Giant Resonances as Oscillations of Two Elastically Coupled Fluids

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The classical equations of motion for elastically coupled, elastic proton and neutron fluids allow two $T=0$ and two $T=1$ propagating modes. The Lamé coefficients evaluated empirically are seen to be very different from each other, yielding the large Poisson ratio of 0.485 for nuclear matter and the $T=0$ and $T=1$ giant-resonance energies, many of which have been seen experimentally.

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In medium and heavy nuclei the collective behavior at an excitation energy of a few megaelectronvolts and above can be characterized as that of a viscoelastic fluid. This was realized when the giant resonances were interpreted as the elastic vibrations[1,2] of the nuclear fluid. If the angular frequency ω of the oscillations and the relaxation time τ of the stresses in the nucleus (which is related to the widths of giant resonances) fulfill $\omega\tau \ll 1$, the medium behaves like a viscous fluid. If $\omega\tau \gg 1$, the medium is endowed with an elastic response to both normal and shear stresses, and can be treated like an elastic solid. One such model dealing with the latter domain was explicitly constructed by Wong and Azziz.[3] However, the resonance energies of most of the isoscalar multipole states were found to lie above the corresponding isovector states.[4] This can be remedied by altering the boundary conditions for the isovector case.[5]

Two fluids can be coupled dissipatively through their velocities. In the elastic regime we can think of two elastic fluids occupying the same volume and coupled elastically. A nonrelativistic Lagrangian density for this system is

$$\mathcal{L} = \tfrac{1}{2}\rho_p \dot{u}_i \dot{u}_i + \tfrac{1}{2}\rho_n \dot{v}_i \dot{v}_i - \mu_p u_{ij}u_{ij} - \mu_n v_{ij}v_{ij} - \tfrac{1}{2}\lambda_p \delta_{ij} u_{kk} u_{ij} - \tfrac{1}{2}\lambda_n \delta_{ij} v_{kk} v_{ij} - \mu_1 (u_{ij}-v_{ij})^2 - \tfrac{1}{2}\lambda_1 \delta_{ij}(u_{kk}-v_{kk})(u_{ij}-v_{ij}),$$

where ρ_p and ρ_n are the proton and neutron densities, respectively, and u_i and v_i are the displacement fields of elements of the fluids relative to Cartesian axes. λ_p, λ_n, μ_p, and μ_n are the coefficients that couple the elements of the fluids to the medium, related to the Lamé coefficients λ and μ by

$$\mu_p = \frac{\rho_p \mu}{\rho_p+\rho_n}, \quad \mu_n = \frac{\rho_n \mu}{\rho_p+\rho_n}, \quad \lambda_p = \frac{\rho_p \lambda}{\rho_p+\rho_n}, \quad \lambda_n = \frac{\rho_n \lambda}{\rho_p+\rho_n}.$$

λ_1 and μ_1 are the coefficients that elastically couple the elements of the fluids to each other. Summation of all repeated indices is assumed. u_{ij} and v_{ij} are the symmetrized Cartesian strain tensors for the proton and the neutron fluids, respectively: $u_{ij}=\frac{1}{2}(\partial_j u_i + \partial_i u_j)$ and $v_{ij}=\frac{1}{2}(\partial_j v_i + \partial_i v_j)$. Inserting this Lagrangian in the Euler-Lagrange equations of motion for continuous systems we obtain the following coupled system of equations of motion:

$$\rho_p \ddot{u}_i = \partial_i \partial_j [(\lambda_p + \lambda_1 + \mu_p + \mu_1) u_j - (\lambda_1 + \mu_1) v_j] + \partial_j \partial_j [(\mu_p + \mu_1) u_i - \mu_1 v_i],$$

$$\rho_n \ddot{v}_i = \partial_i \partial_j [(\lambda_n + \lambda_1 + \mu_n + \mu_1) v_j - (\lambda_1 + \mu_1) u_j] + \partial_j \partial_j [(\mu_n + \mu_1) v_i - \mu_1 u_i].$$

These are only valid in Cartesian coordinates and aside from the coupling terms (with negative sign) are equivalent to the Lamé equation. If we take $\nabla\times(\vec{u}+\vec{v})$, $\nabla\times(\vec{u}-\vec{v})$, $\nabla\cdot(\vec{u}+\vec{v})$, and $\nabla\cdot(\vec{u}-\vec{v})$ as the normal coordinates and assume propagating solutions of the form $\exp ik(x-ct)$, the equations of motion give the speeds of isoscalar and isovector longitudinal waves as

$$C_L = \left\{ \frac{d+e \mp [(d+e)^2 - 4\rho_p\rho_n f]^{1/2}}{2\rho_p\rho_n} \right\}^{1/2},$$

taking minus and plus signs, respectively, with

$$d=\rho_p(2\mu_n+\lambda_n+2\mu_1+\lambda_1), \quad e=\rho_n(2\mu_p+\lambda_p+2\mu_1+\lambda_1), \quad f=(2\mu_p+\lambda_p)(2\mu_n+\lambda_n)+(2\mu_1+\lambda_1)(2\mu_p+\lambda_p+2\mu_n+\lambda_n).$$

