Prediction of a Resonance-Enhanced Laser-Beam Displacement at Total Internal Reflection in Semiconductors

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Lateral Goos-Hänchen shifts can be increased by factors of 10–100 for laser frequency near an exciton-polariton resonance in semiconductors. A "negative" shift can occur for laser frequency ω_L in the pseudogap $\omega_0 < \omega_L < \omega_l$. Surface irregularities can be probed with this effect.

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The use of total external x-ray reflection at a boundary between two optically transparent media to probe some properties of x-ray evanescent waves has recently been discussed.[1] Deviations from simple Fresnel theory suggest that a graded model of the interface is more appropriate than a sharp boundary. Total internal reflection of a laser optical light beam of finite diameter has been studied in connection with phase-matched second-harmonic generation.[2]

In this Letter we predict resonance anomalies in the lateral Goos-Hänchen shift[3] when the frequency of the laser beam is varied through the electronic band gap of an exciton-polariton semiconductor. We predict a positive Goos-Hänchen lateral shift which can be resonance enhanced by a factor up to about 100, and we also predict a large negative Goos-Hänchen shift for laser frequencies in the pseudogap. Lateral shifts result from the interference of different components of the evanescent wave so that deviations from our theory can probe surface irregularities, roughness, etc.

We analyzed a model experimental arrangement shown in Fig. 1. The spatially dispersive semiconductor crystal is set so that a beam is normally incident from vacuum, at angle θ to the interface. Take the incident beam plane polarized, perpendicular to the plane of incidence. The angle of incidence θ can be varied to satisfy the condition of total internal reflection for each plane-wave component of the beam, as the incident frequency is varied. In an isotropic, spatially dispersive medium with dielectric coefficient $\epsilon(\vec{k},\omega)$ given in the "dielectric approximation,"[4] a normally incident plane wave produces two propagating plane-wave modes. These modes are coupled to produce a physical polariton mode. Each of the two constituent modes of the physical polariton has a different propagation constant k_j and therefore a different angle $\theta_{\alpha c}$ ($\alpha = 1, 2$) for total internal reflection.

For a normally incident Gaussian beam the transmitted beam is a superposition of two coupled beams each with a non-Gaussian profile due to the coupling. Upon reflection at the upper interface each of the partial beams produces two coupled beams (Fig. 1). The resulting reflected beam can now be expressed in terms of a beam emanating from a point on the upper reflecting plane interface displaced by a distance $\bar{D}$ (in the plane of incidence) from the geometrical point of incidence.[3] It is convenient to decompose the resulting reflected beam into its four constituents; the major intensity is due to "diagonal" terms.

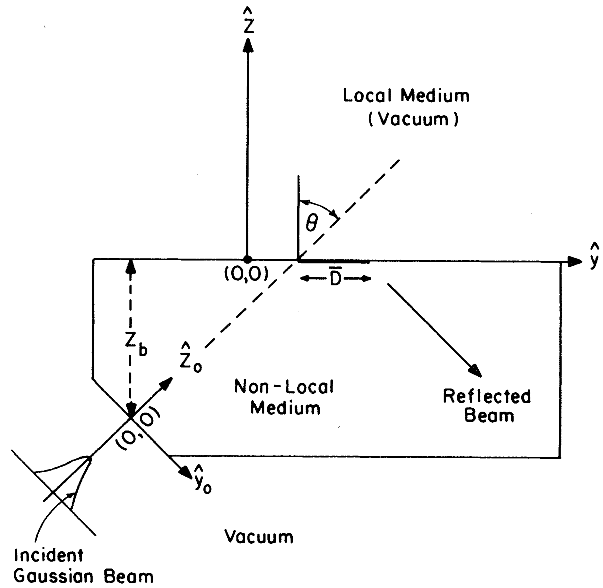

FIG. 1. The laser beam is incident from vacuum at the lower left surface, and propagates to the upper reflecting interface where it is totally internally reflected since $\theta \sim \theta_c$. This is the "Goos-Hänchen" geometry with the "model crystal" cut so as to permit normal incidence at the lower left surface (see Ref. 3).

Details will be given elsewhere,[5] and a *précis* of the theory is given here.

