Dirac-Equation Impulse Approximation for Intermediate-Energy Nucleon-Nucleus Scattering

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(Received 16 March 1983)

In this application of Dirac phenomenology a relativistic impulse approximation is used to describe proton-nucleus elastic scattering at intermediate energies. The results demonstrate the superiority of this relativistic treatment over the nonrelativistic impulse approximation, especially with regard to spin observables. The method has important implications with respect to the extraction of neutron distributions.

PACS numbers: 24.10.-i, 25.40.Cm

Applications of Dirac phenomenology over the past few years have clearly indicated its ability to describe experimental data over a wide range of energy.[1] Analyses at intermediate energies using a Dirac equation with Lorentz scalar and Lorentz four-vector (timelike component only) optical potentials have proven superior to the nonrelativistic treatment, especially with regard to spin observables. For example, if the cross section and one of the two spin observables (analyzing power, A_y, and spin rotation Q) are fitted, the other observable is correctly predicted.[2] This is not true of the nonrelativistic (NR) calculations; in particular these calculations fail to give reasonable agreement with spin observables.[3] It appears that this longstanding difficulty is resolved by using the relativistic treatment described herein. At low energies connection with relativistic Hartree-Fock[4-7] and relativistic Brueckner-Hartree-Fock[8] has been made which provides theoretical understanding of many of the phenomenological results. At intermediate energies a suitable connection between the nucleon-nucleon (NN) and Dirac phenomenologies has been lacking.

Recently McNeil, Ray, and Wallace[9] have presented a relationship between the NN amplitudes parametrized in terms of Pauli matrices and an explicitly invariant representation involving Dirac γ matrices. This representation allows the construction of a Dirac optical potential which is similar in form to the first-order Kerman-McManus-Thaler[10] term [no $(A-1)/A$ scaling] in NR multiple-scattering theory.[11] McNeil, Shepard, and Wallace[12] have shown that at intermediate energies the potential strengths derived in such a way are in good agreement with the results of Dirac phenomenology.[1]

In this work we use the Dirac impulse approximation (IA) to describe $p+{}^{40}\mathrm{Ca}$ and $p+{}^{208}\mathrm{Pb}$ experiments at 500 and 800 MeV and contrast these results with the NR first-order IA obtained using the same NN amplitudes. Our calculations employ an optimally factorized optical potential.[9] The invariant amplitudes are obtained from the Arndt NN phase shifts[13] and are used directly without approximation. Several different treatments of the scalar and vector densities are investigated, and we find the computed observables to be very sensitive to these changes.

The local Dirac IA optical potential is written[12]

$$U_{\mathrm{opt}}(q)=\frac{-4\pi ik}{m}\left\langle 0\left|\sum_{i=1}^{A}\exp(i\vec{q}\cdot\vec{r}_i)F(q)\right|0\right\rangle, \qquad (1)$$

where k is the projectile-nucleus center-of-momentum (c.m.) wave number, $\vec{q}=\vec{k}-\vec{k}'$ is the momentum transfer, and $F(q)$ the invariant NN am-

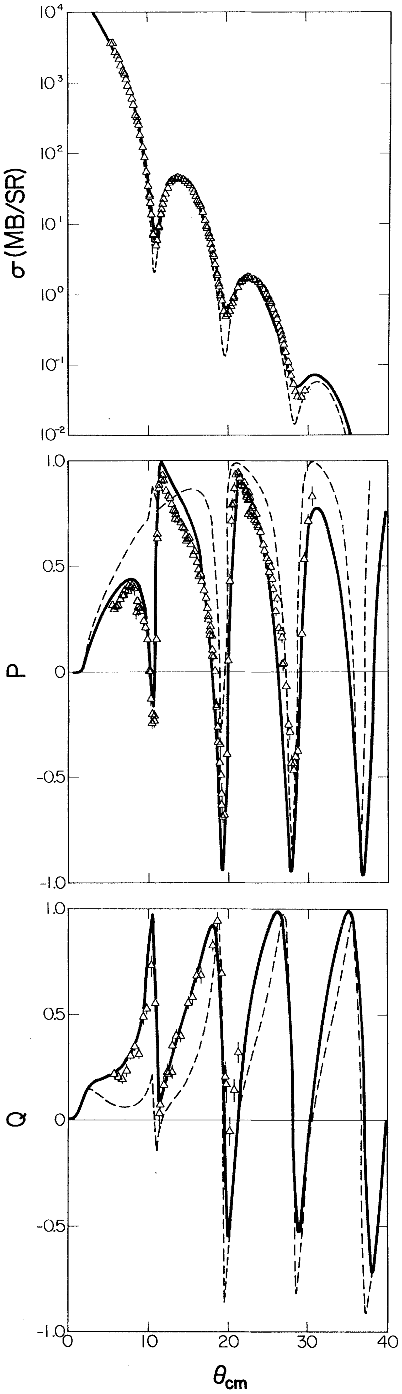

FIG. 1. The calculated cross section, analyzing power, and spin rotation function for $p + {}^{40}Ca$ at 497 MeV with the Dirac impulse approximation, solid curve, and the nonrelativistic impulse approximation, dashed curve.

