had been correctly used already by Field and Fried,[11] who numerically analyzed the quasilinear evolution of electron distribution function for the first time. The electron diffusion tensor is given by

$$\begin{pmatrix} D_{\parallel\parallel} \\ D_{\perp\parallel} \\ D_{\perp\perp}\end{pmatrix}=\sum_{\vec{k}}\frac{8\pi^2 e^2}{m^2 k^2}\mathcal{E}_{\vec{k}}(t)\sum_{n=-\infty}^{\infty}J_n^2\left(\frac{k_\perp v_\perp}{\omega_{ce}}\right)\delta(\omega_{\vec{k}}-k_\parallel v_\parallel-n\omega_{ce})\begin{pmatrix} k_\parallel^2 \\ k_\parallel n\omega_{ce}/v_\perp \\ n^2\omega_{ce}^2/v_\perp^2\end{pmatrix}, \tag{2}$$

where $\mathcal{E}_{\vec{k}}$ is the electric field energy density associated with a mode $\vec{k}$ and J_n is the Bessel function of order n. The electron diffusion tensor includes the Cherenkov resonance ($\omega_{\vec{k}} = k_{\parallel} v_{\parallel}$) and cyclotron resonances, both anomalous ($n < 0$) and normal ($n > 0$) Doppler effects. The evolution of the fluctuation energy $\mathcal{E}_{\vec{k}}$ is described by

$$\partial \mathcal{E}_{\vec{k}} / \partial t = 2 \gamma_{\vec{k}} \mathcal{E}_{\vec{k}}, \tag{3}$$

with the quasilinear growth rate

$$\gamma_{\vec{k}} = - \frac{\epsilon_i}{\partial \epsilon_r / \partial \omega_{\vec{k}}} = - \frac{\epsilon_i^{(i)} + \epsilon_i^{(e)}}{\partial \epsilon_r / \partial \omega_{\vec{k}}}, \tag{4}$$

$$\epsilon_i^{(i)} = - \frac{4\pi e^2}{M k^2} \int d^3 v \left(\frac{\omega_{\vec{k}} - k_{\parallel} v_{\parallel}}{v_{\perp}} \frac{\partial f_i}{\partial v_{\perp}} + k_{\parallel} \frac{\partial f_i}{\partial v_{\parallel}} \right) [(k_{\perp} v_{\perp})^2 - (\omega_{\vec{k}} - k_{\parallel} v_{\parallel})^2]^{-1/2}, \tag{4a}$$

$$\epsilon_i^{(e)} = - \frac{4\pi^2 e^2}{m k^2} \sum_{n=-\infty}^{\infty} \int d^3 v J_n^2 \left(\frac{k_{\perp} v_{\perp}}{\omega_{ce}} \right) \delta(\omega_{\vec{k}} - k_{\parallel} v_{\parallel} - n \omega_{ce}) \left(\frac{n \omega_{ce}}{v_{\perp}} \frac{\partial f_e}{\partial v_{\perp}} + k_{\parallel} \frac{\partial f_e}{\partial v_{\parallel}} \right). \tag{4b}$$

The oscillation frequency $\omega_{\vec{k}}$ is determined through the linear dispersion relation

$$\epsilon_r = 1 - \frac{\omega_{pi}^2}{\omega_{\vec{k}}^2} + \frac{4\pi e^2}{m k^2} \sum_{n=-\infty}^{\infty} \int d^3 v \frac{J_n^2 (k_{\perp} v_{\perp} / \omega_{ce})}{\omega_{\vec{k}} - k_{\parallel} v_{\parallel} - n \omega_{ce}} \left[\frac{n \omega_{ce}}{v_{\perp}} \frac{\partial f_e}{\partial v_{\perp}} + k_{\parallel} \frac{\partial f_e}{\partial v_{\parallel}} \right] = 0, \tag{5}$$

which reduces to the relations

$$1 - \omega_{pi}^2 / \omega_{\vec{k}}^2 + (k_{D\parallel}^2 / k^2)(1 - 2 V_d^2 / v_{e\parallel}^2) = 0$$

for $\omega_{ce} \ll \omega_{pe}$ and

$$1 - \omega_{pi}^2 / \omega_{\vec{k}}^2 + (k_{D\parallel}^2 / k^2)[1 - 2(V_d^2 / v_{e\parallel}^2) \exp(-\lambda) I_0(\lambda)] + (k_{D\perp}^2 / k^2)(1 - T_{e\perp} / T_{e\parallel})[1 - \exp(-\lambda) I_0(\lambda)] = 0$$

