New General Relativistic Effect by Means of Charged-Particle Interferometry

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(Received 7 February 1983)

The general relativistic interaction of the electric field with the gravitational field may be observable in a charged-particle interferometer containing a suitable arrangement of fields. Observation of the effect in the earth's field would constitute a new laboratory test of the equivalence principle, involving e, h, c, and g. The magnitude of the phase shift induced can be comparable to the Colella-Overhauser-Werner phase shift, for electrons.

PACS numbers: 04.80.+z, 03.30.+p, 03.65.-w

The Colella-Overhauser-Werner experiment[1] demonstrated the effect of the gravitational field on matter-wave interferometry for the first time, inspiring several attempts[2-4] to give a description of such phenomena in the framework of general relativity. Also the interference effects due to an external electromagnetic field[5] have been observed in the interferometry of charged particles. In this Letter we would like to point out that rather novel and characteristically general relativistic effects arise in matter-wave interferometry if we consider the simultaneous presence and mutual coupling of *both* electromagnetic and gravitational fields. We believe this new effect to be both of theoretical and possible experimental interest. In the case of electrons, at least, the phase shift we find could be comparable to the Colella-Overhauser-Werner phase shift.

The effect we wish to bring into evidence may be viewed as the change in the electric field due to its coupling to the gravitational field. Although many different configurations of fields and interferometer may be considered, we shall concentrate here on a theoretical "apparatus" with a view towards the greatest theoretical clarity, and leave for later discussions possible improvements and variations lending themselves perhaps more easily to practical realization.

A beam of charged particles (Fig. 1) is coherently split by a beam splitter at the origin O, and the resulting two beams are led around the sides of a rectangle to be recombined and detected at the point C. An electric field $\mathcal{E}$ is parallel or antiparallel to the legs OB and AC, produced by fixed and isolated charges on parallel surfaces as shown. The negative surfaces are equally charged and equally spaced from the central positive surface which has twice this charge. The beams OA and BC are in field-free regions and have the same velocity, in the absence of the gravitational field. The absence of electric field along these legs helps to avoid irrelevant complications due to bending of the beams but is not absolutely essential to the effect. Under the conditions described the phase difference for the two paths is zero and the classical paths take the same time to go from O to C, despite the presence of $\mathcal{E}$. The equal arrival times, in addition to being necessary for coherence reasons, guarantee certain stability properties and allow us to compute the effect of a small perturbation (gravity) by integrating the perturbation along the *unperturbed* paths.[6]

The experiment consists of rotating the apparatus about OA from the horizontal to the vertical position in the earth's gravitational field, and measuring the *change* in phase difference φ for the two beams at C, which we call $\delta\varphi$. First, there will be an effect of the Colella-Overhauser-Werner type in $\delta\varphi$ due to the change of the gravitational potential along BC. We call this $(\delta\varphi)_0$ where

$$(\delta\varphi)_0 = (1/\hbar c^2) gZET, \tag{1}$$

where $E = mc^2/[1-(v/c)^2]^{1/2}$ is the relativistic energy of the beam at O, T the original travel time along BC, g the acceleration of gravity,

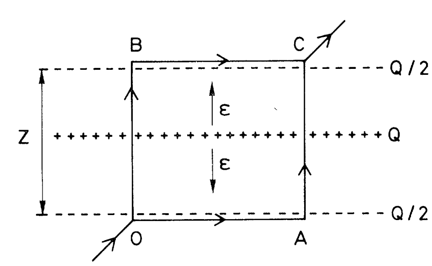

FIG. 1. The interferometer and its electric field configuration.

and Z the height of the interferometer OB. We have written the relativistic energy E instead of simply mc^2 as in the original descriptions to take into account possible high velocities of the beam.[2-4] Verification of this relativistic change of the "gravitationally coupled mass" of the particle would in itself be interesting, but we show below that there is, in addition, a new electrogravitational effect involving $\mathcal{E}$ so that

$$\delta\varphi = (\delta\varphi)_0 - (1/4\hbar c^2) g e Z^2 \mathcal{E} T \tag{2}$$

or expressed relative to $(\delta\varphi)_0$,

$$\delta\varphi = (\delta\varphi)_0 (1 - \tfrac{1}{4} e\mathcal{E}Z/E). \tag{2$'$}$$

It is conceivable, with electrons, that the energy $e\mathcal{E}Z$ can be a reasonable fraction of the electron mass so that the second term in Eq. (2′) may be nonnegligible, compared to 1. For slow neutrons, at least, $(\delta\varphi)_0$ has proved to be accurately measurable.[1,7]

The result will be shown by three different methods: (a) with use of the general relativistic Maxwell equations, (b) by an equivalence-principle argument using special relativity, and (c) by a more general, abstract method involving the action principle.

Before we turn to our derivations, we note that the extra term in Eq. (2) would not arise in a simple nonrelativistic analysis where the phase shift along the path is taken as simply $\int P\cdot dx$. The legs OB and AC cancel by symmetry, and any $\mathcal{E}$-induced change in momentum along BC caused by turning the interferometer would be zero if the electrostatic potential were unaffected by the general relativistic coupling to the gravitational field. On the other hand, it should be remarked that if we had chosen a configuration where the momenta along BC and OA were different in the absence of g, there would be, in general, a nonrelativistic dependence of $(\delta\varphi)_0$ on $g\mathcal{E}$, which might mask our effect. The absence of such an effect is another important feature of the configuration chosen.

