Study of Two-Nucleon Wave Functions in ^{3}He

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The reaction ^{3}He(p, 2p)pn has been studied at 250 and 400 MeV in a quasifree scattering arrangement characterized by $\vec{P}$(recoil) = 0 and various excitation or total energies E(recoil) of the unobserved p-n pair. The ^{3}He spectral function deduced in the framework of the plane-wave impulse approximation is compared to the predictions of Faddeev and variational calculations. Comparisons are also made with p-n relative-motion momentum distributions calculated as the overlap between plane waves for the p-n pair and Irving, Irving-Gunn, and Khanna wave functions for ^{3}He.

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Studies of quasifree scattering from ^{3}He allow nucleon-nucleon correlations to be determined directly if one assumes the plane-wave impulse approximation (PWIA) to be valid. For ^{3}He the experimental momentum distribution as a function of the variables $\vec{P}$ and E can be compared directly with the spectral function obtained from the Faddeev equations and a realistic nucleon-nucleon interaction. Here $\vec{P}$(recoil) is the vector momentum and E(recoil) is the total energy of the unobserved pair of particles in ^{3}He(e, $e'p$) or ^{3}He(p, 2p). The ^{3}He spectral function has been calculated by Dieperink *et al.*[1] from the ^{3}He ground-state wave function obtained by Brandenburg, Kim, and Tubis[2] for the Reid soft-core interaction acting in $^1S_0$ and $^3S_1$-$^3D_1$ states only; and by Meier *et al.*[3] for the Paris potential. Furthermore the ^{3}He spectral function has been calculated by Ciofi degli Atti, Pace, and Salmé[4] using the variational wave function of Nunberg, Prosperi, and Pace[5] with a corrected asymptotic behavior of the harmonic-oscillator basis expansion wave function. Previous ^{3}He(p, 2p) experiments[6] have all been concerned with sections through the spectral-function surface for constant E(recoil). The present experiment is unique in that it defines a section through the spectral-function surface for constant $\vec{P}$ (P = 0). Such measurements have never been made previously.

The experiment was performed with proton beams of 250 and 400 MeV from the TRIUMF accelerator incident on a liquid-^{3}He target. Beam intensities ranged between 0.5 and 5 nA. Normalization of the differential cross sections followed from the sum of left and right counts from the incident-beam polarimeter (containing a ~5-mg/cm^2-thick CH_2 target) located upstream from the liquid-^{3}He target and from an ionization chamber downstream from the liquid-^{3}He target. The target thickness followed from measurements of the temperature of the target cell through germanium resistors embedded in the body of the cell and from its physical dimensions.[7] The thickness so determined was 132 ± 4 mg/cm^2.

Two different detection arrangements were used. At 250 MeV the scattered and ejected protons from the reaction ^{3}He$(p, 2p)pn$ were detected in coincidence by a pair of range telescopes placed (each to an accuracy of 0.1°) symmetrically with regard to the incident-beam direction. Each range telescope consisted of two 0.31-cm-thick plastic scintillators, providing trigger signals and time-of-flight (TOF) information, two sets of multiwire proportional chambers each with vertical and horizontal coordinates, and a 15.2-cm-thick by 12.7-cm-diam NaI(Tl) detector (stopping protons up to ~210 MeV). At 400 MeV the left-hand-side proton was detected by the MRS magnetic spectrometer, while the right-hand-side proton was detected by either one or two range telescopes at the appropriate angles. The two range telescopes consisted each of a 0.08-cm-thick plastic scintillator providing the trigger signal and TOF information, an annular plastic scintillator (with an aperture 5.1 cm in radius) defining the solid angle, an appropriate amount of copper absorber, and a 15.4-cm-thick by 12.7-cm-diam NaI(Tl) detector.

At 250 MeV data were taken at four coplanar symmetric pairs of laboratory angles (40°-40°, 38°-38°, 35°-35°, and 30°-30°) while at 400 MeV data were taken at four coplanar symmetric pairs of angles (40°-40°, 37°-37°, 34°-34°, and 30°-30°) and two coplanar asymmetric pairs of angles (34°-42° and 30°-38°) all containing $\vec{P}$(recoil) = 0. The range of relative momenta of the recoiling n-p system extended to 600 MeV/c. Energy calibrations for the range telescopes were obtained by observing a proton-proton scattering from a CH_2 target.