for $\omega_{ce} \gg \omega_{pe}$, where $k_{D\parallel} = (4\pi n_0 e^2 / T_{e\parallel})^{1/2}$, $k_{D\perp} = (4\pi n_0 e^2 / T_{e\perp})^{1/2}$, $v_{e\parallel} = (2 T_{e\parallel} / m)^{1/2}$, $v_{e\perp} = (2 T_{e\perp} / m)^{1/2}$, $\lambda = (k_{\perp} v_{e\perp} / \omega_{ce})^2 / 2$, and $I_0(\lambda)$ is a modified Bessel function of the 0th order. The parallel and perpendicular temperatures are defined as

$$T_{e\parallel}(t) = (m / n_0) \int d^3 v (v_{\parallel} - V_d)^2 f_e(v_{\perp}, v_{\parallel}, t), \tag{6}$$

$$T_{e\perp}(t) = (m / 2 n_0) \int d^3 v v_{\perp}^2 f_e(v_{\perp}, v_{\parallel}, t). \tag{7}$$

The system of Eqs. (1)–(5) has been solved by the method of finite differences—alternating direction implicit method.[12] Ions are initially Maxwellian with isotropic temperature, $T_i(t=0) = T_{i0}$, while electrons are initially shifted Maxwellian with a drift velocity V_d in the z direction and isotropic temperature, $T_e(t=0) = T_{e0}$. Essential boundary conditions are assumed, i.e., $f_i \to 0$ and $f_e \to 0$ for $v \to \infty$, and the condition $(\partial f_i / \partial v_{\parallel})_{v_{\parallel}=0} = 0$ is imposed to confine the computational range to the positive region ($v_{\parallel} > 0$) for ions. The delta function in Eqs. (2) and (4b) is approximated by a smooth function with a half-width of velocity mesh size. Typical results are shown in Figs. 1–3 with parameters $M/m = 1836$, $T_{i0}/T_{e0} = 0.01$, $V_d = 0.3 v_{e0}$, and $\omega_{ce}/\omega_{pe} = 1$. Initial fluctuation energy density $\mathcal{E}(k_{\perp}, k_{\parallel}) = 0.5 \times 10^{-5} n_0 T_{e0}$ is given to each mode in the domain defined by $k_{\perp}/k_{D0} = [0.2, 1.0]$ and $k_{\parallel}/k_{D0} = [0.05, 1.0]$ with $\Delta k_{\perp}$

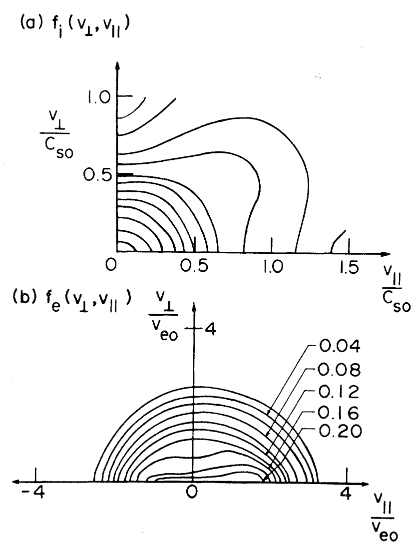

FIG. 1. (a) Space-averaged ion distribution function in velocity space at $t = 60\omega_{pi}^{-1}$. Logarithmically equally spaced contours are drawn. The high-energy tail extends not only in the direction of the electron drift but also in the oblique direction. The velocity is normalized by the initial ion-acoustic velocity, C_{s0}. (b) Space-averaged electron distribution function at $t = 60\omega_{pi}^{-1}$. The velocity is in units of initial electron thermal velocity, v_{e0}.

$=0.2k_{D0}$ and $\Delta k_{\parallel}=0.05k_{D0}$, where $k_{D0}=(4\pi n_0 e^2/T_{e0})^{1/2}$. In computation, the summation over n for the Bessel function runs up to $|n|=5$.

Figure 1(a) shows the ion distribution function at $t=60\omega_{pi}^{-1}$. The formation of a high-energy tail is clearly seen in both directions, parallel and perpendicular to the electron drift. Even by the time of $t=90\omega_{pi}^{-1}$, the bulk ions are only slightly heated ($0.01T_{e0}$ to $0.02T_{e0}$) and energetic tail ions are characterized by anisotropic temperatures, $T_{ih\parallel}\simeq 0.5T_{e0}$ (parallel) and $T_{ih\perp}\simeq 0.1T_{e0}$ (perpendicular). The ion population of the high-energy tail amounts to 6% of the whole population.

