Transverse Mass and Width of the W Boson

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(Received 1 April 1983)

The possibility that both the mass and the width of the W boson can be measured by the Jacobian peak in the transverse mass distribution is proposed and examined.

PACS numbers 14.80.Er, 12.30.Ez, 13.85.Hd

Arnison *et al.* and Banner *et al.* have recently reported the discovery of isolated large-transverse-energy electrons with associated missing energy at the CERN SPS proton-antiproton collider.[1] Such events are candidates for the production and decay of the W boson via the Drell-Yan (DY) reaction $p\bar{p}\to W^{\pm}X$ followed by the leptonic decays $W^{\pm}\to l^{\pm}\,{}^{(}\bar{\nu}{}^{)}_{l}$ (where $l=e$, μ, or τ). If this scenario is correct, it becomes vital to understand the best method to determine both the mass M and the width Γ of the W boson in $p\bar{p}$ collisions to check the predictions of the SU(2) $\times$U(1) model.

The determination of M is difficult because an invariant mass cannot be reconstructed when there is a neutrino in the final state. Several authors have therefore proposed to find M from the differential cross section $\sigma^{-1}d\sigma/dp_T{}^l$ where $p_T{}^l$ is the momentum of the decay lepton perpendicular to the beam direction.[2] In an ideal situation, when the W is produced at rest, $\sigma^{-1}d\sigma/dp_T{}^l$ shows a clear (Jacobian) peak at $M/2$ and a very sharp drop at larger p_T values. In reality, the situation is complicated by the fact that the W usually has a transverse momentum (caused by higher-order QCD corrections where the W recoils against a gluon or a quark) so that the Jacobian peak is badly smeared. This smearing cannot be reliably calculated from theory since QCD corrections do not yield a satisfactory explanation of similar effects seen in dimuon production at low energies. If the hadronic energy in the detector is measured, then the p_T of the W can be determined from $p_T{}^W=|\vec{p}_T{}^l+\vec{p}_T{}^\nu|$ where $p_T{}^\nu=E_T$ is the missing transverse energy. Then one can either exclude events with finite $p_T{}^W$ to see a distinct Jacobian peak or one can include all events and calculate the smearing from some model. In a situation where the total number of events is small, neither of the above approaches is satisfactory.

To avoid the above complication, a new method has recently been proposed[3] to determine the mass of the W accurately, which exploits the properties of the transverse mass distribution $\sigma^{-1}d\sigma/dm_T$ where

$$m_T{}^2=2p_T{}^l p_T{}^\nu(1-\cos\varphi_{l\nu}), \tag{1}$$

and $p_T{}^l$, $p_T{}^\nu$, and $\varphi_{l\nu}$ are the momenta and angle between the leptons in the plane perpendicular to the $p\bar{p}$ collision axis. The transverse-mass method has been applied by Arnison *et al.*[1] to determine a lower bound of 74 GeV/c^2 for M. It was also taken over by Barger *et al.*[4] and subsequently used in their search for the top quark in the Arnison *et al.* data.[5] As was noticed in Ref. 3, $\sigma^{-1}d\sigma/m_T$ has a Jacobian peak at $m_T=M$ which is relatively insensitive to $p_T{}^W$ effects or, in other words, to QCD corrections. Under these circumstances, it is very interesting to investigate whether the transverse-mass method can provide us with a measurement of Γ.

First of all, we want to see how the $p_T{}^W$ influences $\sigma^{-1}d\sigma/dm_T$. In the W rest frame, the electron and neutrino momenta can be defined with respect to a z axis along the $\bar{p}$ beam direction as

$$p^l=(M/2)(1,\sin\theta\cos\varphi,\sin\theta\sin\varphi,\cos\theta),$$

$$p^\nu=(M/2)(1,-\sin\theta\cos\varphi,-\sin\theta\sin\varphi,-\cos\theta),$$

so that $\mu^2=M_T{}^2/M^2=\sin^2\theta\le 1$ and

$$\sigma^{-1}\frac{d\sigma}{d\mu}=\mu(1-\mu^2)^{-1/2}\sigma^{-1}\frac{d\sigma}{d\cos\theta}. \tag{2}$$