The analysis of the 250-MeV data was as previously reported[8] except that the use of a double set of wire chambers in the present experiment allowed for track reconstruction and vertex definition. In addition, the TOF information for both observed protons in the form of a two-dimensional array enabled us to distinguish the events of interest from the background of ^{3}He$(p, 2p)d$ with both protons undergoing nuclear reactions in the stopping detectors. At 400 MeV momentum analysis and TOF information gave a signal for the spectrometer side unencumbered by such background events. For the other proton, being detected in a range telescope, the energy region of interest needed to be corrected for background events from ^{3}He$(p, 2p)d$ (with the second proton undergoing a nuclear reaction in the stopping detector) with use of TOF information. The energy resolutions of the range counter telescopes were in the range 7–9 MeV (full width at half maximum), while the time resolutions varied between 1.2 and 1.6 nsec (full width at half maximum). The sixfold differential cross sections were obtained from a bite 9 MeV by 9 MeV centered around $\vec{P} = 0$. The data are presented in Table I.

In the PWIA the sixfold differential cross section can be written as

$$d^6\sigma/d\Omega_3\, dT_3\, d\Omega_4\, dT_4 = F(d\sigma/d\Omega)_{1/2}{}^{p-p} N\,|\,\varphi(\vec{p}_5, \vec{p}_6)\,|^2 = F'(d\sigma/d\Omega)_{1/2}{}^{p-p} S(\vec{P}, E)\,,$$

with the following index convention: $p + {}^3\text{He} \to p + p + p + n \equiv 1 + 2 \to 3 + 4 + 5 + 6$. Here F and F' are kinematic factors and are defined as follows:

$$F = \frac{p_3 p_4}{4}\,\frac{\pi}{M_{56}}\,\frac{[\{M_{56}{}^2 - (m_5 - m_6)^2\}\{M_{56}{}^2 - (m_5 + m_6)^2\}]^{1/2}}{2M_{56}}\,\frac{(E_3{}' + E_4{}')^2}{p_1}\,\frac{16E_5E_6}{m_4}\,\frac{1}{(\hbar c)^6}\,,$$

with $|\,\varphi(\vec{p}_5, \vec{p}_6)\,|^2$ expressed in fm^6; and

$$F' = \frac{p_3 p_4}{4}\,\frac{(E_3{}' + E_4{}')^2}{p_1}\,\frac{4}{m_4}\,\frac{1}{(\hbar c)^3}\,,$$

with $S(\vec{P}, E)$ expressed in fm^3 MeV^{-1}, and where the primed quantities $E_3{}'$ and $E_4{}'$ refer to the center-of-mass (c.m.) system of particles 3 and 4 and $M_{56} = E$ since $\vec{P} = 0$. The quantity $(d\sigma/d\Omega)_{1/2}{}^{p-p}$ is a half-off-energy-shell p-p differential cross section, N is a spectroscopic factor determined with spin algebra, $|\varphi(\vec{p}_5, \vec{p}_6)|^2$ is the square of the momentum distribution of a p-n pair with momenta $\vec{p}_5$ and $\vec{p}_6$, respectively, and $\vec{S}(P, E)$ is the spectral function. For the present experiment with $\vec{P} = 0$ the p-n pair has $\vec{p}_5 = -\vec{p}_6$. Also with $\vec{P} = 0$ the off-energy-shell dependence of $(d\sigma/d\Omega)_{1/2}{}^{p-p}$ is reduced (the struck proton is at rest before the collision). The half-off-energy-shell p-p scattering amplitude was calculated with use of the expansion for the t matrix[10]:

$$t(\vec{q}_i, \vec{q}_f, T_f) = f(\vec{q}_i, \vec{q}_f)\, t(\vec{q}_f{}', \vec{q}_f, T_f),$$

with $\vec{q}_i$ the c.m. momentum of the interacting

TABLE I. Kinematics, the measured sixfold differential cross sections, and the quantities used in deriving $|\varphi(\vec{p}_5, \vec{p}_6)|^2$ and $S(\vec{P}, E)$.