Figure 1(b) shows the electron distribution function at $t=60\omega_{pi}^{-1}$. Heating in the perpendicular direction as well as the parallel can be clearly seen. If the cyclotron resonance terms in the electron diffusion equation are artificially suppressed, no heating in the perpendicular direction is observed. After $\omega_{pi}t=70$, $f_e(v_{\parallel}, v_{\perp}=0)$ in a drift frame takes an asymptotic form of $f_e \propto \exp(-v_{\parallel}^{\alpha})$, where $\alpha \simeq 4.3-5.0$. It is interesting to note that this value is somewhat larger than 3.6–4.0 observed in a computer simulation[1] and somewhat smaller than 5 predicted in a quasilinear theory[13] both for an unmagnetized plasma.

Figure 2 shows the evolution of the fluctuation energy density and electron temperatures. The fluctuation energy saturates completely by $t=65\omega_{pi}^{-1}$ and slowly decays thereafter. The decay is mainly due to Landau damping through the high-energy ion tail. The electron temperatures, both parallel and perpendicular, grow even during the decay of the fluctuation energy.

The evolution of the electron distribution function may be summarized in the context first proposed by Rudakov.[14,15] First, electrons are turbulently heated (as in the one-dimensional case) through Cherenkov resonance. For the parameters $\omega_{ce} \lesssim \omega_{pe}$, there are a sufficiently large number of electrons which can experience the Doppler effects of cyclotron resonances. Parallel energy of electrons is then transferred to perpendicular energy through scattering caused by the cyclotron resonance. A quasilinear estimate for energy transfer in the ion-acoustic turbulence is $dT_{e\perp}/dt \simeq \alpha\nu^*(T_{e\parallel}-T_{e\perp})$ where $\alpha \simeq 1$ for $\omega_{ce} \lesssim \omega_{pe}$ and ν^* is the anomalous momentum-transfer collision frequency. Our numerical results are in good agreement with the above estimate. For the parameter $\omega_{ce} \gg \omega_{pe}$, the perpendicular scattering of electrons is no more appreciable and the in-

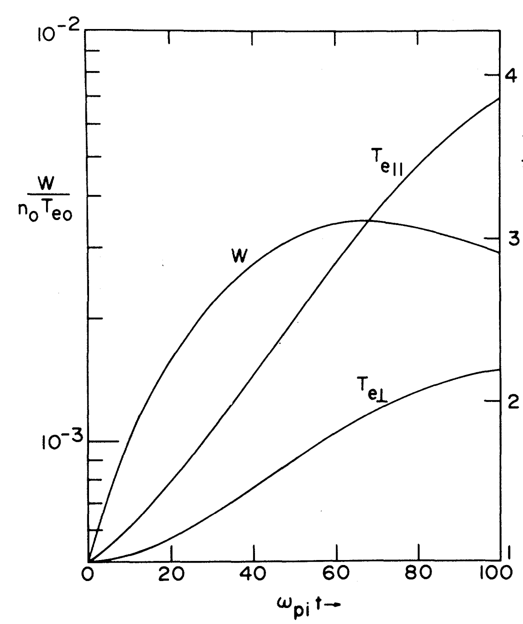

FIG. 2. Time evolution of the fluctuation energy density $W=\Sigma_{\vec{k}}\mathcal{E}_{\vec{k}}$ and parallel ($T_{e\parallel}$) and perpendicular ($T_{e\perp}$) electron temperatures.

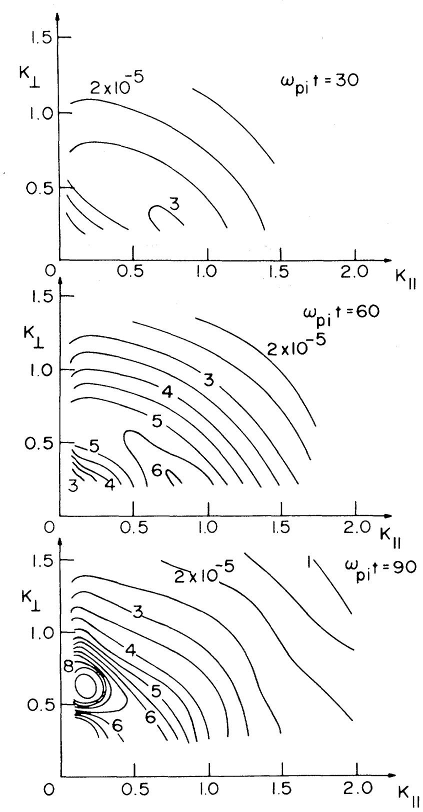

FIG. 3. Time evolution of the fluctuation spectrum $\mathcal{E}(K_{\perp}, K_{\parallel})/n_0T_{e0}$. $K_{\parallel}=k_{\parallel}/k_{D\parallel}$ and $K_{\perp}=k_{\perp}/k_{D\perp}$, where $k_{D\parallel}=(4\pi n_0 e^2/T_{e\parallel})^{1/2}$ and $k_{D\perp}=(4\pi n_0 e^2/T_{e\perp})^{1/2}$.

crease in $T_{e\perp}$ becomes insubstantial.