The presence of the factor $(1-\mu^2)^{-1/2}$ leads to a sharp Jacobian peak at $\mu=1$. The effect of a finite $p_T{}^W$ can be determined by applying a boost along the x direction in the frame perpendicular to the collider axis. The boost parameters were chosen to be $\gamma=E_W/M$ ($E_W{}^2=p_T{}^2+M^2$) and $\alpha=(\gamma^2-1)^{1/2}$; then μ^2 is a function of α:

$$2\mu^2(\alpha)=[(\alpha^2\sin^2\theta\cos^2\varphi+\alpha^2+\sin^2\theta)^2-4\alpha^2\gamma^2\sin^2\theta\cos^2\varphi]^{1/2}+\sin^2\theta+\alpha^2(\sin^2\theta\cos^2\varphi-1) \tag{3}$$

or, inverting this formula,

$$\sin^2\theta = \mu^2(\mu^2+\alpha^2)(\mu^2+\mu^2\alpha^2\cos^2\varphi+\alpha^2\sin^2\varphi)^{-1}. \tag{4}$$

Thus, from a calculation of $d\mu^2/d\cos\theta$, we find

$$\sigma^{-1}\frac{d\sigma}{d\mu} = \mu\frac{(1-\mu^2)^{-1/2}}{(2\pi)}\int_0^{2\pi} d\varphi\, I(\mu,\varphi,\alpha)\sigma^{-1}\frac{d\sigma}{d\cos\theta}, \tag{5}$$

where the function I is

$$I(\mu,\varphi,\alpha) = (\mu^4+\mu^4\alpha^2\cos^2\varphi+2\mu^2\alpha^2\sin^2\varphi+\mu^4\sin^2\varphi)(\mu^2+\alpha^2\sin^2\varphi)^{-1/2}(\mu^2+\mu^2\alpha^2\cos^2\varphi+\alpha^2\sin^2\varphi)^{-3/2}. \tag{6}$$

From (4), we see that m_T decreases as α (or p_T^W) increases, as long as $0<m_T<M$. At the end point $\mu=1$,

$$I(\mu,\varphi,\alpha) = (1+\alpha^2\sin^2\varphi)^{1/2}(1+\alpha^2)^{-1/2}, \tag{7}$$

which is finite for all α. Hence, the Jacobian peak exists for all p_T^W. Further, one can show that the value at the peak for $\alpha=\infty$ is reduced by a factor $2/\pi=0.64$ from the value at $\alpha=0$. To investigate the behavior of (5) near $\mu=0$, we note that $I(\mu,\varphi,\alpha)$ is proportional to μ^{-1} for finite α, so that the point $\mu=0$ is shifted towards a finite value.

After these preliminaries we obtain $\sigma^{-1}d\sigma/dm_T$ for a vector W boson by adding the angular dependence $d\sigma/d\cos\theta=\frac{3}{4}(1+\cos^2\theta)$ to (5) and using (4). Since we are interested in determining Γ we also add a finite width for the W which changes (5) to

$$\sigma^{-1}\frac{d\sigma}{dm_T} = \frac{3}{4}\frac{1}{\pi/2+\arctan(M/\Gamma)}\int_{m^2}^{\infty} ds'\frac{M\Gamma}{(s'-M^2)^2+M^2\Gamma^2}\frac{m_T}{[s'(s'-m_T^2)]^{1/2}}\frac{1}{2\pi}\int_0^{2\pi} d\varphi(2-\sin^2\theta)I,$$

where θ and I are now functions of $m_T/\sqrt{s'}$, φ, and α. The distribution is shown in Fig. 1 for $p_T=0$ GeV and $p_T=50$ GeV with $M=80$ GeV and $\Gamma=2.5$ GeV. α is now given by $p_T/\sqrt{s'}$.

One notes that all the qualitative aspects mentioned in the previous paragraph are evident in the figure. In particular, the p_T modifications are very small near the peak. Instead of a sharp falloff at $m_T=M$ the finite width causes a small tail to appear above the peak. Therefore, in this region, the QCD effects are minuscule so that

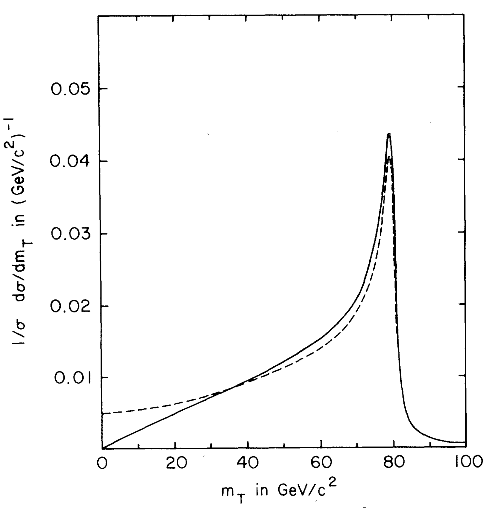

FIG. 1. $\sigma^{-1}d\sigma/dm_T$ for $M=80$ GeV/c^2 and $\Gamma=2.5$ GeV/c^2. The solid line is for $p_T^W=0$ GeV/c, while the dashed line is for $p_T^W=50$ GeV/c.