In our derivations we start from the notion that the phase along a path is the classical action for the path divided by $\hbar$ and consider the first-order perturbation in the action due to "switching on" the gravitational field; that is, turning the system to the vertical position. Since we work to first order in the perturbation, we may integrate the perturbed action functional along the unperturbed, classically meeting, paths.

In the first derivation we take the action for a charged particle in combined gravitational and electromagnetic fields as

$$S = -mc\int ds - \frac{e}{c}\int A_\mu\, dx^\mu, \tag{3}$$

where $ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu})\, dx^\mu\, dx^\nu$. The first term gives[4] the general relativistic Colella-Overhauser-Werner phase shift $(\delta\varphi)_0$, and we proceed to consider the $\int A_\mu dx^\mu$ term. Our charged grids or plates are taken to be isolated so that no charges flow off them in the process of turning the interferometer. From the conservation law for the current[8] $\partial_\mu[(-\|g\|)^{1/2} j^\mu] = 0$ ($\|g\| \equiv \det g$) and the spatial boundedness of the plates we can conclude there is a time-independent total charge $\int d^3x (-\|g\|)^{1/2} j^0$ which does not change as the plates are rotated. Since, neglecting curvature effects, there is no change in the spatial geometry associated with the plates, we can also conclude that the uniform charge density $(-\|g\|)^{1/2} j^0$ is also unchanged, in a coordinate system fixed to the apparatus. Since we have a stationary situation after the interferometer is turned, we take $j^i = 0$. This implies that we can find a solution to the Maxwell equations[8] $\partial_\nu[(-\|g\|)^{1/2} F^{\nu\mu}] = (-\|g\|)^{1/2} j^\mu$ by taking $(-\|g\|)^{1/2} F^{i0}$ to be unchanged[9] and $F^{ij} = 0$. Then, assuming $g_{\mu\nu}$ to be diagonal,[10] lowering the indices, and taking only h_{00} nonzero,[11] we find with $\mathcal{E}$ the original electric field in the horizontal position (F_{0z}) that the effect of switching on the gravitational field is

$$\mathcal{E} \to (1 + \tfrac{1}{2} h_{00})\,\mathcal{E} = (1 + gz/c^2)\,\mathcal{E}. \tag{4}$$

It will be noted that the new $F_{\mu\nu}$ tensor also satisfies the second set of Maxwell equations. The potential A_μ that we wish to insert in Eq. (3) may now be taken to be $A_0 = \int F_{z0}\, dz$, $A_i = 0$. It will have an extra term $\delta A_0 = \frac{1}{8}(g/c^2) Z^2 \mathcal{E}$ along BC and an extra contribution $-\delta A_0$ along OA (we take $z = 0$ at the central plate; the final answer, of course, does not depend on the origin of z). Thus the perturbations add in the phase difference for the two paths, finally giving a phase $(-1/4\hbar c^2) g e Z^2 \mathcal{E} T$. This is the second term in Eq. (2).

For the second method we turn to the time-honored "elevator frame" argument, i.e., the equivalence principle. We imagine that we are stationary and nonaccelerated, observing an experimenter in an accelerating elevator performing the interference experiment. The simple paths of the particles in space-time for the experimenter, become, for us, a more complicated figure in space-time according to the transformations $t \to t' = t + gtz/c^2$, $z \to z' = z + \frac{1}{2} g t^2$, neglecting terms of $O(g^2)$. We now wish to evaluate the change in the phase difference induced by the acceleration, which we can do by evaluating Eq. (3) along the paths we see, using special relativity. The first term in Eq. (3) will give $(\delta\varphi)_0$, as usual. The electromagnetic effect from the second term is easy to evaluate because the acceleration is parallel to the electric field, and parallel components of the electric field do not change under a Lorentz transformation. Thus it is convenient to transform the desired loop integral $\varphi A_\mu dx^\mu$ to an integral over the $(t'z')$ surface $\int F_{0z} dz' dt'$ so that, splitting the integral into two halves with $\mathcal{E}$ constant, we find in each

$$(\delta\varphi)_{\text{electromagnetic}} = \hbar^{-1} e \mathcal{E} \int dz' dt'. \tag{5}$$

A simple way to perform the integral is to introduce a mathematical change of variables $t' \to t'$, $z' \to z$, which restores the shape of the space-time surface to its parallelogram-like form,[12] and also, however, introduces a Jacobian $|\partial x'/\partial x| = 1 + gz/c^2 + O(g^2)$. The g term here represents the effect which after integration indeed yields the second term of Eq. (2).