The time evolution of the spectral distribution $\mathcal{E}(k_\perp, k_\parallel)$ is shown in Fig. 3. A linear dispersion relation determines the energy spectrum for $\omega_{pi} t \lesssim 30$, and the dominant modes are in the direction of electron drift or magnetic field. As was shown, the ion tail extends widely in angles but predominantly toward the drift direction of electrons as the fluctuation grows. The formation of the ion tail makes the wave damp in the direction of the drift. As a result the oblique modes almost perpendicular to $\vec{V}_d$ or $\vec{B}$ become dominant in its nonlinear state as can be seen at $\omega_{pi} t = 90$. Such a pronounced transition of wave spectrum has already been observed for an unmagnetized plasma[8] and does not depend on the strength of the magnetic field,[16] although a particle simulation showed an increasing obliqueness of modes with magnetic field.[17] This may explain the experimental observations known as the anomalous $\vec{k}$ spectrum in the ion-acoustic instability aforementioned.

In conclusion, numerical solutions to the system of quasilinear equations reveal the multidimensional structure of the nonlinear evolution of the current-driven ion-acoustic instability in a magnetic field. The fluctuation energy density saturates at a level of $W/n_0 T_{e0} \approx 3.5 \times 10^{-3}$ accompanied by turbulent heating of electrons and ions. The heating rate of the electrons perpendicular to the magnetic field is large and of the order of the anomalous collision frequency for the parameter $\omega_{ce}/\omega_{pe} \lesssim 1$. The formation of the anisotropic high-energy ion tail is mainly responsible for the stabilization of the instability and the dominance of the cross-field modes.

This work is supported by the Natural Sciences and Engineering Research Council of Canada.

[1]C. T. Dum, R. Chodura, and D. Biskamp, Phys. Rev. Lett. 32, 1231 (1974).

[2]O. Ishihara and A. Hirose, Phys. Rev. Lett. 46, 771 (1981).

[3]K. Appert and J. Vaclavik, Plasma Phys. 23, 763 (1981).

[4]A. Hirose and H. M. Skarsgard, IEEE Trans. Plasma Sci. 5, 66 (1977).

[5]H. de Kluiver, C. J. Barth, H. J. B. M. Brocken, J. J. L. Caarls, B. de Groot, H. W. Kalfsbeek, H. W. Piekaar, A. Ravestein, W. R. Rutgers, B. de Stigter, H. W. H. Van Andel, and H. W. Van der Ven, in *Proceedings of the Seventh International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Innsbruck, Austria, 1978* (International Atomic Energy Agency, Vienna, Austria, 1979), Vol. 2, p. 639.

[6]W. Gekelman and R. L. Stenzel, Phys. Fluids 21, 2014 (1978).

[7]A. Hirose, K. E. Lonngren, and H. M. Skarsgard, Phys. Rev. Lett. 28, 270 (1972).

[8]O. Ishihara and A. Hirose, Phys. Fluids 26, 100 (1983).

[9]S. E. Bodner, J. Plasma Phys. 5, 141 (1971).

[10]A. N. Kaufman, J. Plasma Phys. 8, 1 (1972).

[11]E. C. Field and B. D. Fried, Phys. Fluids 7, 1937 (1964).

[12]J. Killeen and K. D. Marx, in *Methods in Computational Physics*, edited by B. Alder, S. Fernbach, and M. Rotenberg (Academic, New York, 1970), Vol. 9, p. 421.

[13]R. Z. Sagdeev and A. A. Galeev, *Nonlinear Plasma Theory* (Benjamin, New York, 1969).

[14]L. I. Rudakov, in *Proceedings of the Fourth International Conference on Plasma Physics and Nuclear Fusion Research, Madison, Wisconsin, 1971* (International Atomic Energy Agency, Vienna, Austria, 1972), Vol. 2, p. 235.

[15]L. I. Rudakov, Zh. Eksp. Teor. Fiz. 60, 2134 (1971) [Sov. Phys. JETP 33, 1148 (1971)].

[16]A detailed study for various parameters of ω_{ce}/ω_{pe} will be reported separately. But most features of the instability in the nonlinear stage are the same except the weaker perpendicular electron heating for the case of $\omega_{ce}/\omega_{pe} \gg 1$.

[17]D. Biskamp and R. Chodura, in *Proceedings of the Fourth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Madison, Wisconsin, 1971* (International Atomic Energy Agency, Vienna, Austria, 1972), Vol. 2, p. 265.