Critical-Exponent Measurements of a Two-Dimensional Superconductor

A. F. Hebard and A. T. Fiory
Bell Laboratories, Murray Hill, New Jersey 07974
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Voltage-current measurements of an amorphous indium/indium-oxide thin-film composite have yielded values for the exponent $\eta(T)$ describing the power-law decay of the order-parameter correlation function appropriate to the Kosterlitz-Thouless description of phase transitions in two-dimensional systems. A pronounced crossover in the characteristics at the phase-transition temperature is obtained. Theoretical expressions both above and below T_c are verified, and a value of $\eta(T_c) = 0.23$ is measured for a film with normal-state sheet resistance of 3560 $\Omega/\square$ measured at 8 K.

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The suggestion that the ideas of Kosterlitz and Thouless[1] might apply to thin-film superconductors[2-5] has generated a number of experiments[6-11] seeking a detailed confirmation of the theory. As applied to two-dimensional (2D) superfluids, it is the fluctuations in the phase of the order parameter, driven by vortex excitations, which determine the details of the transition. A fundamental result of the theory is that the equilibrium order-parameter correlation function $\langle\psi(\vec{r})\psi^*(\vec{0})\rangle$ decays algebraically to zero with the dependence $r^{-\eta}$. The exponent η is temperature dependent and varies from $\eta = 0$ at $T = 0$ to $\eta = \frac{1}{4}$ at the 2D phase transition temperature T_c. The Kosterlitz-Thouless results apply only because there is a logarithmic interaction between vortices separated by a distance r, of the form $2\pi K_R \ln(r/\xi_c)$ where K_R, the renormalized stiffness coefficient, is proportional to the superfluid density and ξ_c is the vortex core size.[1,5,12] For thin-film superconductors, this logarithmic interaction takes place for $\xi_c < r < \Lambda$, where the 2D magnetic screening length Λ can be on the order of 1 cm for thin-film superconductors with a suitably high normal-state resistivity.[2]

Theoretically, the stability of the superfluid is determined by the condition $\eta = k_B T/2\pi K_R \leq \frac{1}{4}$ where η has the universal value of $\frac{1}{4}$ at $T = T_c$.[13] The critical temperature T_c, less than the mean-field transition temperature T_{c0}, demarcates two regions of phase space with markedly different physical behavior. Below T_c and in the absence of finite-size effects[5] the electrical response is dominated by the presence of bound pairs of thermally excited vortices of opposite circulation. Above T_c, where the order-parameter correlation function decays exponentially, the response is dominated by a plasma of thermally excited free vortices. Experimental results using dc techniques have been reported for a variety of 2D superconducting systems including granular Al,[6] granular NbN,[7] quench-condensed Hg-Xe,[8] and periodic arrays of proximity-coupled superconductors[9,10] and Josephson junctions.[11] Nonlinear voltage-current (V-I) measurements[8-10] below T_c or resistance transitions[6,7,10,11] above T_c have been found to be consistent with the theoretical predictions. In addition, extrapolation to zero frequency of finite-frequency measurements of K_R have been used to infer T_c for both thin-film superconductors[14] and superfluid helium.[15] The calculated ratio $k_B T_c/2\pi K_R(T_c)$ is in reasonable agreement (~25%) with theory for both of these experiments. For granular systems it has been demonstrated that film inhomogeneity can be critically important,[6] although it is believed that this problem is somewhat alleviated with the more controlled fabrication of array structures.[9-11]

In this Letter, measurements on a thin-film indium/indium-oxide composite superconductor are reported which show good agreement with theory both *above and below* T_c. Values for $\eta(T)$ are determined from nonlinear V-I characteristics for $T < T_c$ and a pronounced crossover from nonlinear towards linear dependence for $T > T_c$ is used to establish T_c. Parameters such as the normal-state resistance R_n and ξ_c are extracted from the data and found to be reasonable. The excellent agreement with theory is obtained because of the use of indium/indium-oxide composites which have an amorphous metallic component[16] and which appear to be homogeneous over the longest length scales (0.05 cm) probed by this experiment. The 100-Å-thick film used for study is 0.01 cm wide by 0.05 cm long. The ambient magnetic field is less than 0.01 Oe.

