Meson-Exchange Hamiltonian for NN Scattering up to 1 GeV and Δ-Nucleus Dynamics

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(Received 21 March 1983)

A Hamiltonian for π, N, and Δ is constructed from meson-exchange mechanisms. The NN interaction in the model is directly derived from the Paris potential. Good fits to pp scattering data up to 1 GeV, including the "resonant" $^1D_2$ and $^3F_3$ partial waves, have been obtained when a $\pi N \leftrightarrow \Delta$ interaction is introduced in a dynamical three-body approach. The model can be used for a many-body study of Δ-nucleus dynamics.

PACS numbers: 21.30.+y, 13.75.Cs, 13.75.Gx

In recent years, one of the major developments in nuclear physics has been to extend[1] the conventional nuclear many-body theory to include nonnucleonic degrees of freedom, notably the π and the Δ isobar. Such an extension is not only obviously needed to describe nuclear reactions induced by intermediate energy pions, nucleons, and electrons, but is probably also needed to resolve various problems encountered[1] in the study of low-energy nuclear phenomena. In addition, many theoretical studies,[2] motivated by polarization measurements[3] of NN scattering, have shown that the dibaryon resonances, if they exist, can be extracted from the NN data only when the dynamics involving N, π, and Δ is treated properly. Clearly, all of these problems, both at low and intermediate energies, are related to the same dynamics, and must be investigated within a unified theoretical framework. This can be achieved in a straightforward manner, by constructing a many-body Hamiltonian[4] for π, N, and Δ within the framework of particle quantum mechanics.[5]

The model Hamiltonian must contain the following interaction mechanisms: (a) The Δ isobar is related to πN scattering through resonance formation; (b) π production from NN collisions is dominated by Δ excitation $NN \rightarrow N\Delta \rightarrow \pi NN$. Naturally, an acceptable model should begin with satisfactory descriptions of πN scattering up to ~300 MeV and NN scattering up to ~1 GeV. A phenomenological separable model of this type was previously constructed[4] and has been used[6] to obtain a qualitative understanding of the Δ-nucleus interactions as revealed by pion experiments. In this paper, I show that the approach of Ref. 4 can be extended to obtain a model Hamiltonian which not only fulfills the requirements stated above, but is also directly related to the most successful meson theory of the NN force: the Paris potential.[7]

To illustrate the theory, it is sufficient to consider only πN and NN scattering. Following the formulation of Ref. 4, the model Hamiltonian for these two processes can be written as

$$H = H_0 + v + h, \tag{1}$$

where H_0 is the sum of relativistic free-energy operators $E_N(p)$, $E_\Delta(p)$, and $E_\pi(p)$ for N, Δ, and π, respectively, v is the sum of all direct two-body interactions between NN, $N\Delta$, and $\Delta\Delta$ states, and h is a $\pi N \leftrightarrow \Delta$ vertex interaction in the P_{33} channel. (Other weak πN interaction are omitted for simplicity.) It is easy to write down[4] in this model the P_{33} scattering amplitude in the πN c.m. frame:

$$t_\alpha(q, q_0, w) = h(q)h(q_0)/[w - m_\Delta - \Sigma_\Delta(w)], \tag{2}$$

where q and w are the πN relative momentum and collision energy, and

$$\Sigma_\Delta(w) = \int_0^\infty \frac{q^2 dq\, |h(q)|^2}{w - E_\pi(q) - E_N(q) + i\epsilon} \tag{3}$$

is the Δ self-energy calculated from $\pi N \leftrightarrow \Delta$ coupling $h(q) = g_\Delta(q/\mu)\, [\Lambda_\Delta^2/(\Lambda_\Delta^2 + q^2)]^2$. m and μ are respectively the masses of N and π. By fitting P_{33} phase shifts up to ~300 MeV, we find that $2(m+\mu)g_\Delta^2 = (0.98)^2$, $\Lambda_\Delta = 358$ MeV/c, and the bare mass of the Δ is $m_\Delta = 1280$ MeV.