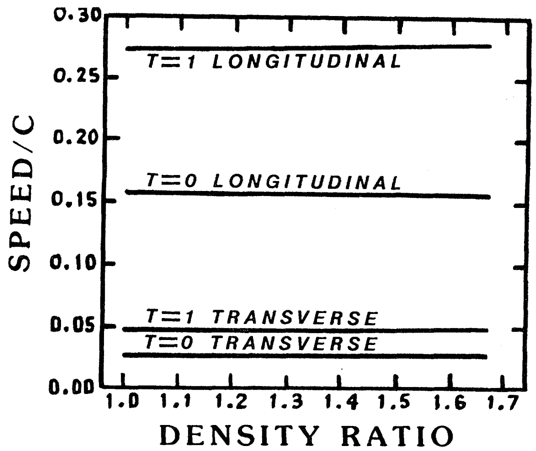

FIG. 1. Speeds of propagation of the different modes vs density ratio of the two fluids.

The speeds of isoscalar and isovector transverse waves are

$$C_T = \left\{\frac{a+b\mp[(a+b)^2-4\rho_p\rho_n g]^{1/2}}{2\rho_n\rho_p}\right\}^{1/2}$$

where

$$a=\rho_p(\mu_n+\mu_1),\quad b=\rho_n(\mu_p+\mu_1),$$

$$g=\mu_p\mu_n+\mu_1\mu_p+\mu_1\mu_n\,.$$

Just like in normal nuclear matter, four types of waves can propagate. This is due to the shear elasticity instead of the spin degree of freedom. Also, the isovector modes are built on top of the corresponding isoscalar modes and will always be higher in energy.

The equations of motion can be solved using the method of Lamb,[6] employing the boundary condition that the total stress must vanish at the nuclear surface. This leads to the wave eigenvectors for the electric and magnetic multipoles. The magnetic wave eigenvectors are independent of the ratio of λ to μ. The electric wave eigenvectors are sensitive to this ratio, more so in the case of smaller multipoles and crucially in the case of the electric monopole. The bulk modulus k can be deduced from the incompressibility of nuclear matter K_∞ by $k=n_0K_\infty/9$. If we take K_∞ to be 200 MeV and a density of 0.16 nucleons fm^{-3}, we get $k\sim 3.56$ MeV fm^{-3}. At this point two assertions are made: (1) Because of charge independence, $\mu_1\sim\frac{1}{2}\mu$ and $\lambda_1\sim\frac{1}{2}\lambda$. (2) Experimentally,[7] the isovector magnetic dipole resonance lies at an energy $\sim 45A^{-1/3}$ MeV. Since the magnetic modes are unaffected by surface tension or the Coulomb energy, we can now empirically evaluate the Lamé coefficients: $\lambda=3.482$ MeV fm^{-3} and $\mu=0.110$ MeV fm^{-3}. Also, the speeds of the two isoscalar modes of propagation in the elastic regime can be seen in Fig. 1 to be unaffected by the ratio of proton to neutron densities and very slightly affected in the isovector cases.

The energies of the isoscalar and isovector $M1$, $M2$, $E0$, $E1$, $E2$, and $E3$ resonances were calculated with the assumption that $R=1.2A^{1/3}$ and some are shown in Tables I and II. The numbers on the left indicate the harmonics, where one can see that the largest wave eigenvectors occur for the $E0$ and $M1$ modes. Only with these widely differing values of λ and μ can the experimental energies of the $T=1$ $M1$ and $T=0$ $E0$ be made compatible. These values yield a very large Poisson ratio σ for nuclear matter: 0.485 (an incompressible medium has $\sigma=\frac{1}{2}$). In this model the surface tension, Coulomb energy, and effective mass were not taken into account, yet many of these resonances have been seen experi-

TABLE I. Some isoscalar multipole energies in $A^{1/3}$ MeV of the lower harmonics lying above $40A^{-1/3}$ MeV.

	$M1$	$M2$	$E0$	$E1$	$E2$	$E3$
1			80			
2	40		162			
3	55	47	243	48		46
4	69	61		62	54	60
5	83	76		76	68	75
6		90		90	83	89
7		104		104	97	103
8				115	111	117

TABLE II. Some isovector multipole energies in $A^{1/3}$ MeV with a density ratio of 1.5 lying above $40A^{-1/3}$ MeV.

	$M1$	$M2$	$E0$	$E1$	$E2$	$E3$
1	45		141			
2	71	56	284	58	43	55
3	96	82		84	69	80
4	121	108		109	95	106
5	146	133		134	120	131
6	171	158		158	145	156
7					170	181
8					195	206

mentally. This is partly because, as is pointed out in Ref. 3, the surface tension and Coulomb energy have a very small effect on the electric multipoles. It would be interesting to obtain from microscopic calculations of the sort performed in Refs. 3 and 8 the present coupled system of equations of motion leading to a large Poisson ratio for nuclear matter.

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