The power-law dependences of the curves in

Fig. 1 are a direct consequence of the current-induced unbinding of vortex pairs, a process indicated schematically in the upper left portion of the figure. Theory predicts a V-I dependence for $T < T_c$ of the form[5]

$$V = 2R_n\, I_0[a(T) - 3](I/I_0)^{a(T)}, \qquad (1)$$

where $a(T) = 1 + 0.5/\eta(T)$ is a power-law exponent, $I_0 = wek_B T_c/\hbar\xi_c$ is a critical current, and w is the film width. If we assume a mean-field temperature dependence of the superfluid density, the exponent $a(T)$ has an approximate dependence $1 + 2(T_{c0} - T)/(T_{c0} - T_c)$, except near T_c, where theory predicts a square-root cusp dependence of the form $a(T) = 3 + \pi[(T_c - T)/b(T_{c0} - T_c)]^{1/2}$.[5,8] The constant b will be determined below from experiment and is nonuniversal. Most importantly, one can deduce values for $\eta(T)$ from measurements of $a(T)$. One would therefore expect to observe a rapid crossover in behavior at $T = T_c$ where vortices with the largest separations unbind and $\eta(T_c) = \frac{1}{4}$, implying $a(T_c) = 3$.

The data in Fig. 1 illustrate, in essence, just such a behavior. The transition temperature T_c is defined as the highest temperature at which no deviation from power-law behavior at low currents is observed. This criterion is satisfied by curve d, measured at $T = T_c = 1.903$ K, which has a slope $a(T_c) = 3.281(3)$ calculated from data extending over *six* decades in voltage. For the lower-temperature data, the straight-line portions below 10^{-5} V were used to obtain $a(T)$. The plot in Fig. 2 of $a(T)$ vs T reveals linear behavior with an extrapolated value of 3 occurring at $T_{cx} = 1.939$ K. At this temperature (curve a of Fig. 1) the voltage at small currents is linear in current, which is the expected response for thermally excited free vortices, and so T_{cx} is clearly above T_c. It is important to note that the power-law dependences of the data in Fig. 1 extend over a large enough range and show a sharp discontinuity in slope as a function of T (compare c, which is at a temperature just 6 mK higher than d) in agreement with theoretical expectations of a sharp phase transition. This pronounced crossover in behavior establishes T_c to within approximately ± 3 mK and enables us to make an *independent measurement* of $a(T_c)$, in contrast to previous experiments[8-10] where T_c is defined as that

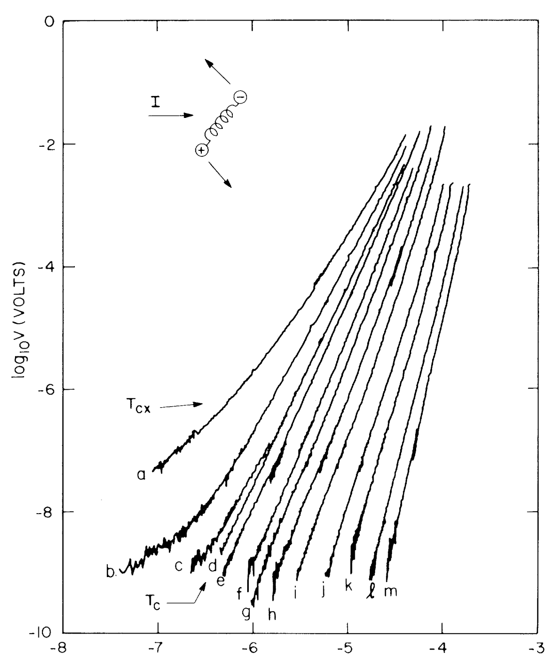

FIG. 1. Plot on logarithmic axes of the V-I characteristics taken at thirteen successively lower temperatures ranging from 1.939 K (curve a) to 1.460 K (curve m). The curves are low-frequency (24.2 Hz) digitized data representing approximately 200 points per voltage decade. The nonlinear pair-breaking process is illustrated schematically in the inset. T_c and T_{cx} are discussed in the text.