T_1 (MeV)	θ_3 (deg.)	θ_4 (deg.)	$\lvert\vec{p}_5-\vec{p}_6\rvert$ (MeV/c)	E (MeV)	$d^6\sigma/(d\Omega_3 dT_3 d\Omega_4 dT_4)$ μb/(sr^{2}MeV^2)	$(d\sigma/d\Omega)^{p-p}$ [a] (mb/sr)	$(d\sigma/d\Omega)_{1/2}^{p-p}$ (mb/sr)
250	40	40	235.6	1892.5	7.29 ± 0.67	3.75	3.60
400	40	40	249.5	1894.3	3.43 ± 0.46	3.82	3.55
250	38	38	316.7	1904.4	0.532 ± 0.181	3.73	3.88
400	34	42	365.6	1913.1	0.516 ± 0.063	3.85	3.39
250	35	35	399.0	1919.7	0.159 ± 0.067	3.83	3.76
400	37	37	409.8	1922.0	0.210 ± 0.062	3.85	3.55
250	30	30	487.0	1939.9	0.070 ± 0.021	4.67	3.12
400	30	38	504.3	1944.4	0.057 ± 0.018	3.82	3.39
400	34	34	507.7	1945.2	0.074 ± 0.025	3.76	3.75
400	30	30	597.3	1970.5	0.029 ± 0.014	3.75	3.69

[a] $(d\sigma/d\Omega)^{p-p}$ is on the on-energy-shell p-p differential cross section obtained with use of the final-state energy approximation from experimental phase shifts (Ref. 9).

p-p system in the initial state and with $\vec{q}_f$ and T_f the c.m. momentum and energy, respectively, of the interacting p-p pair in the final state. The quantity $t(\vec{q}_f{}', \vec{q}_f, T_f)$ is the on-energy-shell t matrix while $f(\vec{q}_i, \vec{q}_f)$ is the half-off-energy-shell extension function. The calculations were done with the Reid soft-core interaction.[11] For the kinematics of the present experiment the difference between the half-off-energy-shell p-p differential cross section and the quite often used final state on-energy-shell approximation was never more than 30% and in general was less than 10%.

The data represented as $(d^6\sigma/d\Omega_3\, dT_3\, d\Omega_4\, dT_4)/[F(d\sigma/d\Omega)_{1/2}{}^{p-p}N]$ are plotted in Fig. 1 as a function of $|\vec{p}_5-\vec{p}_6|$, twice the momentum for the relative motion in the c.m. system of the unobserved p-n pair. The error bars shown reflect counting statistics only. The data are subject to an overall normalization uncertainty of ±10%. It should be noted that all the data, which were taken under various kinematic conditions, present a smooth distribution pointing to the validity of the PWIA to first order.

The data are compared with p-n relative-motion momentum distributions calculated as the overlap between plane waves for the neutron and proton and Irving, Irving-Gunn, and Khanna wave functions for ^{3}He (Ref. 12). It was assumed that the $T=0$ component of the p-n relative-motion wave function mainly contributes to the reaction ^{3}He$(p, 2p)d$ and therefore N was set equal to $\frac{1}{2}$. This assumption is strictly valid only for small p-n relative energies. There appears to exist reasonably good agreement in magnitude and shape with the theoretical distributions for the Irving and Irving-Gunn wave functions. However, the use of plane waves should fail for small p-n relative momenta. The agreement with the theoretical distribution for the more realistic Khanna exponential wave function (with $\beta = 307.5$ MeV) is somewhat poorer.