The baryon-baryon interaction v is assumed to be determined by meson-exchange mechanisms at $r > 1$ fm. The short-range part of v is treated phenomenologically, as usual. To describe the NN force theoretically, we choose the Paris potential as a means to build into our theory as much as possible the present knowledge of meson-exchange mechanisms. Our major task is therefore to devise a consistent procedure so that the $NN \rightarrow NN$ part of v can be derived from the Paris potential, when the Δ and π are explicitly present in the model Eq. (1).

The $NN \leftrightarrow N\Delta$ and $NN \leftrightarrow \Delta\Delta$ parts of v are calculated from one-pion and one-rho exchange. The $N\Delta \leftrightarrow N\Delta$, $\Delta\Delta \leftrightarrow \Delta\Delta$, and $N\Delta \leftrightarrow \Delta\Delta$ parts of v are put equal to zero to minimize the complications of the model. However, note that the vertex interaction h for $\pi N \leftrightarrow \Delta$ of Eq. (1) automatically generates a one-pion-exchange $N\Delta \leftrightarrow \Delta N$ transition. Thus this simplified model does contain the most important ingredient for a successful description of pion scattering from nucleons.

It is straightforward to derive[4] from Eq. (1) the NN scattering equation in the channel space $NN \oplus N\Delta \oplus \Delta\Delta \oplus \pi NN \oplus \pi N\Delta$. With above simplifications of the baryon-baryon interaction v, the $NN \to NN$ scattering equation for each partial-wave eigenchannel $\alpha = JST$ can be cast in the c.m. frame into

$$T_{l',l}{}^{\alpha}(p',p,E) = V_{l',l}{}^{\alpha}(p',p,E) + \sum_{l''} \int \frac{p''^2 dp''\; V_{l',l''}{}^{\alpha}(p',p'',E) T_{l'',l}{}^{\alpha}(p'',p,E)}{E - 2E_N(p'') + i\epsilon}, \tag{4}$$

where the effective energy-dependent NN interaction $V_{l',l}{}^{\alpha}$ contains effects due to the coupling to $N\Delta$, $\Delta\Delta$, and πNN states. The most difficult part of the whole approach is to calculate the NN interaction in πNN states. In the separable model,[4] such effects have been investigated and found to have ~20% effect on ${}^1D_2$ phase shifts at energies above 500 MeV. However, they are completely negligible in all other NN partial waves. For our present purpose, we ignore this three-body effect. In this approximation, we can derive rigorously $V^{\alpha}(E)$ from Sec. IV.A of Ref. 4:

$$V_{l',l}{}^{\alpha}(p',p,E) = v_{l',l}{}^{1,\alpha}(p',p) + \sum_{\substack{i=2,3\\ l''S''}} \int p''^2 dp''\, \frac{v_{l',l''S''}{}^{i,\alpha}(p',p'') v_{l''S'',l}{}^{i,\alpha}(p'',p)}{E - E_i(p'') - E_{\Delta}(p'') - \Sigma_{\Delta}[w_i(E,p'')]}, \tag{5}$$

where $i = 2, 3$ denote respectively the $N\Delta$ and $\Delta\Delta$ intermediate states with $E_2 = E_N$ and $E_3 = E_{\Delta}$, and $v_{l',l''S''}{}^{i,\alpha}$ with $i = 1, 2, 3$ is respectively $NN \to NN$, $NN \leftrightarrow N\Delta$, and $NN \leftrightarrow \Delta\Delta$ interactions. An important feature of Eq. (5) is the appearance of the Δ self-energy Σ_{Δ} which *depends on both the collision energy E and the intermediate relative momentum* $\vec{p}''$. Such an energy and momentum dependence is an essential dynamical effect resulting from any three-body formulation of the πNN system,[4,8] and is precisely defined in our approach from the $\pi N \leftrightarrow \Delta$ vertex function in Eq. (3). If baryons in πNN and $\pi N\Delta$ states are treated nonrelativistically, one finds[2] that

$$w_i(E,p'') \to E - m_i - (\vec{p}''^2/2m_i) - \vec{p}''^2/2[m + E_{\pi}(q)]$$

with $m_i = m, m_{\Delta}$ for $i = 2, 3$. For a given p'', $w = w_i(E,p'')$ is substituted into Eq. (3) to evaluate Σ_{Δ} in Eq. (5).