On a plane of constant z_0 take the incident field with Gaussian cross section as

$$\vec{E}^0(x_0,y_0,z_0)=\hat{x}\exp[-(x_0^2+y_0^2+z_0^2)/2w_r^2]=\int_{-\infty}^{\infty}\int \hat{x}E^0(p,q)\exp[ik_0(px_0+qy_0+m_0z_0)]dq\,dp. \tag{1}$$

The last expression is the angular spectrum representation with $k_0=\omega/c$, and

$$m_0=(1-p^2-q^2)^{1/2} \text{ if } q^2<1, \quad m_0=i(q^2+p^2-1)^{1/2} \text{ if } q^2>1. \tag{2}$$

$E^0(p,q)$ is defined from the Fourier transform of the amplitude $E^0(x_0,y_0,0)$:

$$E^0(p,q)=I(0)\exp[-(p^2+q^2)/2f^2]/(2\pi f^2)^{1/2}, \tag{3}$$

where $I(0)$ is constant and $f=1/w_rk_0$. For a narrow optical beam, $f\sim10^{-3}$ and so p and q are small; for perpendicular polarization we take $p=0$. The transmitted field $\vec{E}^t(y_0,z_0)$ and the reflected field at the upper interface $\vec{E}^R(y,z)$ will also be written in angular spectrum representation. Thus if k_j $(j=1,2)$ for upper and lower polariton branches, respectively, is a solution of the dispersion equation $k_j^2=(\omega/c)^2\epsilon(k_j,\omega)$ and $n_j=k_j(c/\omega)$, then

$$\vec{E}^t(y_0,z_0)=\int_{-\infty}^{\infty}\hat{x}\sum_{j=1}^{2}E_j^t(q)\exp[ik_0(qy_0+\sigma_jz_0)]dq, \tag{4}$$

with

$$\sigma_j=(n_j^2-q^2)^{1/2} \text{ if } q^2<\mathrm{Re}(n_j^2), \quad \sigma_j=i(q^2-n_j^2)^{1/2} \text{ if } q^2>\mathrm{Re}(n_j^2). \tag{5}$$

The partial field amplitudes $E_j^t(q)$ are determined from mode-coupling conditions of the form

$$\sum_{j=1}^{2}\alpha_j^t(q)E_j^t(q)=0, \tag{6}$$

where the $\alpha_j^t(q)$ are obtained by use of both the Maxwell equations and the appropriate additional boundary conditions[4]; the effect of changing the additional boundary conditions will be discussed elsewhere.[5] We find

$$E_j^t(q)=\frac{2m_0E^0(q)}{(m_0+\sigma_j)-(\alpha_j^t/\alpha_{3-j}^t)(m_0+\sigma_{3-j})}\equiv I_j(0)E^0(q). \tag{7}$$

For the reflected field write

$$\vec{E}^R(y,z)=\int_{-\infty}^{\infty}\hat{x}\sum_{i,j=1}^{2}E_{ij}^R(v_i)\exp[ik_0(v_iy-w_{ij}z)]dq, \tag{8}$$

with $v_i=q\cos\theta+\sigma_i\sin\theta$ and w_{ij} given by identical expressions as in Eq. (5), with the substitutions $\sigma_j\to w_{ij}$ and $q\to v_i$ $(i=1,2)$. The $E_{ij}^R(v_i)$ are determined by mode-coupling conditions

$$\sum_{j=1}^{2}\alpha_{ij}^R(v_i)E_{ij}^R(v_i)=0. \tag{9}$$

We then find

$$E_{ij}^R(v_i)=\frac{(\omega_i-\omega_{iT})}{\Delta_{ij}}E_i^t(v_i)\exp(iz_b\omega_i), \tag{10}$$

with

$$\Delta_{ij}=(\omega_{ij}-\omega_{iT})-\alpha_{ij}^R(\omega_{i,3-j}+\omega_{iT})/\alpha_{i,3-j}^R. \tag{11}$$

In Eqs. (10) and (11), ω_{iT} is given by identical expressions as in Eq. (5) with the substitutions $n_j\to n_T=1$ (for vacuum exterior to the crystal), and $q\to v_i$; z_b is defined in Fig. 1. The diagonal parts of the reflected beam can now be expressed in a coordinate system (y_r,z_r) obtained by rotating the angle of incidence θ. The result is

$$\vec{E}_{jj}^R(y,z)=\hat{x}C\exp(-ik_0n_jz_r)\exp(-y_r^2/2w_r^2)[1+e_c]. \tag{12}$$

A final transformation brings (12) to the desired form, by exponentiating $1+e_c$ as $\exp[\ln(1+e_c)]$ and expanding out the logarithm. We adapted the procedure of Horowitz and Tamir[6] in these steps. The diagonal reflected fields can now be written

$$\vec{E}_{jj}^{R}(y,z) = \hat{x} C \exp\{-(y_r - \bar{D}_{jj}\cos\theta)^2/\bar{\omega}^2\} \exp[i(k_j z_r + f)]. \tag{13}$$