The data represented as $(d^6\sigma/d\Omega_3\, dT_3\, d\Omega_4\, dT_4)/[F'(d\sigma/d\Omega)_{1/2}{}^{p-p}]$ are compared with various theoretical predictions for the ^{3}He spectral function $S(\vec{P}=0, E)$ in Fig. 2. It is apparent that the data qua shape agree fairly well with the spectral functions of Refs. 3 and 4. In these calculations both $T=0$ and $T=1$ contributions have been added. The experiment shows clearly an excess of p-n relative momenta greater than ~200 MeV/c amounting to a factor of 4 approximately. Discrepancies have also been observed[13] (although in a different part of phase space) in a comparison between ^{3}He$(e, e'p)d$ results and the spectral functions of Refs. 1 and 4. In this case both the

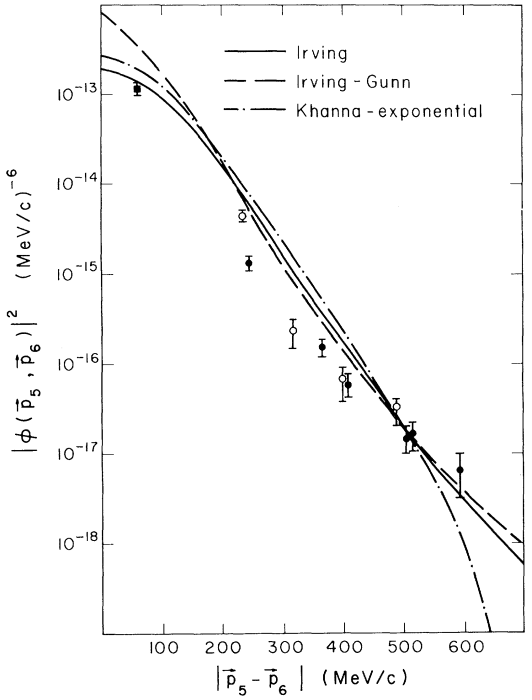

FIG. 1. Differential cross sections $d^6\sigma/d\Omega_3\, dT_3\, d\Omega_4\, dT_4$ divided by the factor $F(d\sigma/d\Omega)_{1/2}{}^{p-p}N$ as a function of the relative momentum of the unobserved p-n pair. The 250-MeV results are presented as open circles, while the 400-MeV results are presented as filled circles. The 155-MeV datum of Frascaria *et al.* is presented as a filled square. The solid, dashed, and dot-dashed curves correpond to p-n relative-motion momentum distributions as indicated.

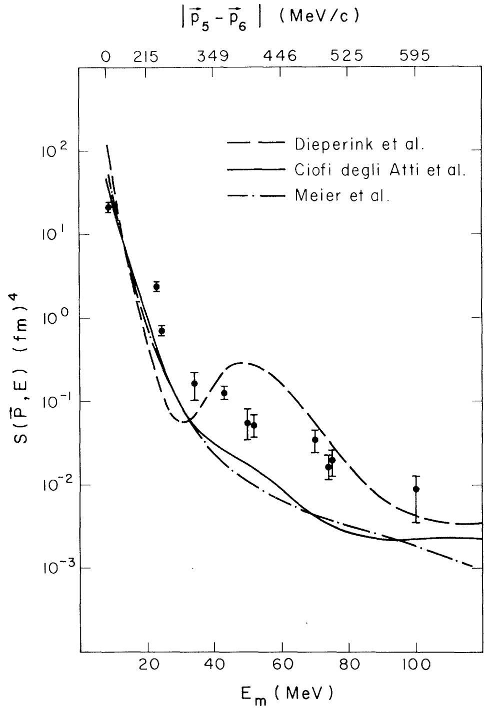

FIG. 2. Differential cross sections $d^6\sigma/d\Omega_3\, dT_3\, d\Omega_4 dT_4$ divided by the factor $F'(d\sigma/d\Omega)_{1/2}{}^{p-p}$ as a function of the relative momentum of the unobserved p-n pair and the missing energy E_m. The quantity $E_m = E - m_2 + m_4 = E_x - Q$, with E_x the excitation energy of the p-n pair and Q the reaction Q value. The calculated spectral functions are as indicated.

theoretical predictions overestimate the experimental result. However, reasonably good agreement was obtained in comparing ^{3}He$(e, e'p)pn$ results with the same spectral functions integrated over missing energies up to 20 MeV.[13] As a first step in investigating these discrepancies one should further examine the various distortions such as those due to multiple scattering and final-state interaction effects. Note that the p-n excitation energies considered here are still far from a real N-Δ pair. Only after the above effects have been studied in more detail can further conclusions be drawn about the spectral functions and the p-n relative-motion wave function.

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