plitude given by[9]

$$F(q) = F_S + F_V \gamma_1^{\mu} \gamma_{2\mu} + F_P \gamma_1^{5} \gamma_2^{5} + F_A \gamma_1^{5} \gamma_1^{\mu} \gamma_2^{5} \gamma_{2\mu} + F_T \sigma_1^{\mu\nu} \sigma_{2\mu\nu}. \qquad (2)$$

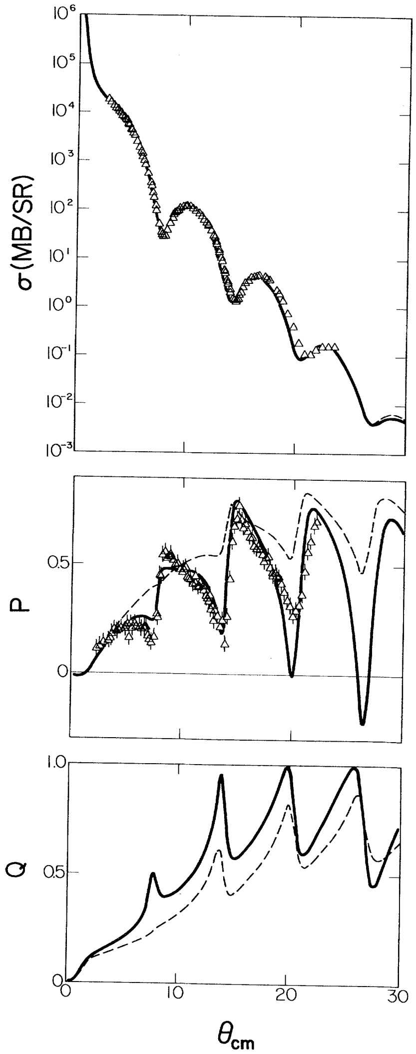

FIG. 2. Same as Fig. 1 for $p + {}^{40}Ca$ at 800 MeV. The Dirac and NR IA cross sections are essentially equal.

For a spin-saturated nucleus, Eq. (1) simplifies considerably to

$$U_{opt}(q) = \frac{-4\pi i k}{m}[F_S(q)\tilde{\rho}_S(q) + \gamma^0 F_V(q)\tilde{\rho}_V(q)], \qquad (3)$$

where $\tilde{\rho}_S$ and $\tilde{\rho}_V$ are the scalar and vector densities, respectively.[4,12] The Fourier transform of Eq. (3) yields the Dirac optical potential in coordinate space. These potentials are used in the Dirac equation

$$\{\vec{\alpha}\cdot\vec{p} + \beta[m + U_S(r)] + [U_0(r) + V_C(r)]\}\psi(\vec{r}) = E\psi(\vec{r}), \qquad (4)$$

where U_S and U_0 are the scalar and vector potentials, respectively, V_C is the Coulomb potential obtained from the empirical charge density, E is

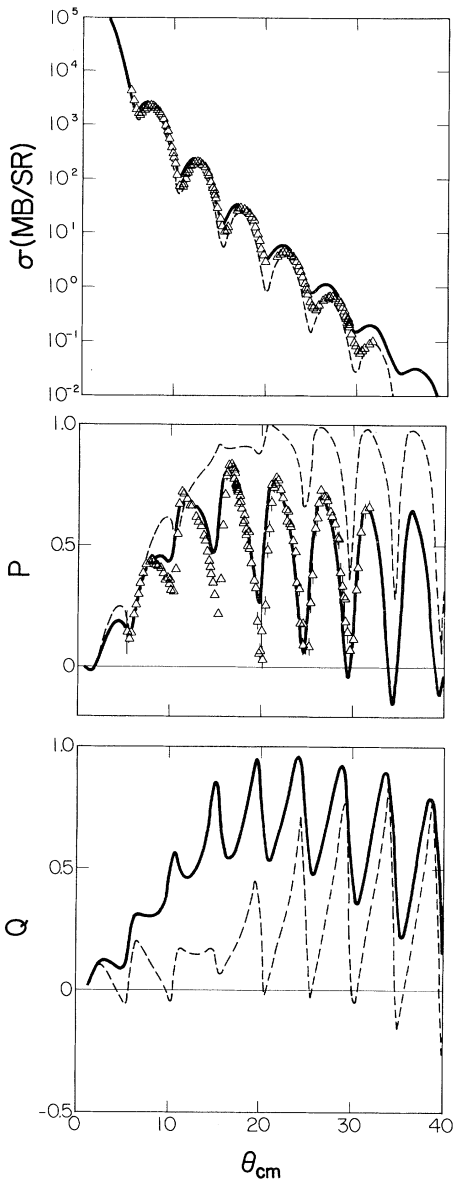

FIG. 3. Same as Fig. 1 for $p+{}^{208}$Pb at 497 MeV.