Our final derivation starts from a more abstract approach to the action principle,[8] using the fact that the total change in the action induced by a small change of the gravitational field ("turning on" g) is given by

$$\delta S = (1/2c) \int d^4x \, T_{\mu\nu}(x) h^{\mu\nu}(x), \tag{6}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of the entire system, particle plus electric fields. In this formulation the difference in phase for the two paths results from the different spatial distributions of the energy-momentum density according to which path the particle takes. Since we take only h_{00} nonzero, Eq. (6) becomes, in fact, $\delta S = (g/c^2) \int dt \, d^3x \, T_{00} z$ so that

$$\delta\varphi = (g/\hbar c^2) \int dt \, d^3x \, (T_{00}{}^{BC} - T_{00}{}^{OA}) z, \tag{7}$$

where $T_{00}{}^{BC}$ means the energy density of the system when the particle is on the leg BC. In writing Eq. (7) we have also used the cancellation of the legs OB and AC.

The T_{00} in Eq. (7) in principle contain the energy densities of the whole apparatus, or the whole world for that matter, but of course only those energies which change when the path is altered enter into the difference of energy densities appearing in the equation. If the whole energy associated with the particle were simply concentrated in a delta-function-like manner[8] at the position $x(t)$ of the particle like $T_{00}(\vec{x}, t) \sim \delta^3(\vec{x} - \vec{x}(t))$, then the integral could be performed trivially. This can be taken to be true for the kinetic energy E and simply yields $gZET/\hbar c^2$, or $(\delta\varphi)_0$, the relativistic Colella-Overhauser-Werner phase shift. In addition however, there is an energy density, $\frac{1}{2}(\text{electric field})^2$, which must be examined. The total electric field may be written as the sum of that produced by the apparatus, $\vec{\mathcal{E}}$, and that from the particle $\vec{\mathcal{E}}_e$. The term in the electric field energy density which varies when the path is altered is then $\vec{\mathcal{E}} \cdot \vec{\mathcal{E}}_e$. [The $\mathcal{E}_e{}^2$ term is a self-energy and may be considered as already taken into account in $(\delta\varphi)_0$.] Thus the term we wish to evaluate is $(gT/\hbar c^2) \int d^3x (\vec{\mathcal{E}}_e{}^{BC} - \vec{\mathcal{E}}_e{}^{OA}) \cdot \vec{\mathcal{E}} z$. Upon evaluating this integral we find $(\hbar c^2)^{-1} gT \frac{1}{4} e\mathcal{E}Z^2$, the sought for expression.

Our effect has a number of intriguing features. It involves e, h, g, and c, together, for the first time in a possible test of general relativity. If the configuration of charges in Fig. 1 is reversed, this effect, the second term in Eq. (2), should also reverse in sign. This may help to isolate it experimentally as will also the ability to vary $\mathcal{E}$ in general.

The electromagnetic field is a purely relativistic system and its coupling to gravity, in general, can only be described by general relativity. However, when curvature effects are negligible, equivalence-principle arguments, like the "elevator frame" may be used. Hence, observation of the effect in the earth's field could be viewed as confirming the equivalence principle, particularly with respect to the electromagnetic field. It would thus be on par, as a test of general relativity, with, say, the gravitational red shift, which can also be arrived at by equivalence-principle arguments, but in this case involving the action of matter, electromagnetism, and gravity together quantum mechanically.

The characteristic parameter for our effect in general is the product of the gravitational and electrostatic potentials across the apparatus times the traversal time T; for the earth's field, $(\hbar c^2)^{-1} (gz)(e\mathcal{E}z)T$. If we write $T = (\text{length})/v$ the effect is then proportional to the area enclosed by the interferometer, for a given velocity and electrostatic potential. If we measure the potential in units of hundreds of kilovolts, this characteristic parameter is $\frac{1}{2} 10^{-8} (v/c)^{-1}/(100 \text{ kV})$ cm^2. Phase shifts of this order of magnitude are indeed measurable in optical and neutron interferometry (where v/c is small, of course). We are not sure if this would be true for charged-particle interferometry as required here. Furthermore, for this or a similar apparatus, there would be great experimental problems to overcome, such as mechanical instability, spurious length changes, stray magnetic flux, etc. But we hope that further ideas along this line and experimental ingenuity may permit a realization of this attempt to explore further the nature of gravitation.

We would like to thank J. Ehlers for a critical reading of the manuscript.

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[8]L. Landau and E. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley, Reading, Mass., 1967), 2nd ed. See Sec. 88 for Maxwell's equations and Sec. 94 for the action principle.

[9]We use a metric tensor with signature $(+---)$ and Latin indices for spatial components.

[10]This implies that we have eliminated the Sagnac effect by orienting the plane of the interferometer so that the normal component of the earth's angular velocity is zero.

[11]The statements concerning the spatial geometry and a coordinate system "fixed to the apparatus" so that only h_{00} is nonzero depend on the reasonable assumption that the apparatus may be regarded as rigid as it is slowly turned, that is, that the metric length BC is unchanged, neglecting curvature effects. The reader preferring to use another coordinate system might then have to adjust the coordinate distance BC to keep the metric distance constant.

[12]The figure in the z-t plane is not quite a parallelogram because of the acceleration produced by $\mathcal{E}$. The lines corresponding to motions O to B and A to C are simply space-time translations of each other, however.