In Fig. 1, $\Sigma_{\Delta}(w(E,p''))$ for the $N\Delta$ state is plotted as a function of E and p''. First we note that at energies below pion production threshold $E < 280$ MeV in the laboratory frame, the imaginary part of Σ_{Δ} vanishes. In this case, $V_{l',l}{}^{\alpha}$ of Eq. (5) is real and leads to pure elastic scattering from solving Eq. (4). Second, the momentum dependence of Σ_{Δ} is significant. Even in the low-energy region < 300 MeV, it cannot be represented by a constant corresponding to the usual coupled-channels model[9] with a stable Δ. The Σ_{Δ} for $\Delta\Delta$ has a similar feature except that its imaginary part becomes significant only when $E \simeq 1$ GeV is reached. Clearly, the success of our model depends partly on whether this energy and momentum dependence originating from $\pi N \leftrightarrow \Delta$ has correct dynamical consequences in NN scattering.

We now derive v^1 from the Paris potential. Obviously, a portion of the 2π-exchange part of the Paris potential corresponds to the intermediate $N\Delta$ and $\Delta\Delta$ excitations mediated by *uncorrelated*

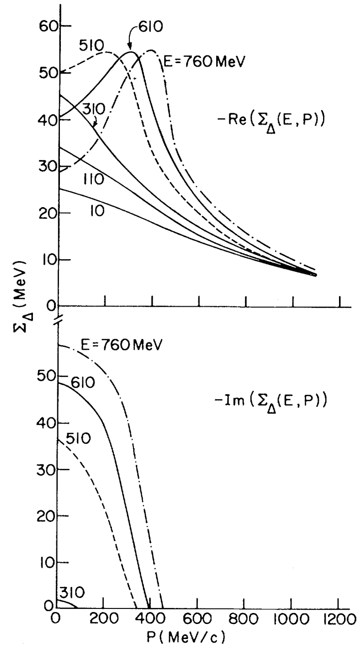

FIG. 1. The Δ self-energy $\Sigma_{\Delta}(E,p)$ in the $N\Delta$ intermediate state in Eq. (5).

direct 2π exchange. Our procedure is to assume that this part of the Paris potential is equal to the second term in Eq. (5) evaluated at a *fixed* energy E_s *below the pion production threshold.* The matrix element of v^1 in our model is then defined as

$$v_{l',l}^{1,\alpha}(p',p) \equiv [V_{l',l}^{\alpha}(p',p)]_{\rm Paris} - [\text{second term of Eq. (5)}]_{E=E_s}\,. \tag{6}$$

It is clear from our discussion of Fig. 1 that the second term of Eq. (6) is real and hence the resulting $v_{l',l}^{1,\alpha}(p',p)$ is the matrix element of a *nonlocal* but *Hermitian* operator. Thus, our construction of v^1 is rigorous within particle quantum mechanics.[5] In fact, the definition of Eq. (6) is sufficient for the application of the model in subsequent many-body calculations.

It is clear that our model based on Eqs. (5) and (6) reproduces the Paris phase shifts at $E=E_s$. As seen in Fig. 1, the energy and momentum dependence of Σ_Δ are very similar at low energy. Thus, our model and the Paris potential should give very similar NN scattering at low energy. When the energy increases, Σ_Δ becomes complex and our model will show its power to predict NN inelasticity which cannot be described by the Paris and other low-energy NN potentials.

Transition potentials v^2 and v^3 are calculated from π and ρ exchange. A form factor $(\Lambda^2-\mu^2)/(q^2+\Lambda^2)$ is introduced in each meson-$N\Delta$ vertex. The resulting transition potentials are given in Eqs. (22)–(25) of Niephaus *et al.*[9] The coupling constants are taken from row (a) of their Table IV. The scattering equation (4) is solved in momentum space with a standard matrix method to treat relativistic kinematics.