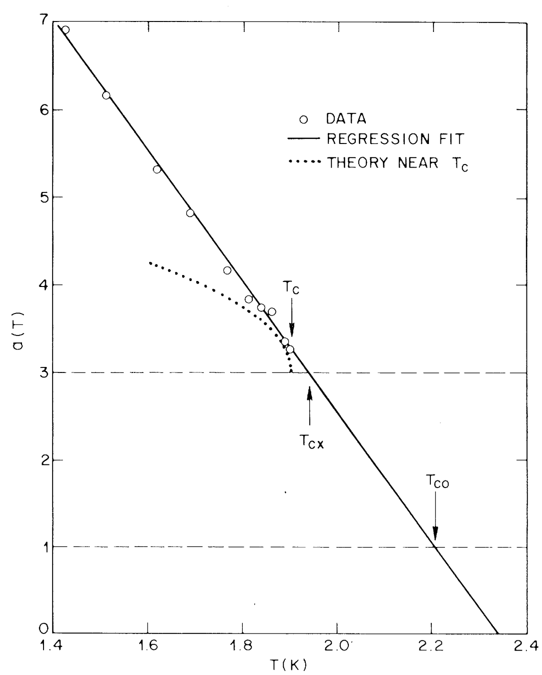

FIG. 2. Plot of power-law slopes extracted from curves d–m of Fig. 1 as a function of temperature. The temperatures T_{cx}, T_c, and T_{c0} together with the theoretical fits are discussed in the text.

temperature where $a(T)$ is exactly 3. An extrapolation of $a(T)$ to unity in Fig. 2 gives the additional result $T_{c0} = 2.206$ K, in reasonable agreement with the value $T_{c0} \simeq 2.3$ K determined from a fit of the Aslamosov-Larkin theory of paraconductivity[5] to the resistive transition in the region $T > T_{c0}$. This fit, carried out over a temperature range of 1.8 K, ignores the temperature dependence of R_n and gives a normal-state sheet resistance of 3735 Ω/□.

For $T > T_c$ the presence of thermally excited free vortices, indicated schematically in the inset in Fig. 3, gives rise to linear V-I characteristics with a resistance R which has the theoretical dependence near T_c of the form[5]

$$R = 10.8 b R_n \exp\{-2[b(T_{c0} - T_c)/(T - T_c)]^{1/2}\}. \quad (2)$$

The nonuniversal constant b is the same parameter which appears in the square-root cusp of the exponent in Eq. (1). The data in Fig. 3 used to test this functional dependence were taken at low enough currents to assure that nonlinear pair breaking was not occurring. Use of the previously determined values for T_c and T_{c0}, the slope

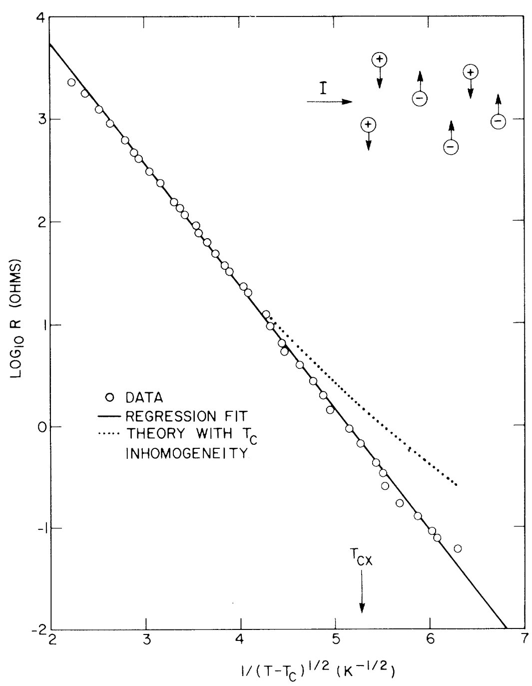

FIG. 3. Plot for $T > T_c$ of the logarithm of the resistance as a function of $(T - T_c)^{-1/2}$ where $T_c = 1.903$ K. These data represent the flux-flow resistance (linear V-I characteristics) of thermally excited vortices, a process indicated schematically in the inset. The theory fits are discussed in the text.