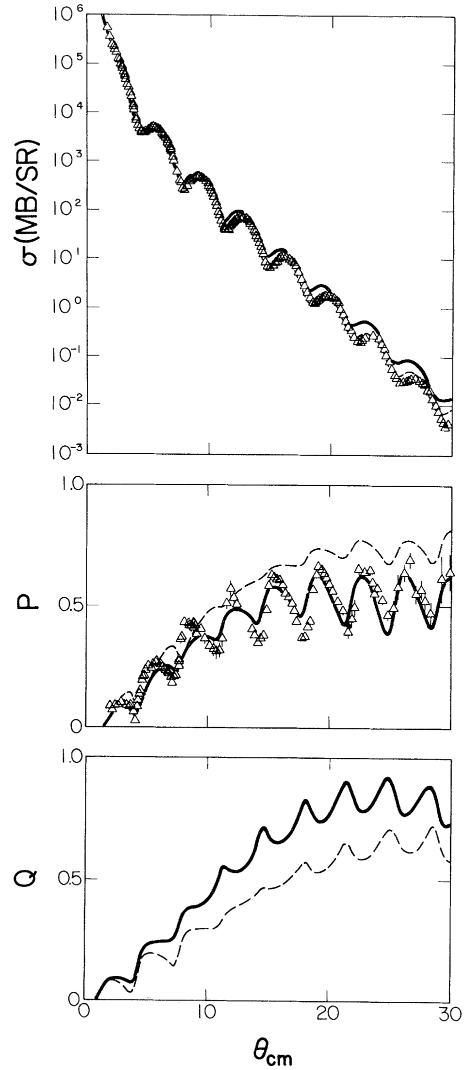

FIG. 4. Same as Fig. 1 for $p+{}^{208}$Pb at 800 MeV.

the c.m. energy, and m is the proton mass.

The calculations shown by the solid curves in Figs. 1–4 use Eqs. (2) and (3) to obtain the Dirac optical potentials. The scalar and vector neutron and proton densities are calculated with the relativistic Hartree approximation of Ref. 4. We find quite good agreement with experiment,[14] especially for the spin observables. Similar agreement has been obtained by Shepard, McNeil, and Wallace[15] for $p+{}^{40}$Ca at 500 MeV with Yukawa forms for the invariant amplitudes.

The NR results are shown by dashed curves in Figs. 1–4. In every case the Dirac IA is superior to the NR first-order IA, particularly for the spin observables. In order to appreciate the origin of this effect, we note some important differences between the relativistic and nonrelativistic treatments. In the NR case the multiple-scattering optical potential is determined from the NN c.m. amplitudes[9]

$$(2ik_c)^{-1}f_c = A + B\vec{\sigma}_1\cdot\vec{\sigma}_2 + iqC(\sigma_{1n}+\sigma_{2n}) + D\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q} + E\sigma_{1z}\sigma_{2z}, \qquad (5)$$

with use of the same vector densities as in the Dirac IA. For spin-saturated target nuclei the first-order spin-independent and spin-orbit optical potentials depend linearly on the A and C amplitudes, respectively. By contrast, the potential which induces spin precession in the Dirac case depends nonlinearly on U_0 and U_S (Ref. 1) and thus nonlinearly on all five of the NN amplitudes in Eq. (5), particularly the single–spin-flip amplitude C. Dirac calculations in which the B, D, and E amplitudes are omitted are negligibly different from the results shown in Figs. 1–4. Therefore it is the nonlinear contributions of the A and C amplitudes which are primarily responsible for the improved Dirac IA prediction.

We have also calculated the Dirac IA optical potentials using the densities of Ref. 4 scaled such that the vector proton density agrees with the point proton density obtained from the empirical charge distribution[11] and that the vector neutron density agrees with Hartree-Fock-Bogoliubov calculations.[16] This results in slightly improved χ^2 but little change in Figs. 1–4. Calculations for 500-MeV $p+{}^{40}Ca$ with identical scalar and vector densities produce A_y and Q values noticeably inferior to those shown in Fig. 1, but still in better agreement with experiment than the NR IA. These calculations indicate a clear sensitivity to the densities used.

Because of its predictive ability and sensitivity to densities, the Dirac approach is potentially a powerful tool for nuclear structure studies. Estimates of the neutron-proton root mean square radius difference for ${}^{40}Ca$ deduced from the Dirac IA calculations yield values closer to the expected value[16] and considerably less energy dependent than that obtained in second-order Kerman-McManus-Thaler IA analyses.[14]

The results of this and other[12,15] investigations demonstrate the dramatic success of the Dirac IA optical potential above 500 MeV. Clearly a formal discussion of relativistic multiple-scattering theory is called for by this work.

We thank J. A. McNeil, B. Mulligan, C. Shakin, J. Shepard, and S. J. Wallace for useful discussion. The support of the National Science Foundation under Grants No. Phy 81-07397 and No. Phy 81-07395 and the U. S. Department of Energy is gratefully acknowledged. One of us (B.D.S.) is a recipient of an Alfred P. Sloan Research Foundation Fellowship.

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