This exhibits the diagonal Goos-Hänchen shifts which are denoted by $\bar{D}_{jj}$ for upper and lower polariton branches ($j=1,2$). These shifts are given by

$$\bar{D}_{jj} = -\frac{1}{2\cos\theta}\,\mathrm{Re}\left\{\frac{A_j^{\,j}(\theta) Z_j}{1 + A_j^{\,j}(\theta)[Y_j - \delta_j^{1/2}]}\right\}, \tag{14}$$

where

$$Z_j \equiv f^{1/2} w_r \exp(-i\pi/4 + \gamma_j^2/4) D_{-1/2}(\gamma_j), \quad Y_j \equiv f^{1/2}\exp(-i\pi/4 + \gamma_j^2/4) D_{1/2}(\gamma_j), \tag{15}$$

and

$$A_j^{\,j}(\theta) \equiv 2^{1/2} i n_T^{1/2} \cos^{1/2}\theta \left(\frac{1}{k_0} - \frac{\alpha_R^{[i,j,(3-j)]} - 1}{k_1}\right); \quad \gamma_j \equiv i\delta_j k_0 w_r; \quad \alpha_R^{[i,j,(3-j)]} \equiv \alpha_{i,j}^{\,R}/\alpha_{i,3-j}^{\,R}. \tag{16}$$

$D_\nu(\gamma)$ is the Weber (parabolic-cylinder) function of order ν and argument γ, $w_r = (k_0 f)^{-1}$ is the beam width, and $\delta_j \equiv n_j(\sin\theta - \sin\theta_{jc})\sec\theta$ is the angular distance from the critical angle θ_{jc}. In Eq. (13), $\bar{\omega} \sim w_r$.

We calculated the Goos-Hänchen shifts $\bar{D}_{jj}$ as functions of angle θ near θ_{jc} for fixed frequency $\omega_L = \omega_0(1+\alpha) \gtrsim \omega_l$, slightly above ω_l still in the resonance region; and as a function of ω_L near resonance, adjusting the incident angle so that $\theta = \theta_{jc}$ at each ω_L. Computations were carried out with use of numerical parameters for CdS and GaAs. The beam parameter f was varied from 10^{-4} to 10^{-2}. Results for $\bar{D}_{22}(\theta)$ are shown in Fig. 2 for both CdS and GaAs. Note the large but nondivergent enhancement as $\theta \to \theta_{2c}$. These results agree generally with Horowitz and Tamir's earlier analysis,[6] although the magnitudes are resonance enhanced. In Fig. 3 we exhibit the variation of $\bar{D}_{22}(\omega)$ near resonance, when θ is held at θ_{2c}. The striking new results here are the additional resonant enhancement near ω_l and the negative values of the beam displacement in the pseudogap $\omega_0 < \omega_L < \omega_l$; this just precedes the swing up to a maximum positive $\bar{D}_{22}$ in CdS. We believe that these are the first calculations of negative shift for realistic condensed media, although suggestion of possible negative shifts was made earlier[7] in a different context. Elsewhere we discuss the shifts for the upper polariton branches and the effect of changing the laser beam width, and using various additional boundary conditions.[5] It turns out that quantitative distinction between different additional boundary conditions is possible from measurement of the $\bar{D}_{jj}$. Results for GaAs material parameters also show resonant enhancement, and anomalies.

The unusual dispersion we find in the lateral Goos-Hänchen shift occurs because of interfer-