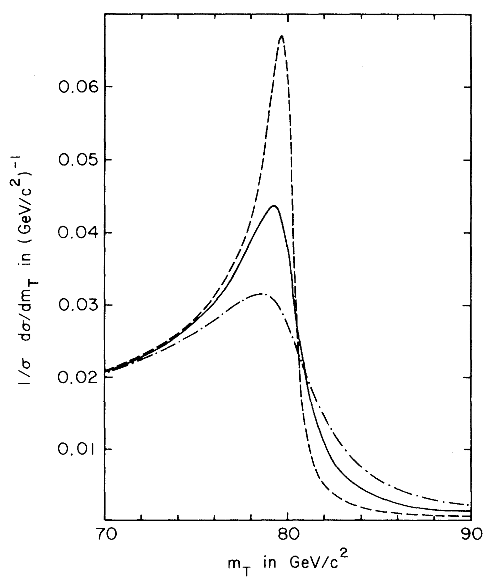

FIG. 2. $\sigma^{-1}d\sigma/m_T$ for $M=80$ GeV/c^2 and $p_T^W=0$ GeV/c^2. The dashed, solid, and dot-dashed lines refer to $\Gamma=1$, 2.5, and 5 GeV/c^2, respectively.

TABLE I. Results of the computer experiment. Δ is the resolution in p_T^{ν}; N is the number of generated events for the maximum-likelihood fit. Γ_{obs} is explained in the text. The input values were $M = 80$ GeV/c^2 and $\Gamma = 2.5$ GeV/c^2. Errors are based on the half height of the peak.

Δ	N	M (GeV/c^2)	Γ_{obs} (GeV/c^2)	Γ (GeV/c^2)
2	200	80.5 ± 0.7	2.3 ± 0.9	2.0 ± 1.0
5	200	80.5 ± 0.7	2.7 ± 1.0	< 2.7
5	1000	80.1 ± 0.3	3.5 ± 0.6	2.4 ± 0.9
10	1000	80.5 ± 0.5	5.1 ± 0.8	< 3.2

one can hope to measure Γ in this region. To illustrate this we show three distributions in Fig. 2 for different choices of Γ. As Γ is increased the height of the Jacobian peak is lowered and it broadens out. Thus the shape of the curve above $m_T = M$ is more sensitive to finite-width effects than to QCD corrections.

As a further check to determine the type of accuracy with which one could hope to measure Γ, we have written an event generator where we vary the experimental resolution on p_T^{ν}. Since p_T^W does not influence the m_T distribution near the value $m_T = M$, we have set $p_T^W = 0$. As input values, we have chosen $M = 80$ GeV/c^2 and $\Gamma = 2.5$ GeV/c^2. The error in p_T^{ν} was chosen according to a Gaussian distribution and the full width at half maximum Δ represents the experimental resolution. Errors in p_x^{ν} and p_y^{ν} were picked independently. The "measured" values of M and Γ were then determined via a maximum-likelihood fit. The results are in Table I. The fit was done to distributions that did not include the experimental uncertainties so that the measured width is a combination of the Breit-Wigner width and the Gaussian width of the measurement. If one approximates this combination by $\Gamma_{obs}^2 = \Gamma^2 + \Delta^2/4$ one obtains the values for Γ in the last column. More accurate numbers would need a maximum-likelihood fit to distributions that incorporate all measurement uncertainties. From the values shown it is clear that an accurate determination of M is possible. The accuracy with which Γ can be determined depends critically on the experimental resolution and the number of events available.

We conclude that the m_T distribution provides us with an excellent tool to measure M and Γ since the corrections due to $p_T \neq 0$ are so small. This measurement is not sensitive to any form of perturbative or nonperturbative QCD effects in contrast to the usual p_T^{l} spectrum determination.

Finally, we would like to thank P. Grannis and M. Marx for useful discussions. This work was supported in part by the National Science Foundation under Grant No. PHY 81-09110.

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