With the above constructions, the model only has two free parameters: E_s and Λ. For $\Lambda < 1000$ MeV/c the calculated low-energy NN phase shifts are found to be insensitive to E_s if E_s is chosen to be <100 MeV. However, we found that the calculated inelasticity is very sensitive to the cutoff parameter Λ. The best results for pp scattering in all partial waves are obtained with $E_s = 10$ MeV and $\Lambda = 650$ MeV/c. It is seen from Fig. 2 that the model gives excellent descriptions of Arndt's phase shifts[10] up to 1 GeV in most partial waves (see Ref. 10 for new phase parametrization). In particular, the "resonant" $^1D_2$ and $^3F_3$ are well described by the model. The main discrepancy is in the inelastic parameter ρ of $^3H_4$ which was set to zero in the analysis of Ref. 10. Calculations by Green and Sainio[11] and others also predict nonzero ρ in this and other peripheral waves. Their values, however, only qualitatively agree with the present work.

I conclude that the present model defined by Eqs. (1)–(6) contains correct meson-exchange mechanisms for NN scattering up to 1 GeV. A dynamical three-body treatment of $\pi N \to \Delta$ coupling and the use of the Paris potential are essen-

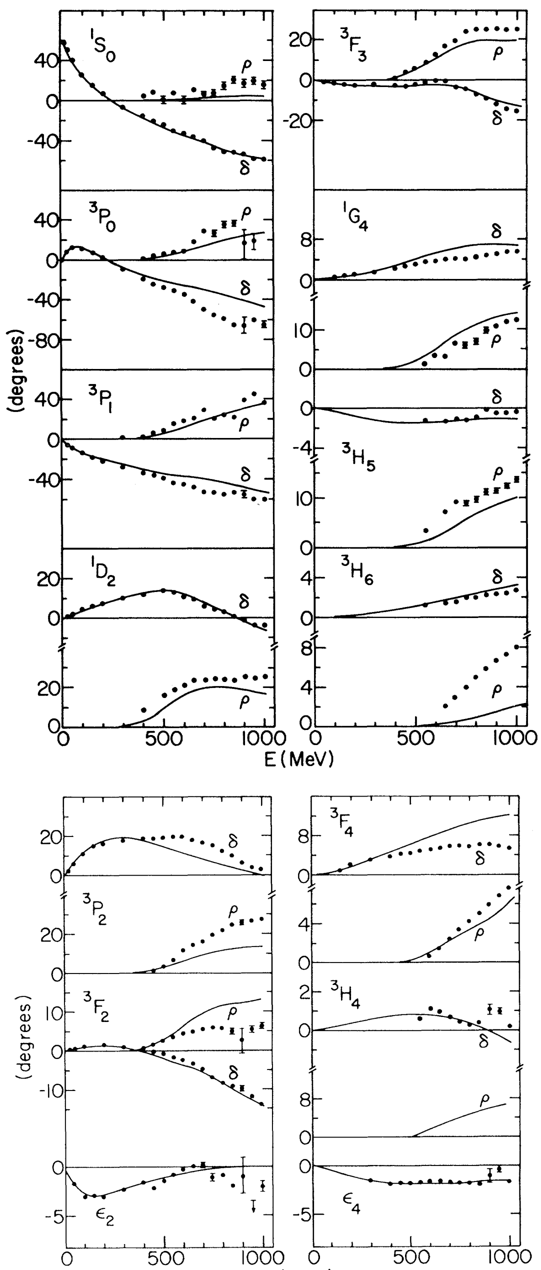

FIG. 2. The calculated pp scattering phase shifts compared with Arndt's phase shifts (Ref. 10).

tial for the success of the model. The model is a sound starting point for further refinements, such as including effects of NN interaction in πNN states, and including the $N^*(1470)$ resonance so as also to describe, realistically, $T=0$ NN scattering. Finally, it is necessary to emphasize that the model can be used, as demonstrated in Refs. 6 and 12, to carry out many-body calculations. Therefore, we have a realistic unified approach to study Δ-nucleus dynamics both at low and intermediate energies.

I thank B. Day for useful discussions. This work was supported by the U. S. Department of Energy under Contract No. W-31-109-ENG-38.

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