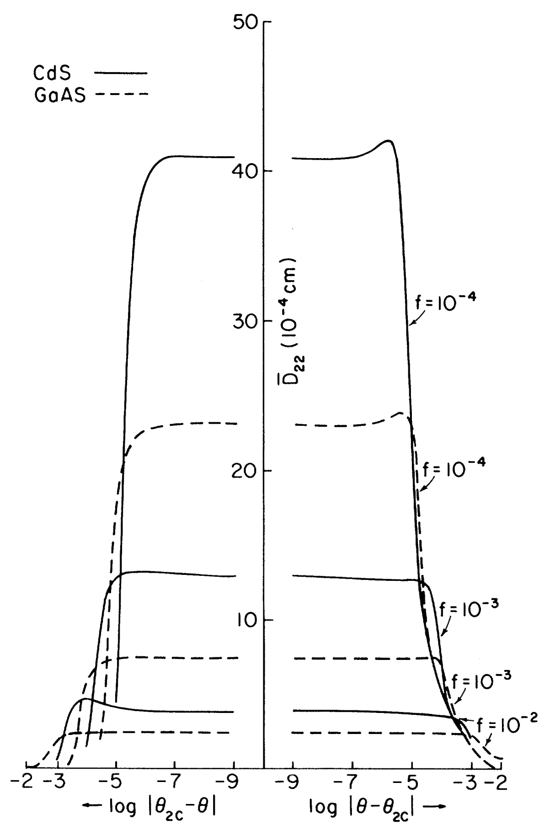

FIG. 2. Goos-Hänchen shift $\bar{D}_{22}$ plotted against $\log(|\theta - \theta_c|)$. Material parameters used (see Ref. 4): for CdS, $\epsilon_\infty = 8.0$, $\hbar\omega_0 = 2.55$ eV, $m^* = 0.9 m_e$, $4\pi\alpha_0 = 0.0125$, and $\Gamma/2\omega_0 = 10^{-5}$. For GaAs, $\epsilon_\infty = 12.55$, $\hbar\omega_0 = 1.55$ eV, $m^* = 0.6 m_e$, $4\pi\alpha_0 = 0.0013$, and $\Gamma = 0.1$ cm^{-1}.

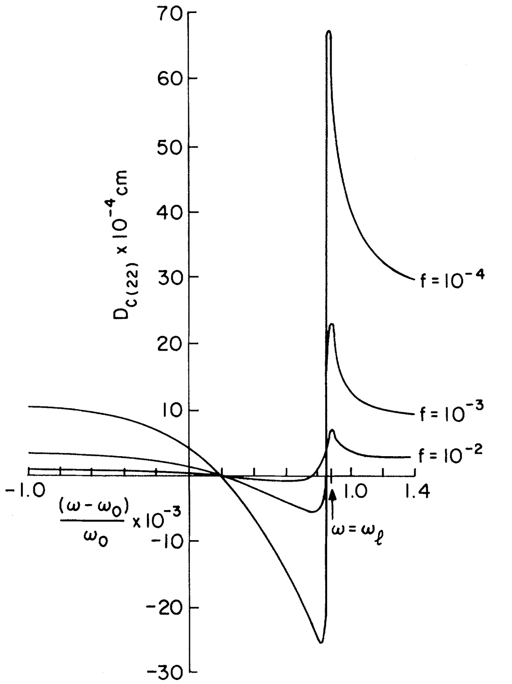

FIG. 3. $\bar{D}_{c(22)}$ plotted against $(\omega-\omega_0)/\omega_0$ in the resonance region for CdS parameters. Parameters as in Fig. 2. The longitudinal frequency ω_l is indicated.

ence at the upper reflecting interface between the "geometrically" reflected fields and the "lateral" wave field. For the usual Goos-Hänchen case this occurs because the relative phases of the two fields is $3\pi/4$ on the trailing edge, giving destructive interference, and $\pi/4$ on the leading edge, leading to constructive interference[8]; when the beam is re-formed a net positive shift occurs toward the leading edge. The presence of several coupled polariton fields each of which gives a (frequency) dispersive phase shift complicates these interference effects and can lead to a negative beam shift. Work is in progress to obtain analytical expressions for these shifts and will be reported elsewhere.

Resonance enhancement of the displacement by factors of up to 100 can make this effect much more accessible experimentally than the usual, nonresonance case where multiple-reflection techniques are needed.[3] Use of the enhanced lateral shift may be practical to probe surface features. The change from positive to negative shift as ω_L is tuned through resonance can have interesting applications and should be experimentally verified. Finally, although we confined our report here to the longitudinal displacements, resonance enhancement of the transverse displacement[9] is to be expected. The enhancements will be very useful to the design of experiments probing fundamental photon properties.[10]

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$$\epsilon(\vec{k},\omega) = \epsilon_\infty + (4\pi\alpha_0\omega_0^2)/(\omega_0^2 - \omega^2 - i\omega\Gamma + Dk^2);$$

and ϵ_∞ is the background dielectric constant, $4\pi\alpha_0$ the oscillator strength, Γ the damping constant, ω_0 the "bare" resonance frequency, $D = \hbar\omega_0/m^*$, and m^* is the exciton mass. See J. L. Birman, in *Excitons*, edited by E. Rashba and M. Sturge (North-Holland, Amsterdam, 1982), Chap. 2, and in *Light Scattering in Solids*, edited by J. L. Birman, H. Z. Cummins, and K. K. Rebane (Plenum, New York, 1979).

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