and intercept of the regression fit to the data, which extends over four decades in resistance, yields the parameters b and R_n. The goodness of fit in the data of Fig. 3 is insensitive to the ± 3-mK uncertainty in T_c although the uncertainties in $b = 6.28^{-0.98}_{+1.05}$ and $R_n = 20\,850^{-7700}_{+11900}$ Ω are large. This value for R_n is in agreement with the resistance of 17 800 Ω (3560 Ω/□) measured at 8 K in the paraconductivity regime which may be somewhat fortuitous because of the theoretical approximations affecting the prefactors in Eqs. (1) and (2).[5]

Additional parameters can now be extracted from the data. With use of R_n and the Ginzburg-Landau temperature dependence for ξ_c, a nonlinear optimal fit of curves d–m of Fig. 1 by the theoretical form of Eq. (1) gives the values $I_0(T_c) = 0.77$ mA and $\xi_c(T_c) = 52$ Å. This rather short coherence length is typical for amorphous materials. If we know ξ_c, b, T_c, and T_{c0}, it is a straightforward matter to calculate the pair correlation length ξ^+ which represents the average distance between thermally excited vortices. Interestingly, at the lowest temperature (1.928 K) for which linear V-I data could be obtained, ξ^+ is calculated to be 6.7 μm, a factor of 1000 larger than ξ_c but still appreciably smaller than the 100-μm width of the film.

To ascertain the effects of large-scale film inhomogeneity,[6] a linear gradient in T_c along the length and width of the film was modeled into Eqs. (1) and (2). For a gradient of $(T_{cx} - T_c)/(0.05 \text{ cm}) = 0.72$ K/cm, a less than 2% effect on the measured slopes of Eq. (1) was found. The effect on Eq. (2), however, for this same gradient along the film length is appreciable as shown by the dotted line in Fig. 3. Clearly, such a large gradient in T_c is not present, and the difference between T_c and T_{cx} is not explained by inhomogeneity of this type.

Once we have obtained b, the square-root cusp component of the theoretical behavior for $a(T)$ near T_c is revealed by the dotted curve in Fig. 2. There are no adjustable parameters since the previously stated values for b, T_c, and T_{c0} were used. The good qualitative agreement between the "size" of the cusp and the data reflects the importance of renormalization effects near T_c. The discrepancy between theory and the data at lower temperatures arises because the square-root dependence is valid only near T_c and also does not include the mean-field temperature dependence of the superfluid density. The influence of the finite measurement length near T_c can be estimated by considering the lowest measuring current, $I = 0.46\ \mu A$ or curve d in Fig. 1, which corresponds to depairing of vortices with separation greater than $r_c = 11\ \mu m$.[17] The correction to $a(T_c)$ is theoretically given as $[\ln(r_c/\xi_c)]^{-1} \approx 0.13$.[17]

The critical exponent $\eta(T_c) = 0.5[a(T_c) - 1]^{-1}$ obtained from curve d is 0.219. Application of the theoretical finite-length correction to $a(T_c)$ gives $\eta(T_c) = 0.23$. This latter value is to be compared with the theory for the asymptotic, long-length limit, $\eta(T_c) = 0.25$. The difference of 0.02 remains unexplained.

In conclusion, critical-exponent measurements of a 2D superconductor have been made and good agreement with theory both above and below T_c has been obtained. The results reported here are also consistent with ac[14] and dc measurements on two additional films.[18] The deviation of $\eta(T_c)$ from the theoretical value, after correction for finite length effects, is small. An additional concern in accounting for any systematic deviations is that $\eta(T_c)$ may in fact be renormalized as the normal-state sheet resistance increases towards 30 000 Ω, a region where superconductivity has been found to be severely weakened by localization effects.[19]

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