Bound States of Negative Pions and Neutrons

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By use of standard forms for the various interactions of a system of N neutrons and Z negative pions, it is shown that for a large variety of values of N and Z the system can become bound. This gives rise to a structure similar to that of an ordinary nucleus, but in which the protons are replaced by negative pions.

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Ericson and Myhrer[1] have pointed out that the real part of the pion-nucleus optical potential is sufficiently strong that a negative pion may become bound by the strong interaction of the nucleus. However, as has been noticed by Friedman, Gal, and Mandelzweig,[2] the imaginary part of the optical potential has the effect of producing very large widths for these states (of the order of 2000–3000 MeV), so that they cannot be observed experimentally. We will study in this paper the case in which a negative pion is bound not by a nucleus but by a piece of neutron matter, so that in this case the optical potential is real, since the pion absorption process, which is responsible for the imaginary part, cannot take place when there are only neutrons. Therefore, these states will be stable, with a lifetime similar to that of the charged pion. Another important difference between the case when the π^- is bound in a nucleus and when it is bound in a piece of neutron matter is the fact that at threshold the p-wave spin-averaged pion-neutron amplitude is 10 times stronger than the corresponding pion-proton amplitude, so that the conditions for binding are much more favorable when there are only neutrons.

Of course, a system of N neutrons alone cannot form a bound state, and as a matter of fact at normal nuclear densities they are unbound by a large amount, but we will show that the energy that is lacking can be provided by the pions. This situation is similar to that of ordinary nuclei, where if we take N neutrons at normal nuclear matter density, the self-energy of the system is very large and positive (which means that it is unbound); however, the optical potential of a proton in this piece of neutron matter is very strong, so that if we let it, the proton can become bound by the optical potential, and the self-energy of the system decreases. If we now keep adding more protons (taking due regard of the Pauli principle), eventually the sum of all the accumulated binding energies is larger than the self-energy of the neutrons, and the system becomes a bound state of N neutrons and Z protons, which is an ordinary nucleus. We will see in this paper that the same mechanism will give rise to bound states of N neutrons and Z negative pions.

In order to be able to describe this system theoretically, we need to know (a) the interaction of a pion with the distribution of neutrons, (b) the interaction between the neutrons themselves, (c) the interaction between the pions themselves, and (d) the Coulomb interaction. In the case of part (a), we will use the optical potential of pionic atoms extrapolated to the case of zero proton density and corrected for the finite size of the pion-nucleon interaction. Part (b) will be treated in a local density approximation, with use of the recent results of Jackson *et al.*,[3] for the self-energy of infinite neutron matter, where they have used for the neutron-neutron interaction the Bethe-Johnson potential.[4] We will simply neglect part (c) since, as the two pions are in a state with isospin 2, we have that the $\pi\pi$ interaction is extremely weak (both the $l=0$ and $l=2$ phase shifts[5] for the case $I=2$ are smaller than 1 deg for invariant energies of up to 800 MeV). Finally, part (d) will be taken into account in the standard fashion.

Let us consider first the interaction of the pion with the N neutrons. The level shifts and widths of pionic atoms have traditionally been fitted with the approximate Klein-Gordon equation[1,2]

$$\nabla^2\psi(\vec{r}) + k_0{}^2\psi(\vec{r}) - 2\omega V(\vec{r})\psi(\vec{r}) = 0, \tag{1}$$

where the optical potential $V(\vec{r})$ is of the Kisslinger form

$$(2\mu/4\pi)\,V(\vec{r}) = q(r) - \nabla\cdot p(r)\nabla\,. \tag{2}$$

The local and velocity-dependent parts $q(r)$ and $p(r)$ are given in terms of the neutron and proton densities $\rho_n(r)$ and $\rho_p(r)$ as[2,6]

$$q(r) = -(1+\mu/M)\{b_0[\rho_n(r)+\rho_p(r)] + b_1[\rho_n(r)-\rho_p(r)]\} - (1+\mu/2M)B_0 4\rho_n(r)\rho_p(r), \tag{3}$$

$$p(r) = -(1+\mu/M)^{-1}\{c_0[\rho_n(r)+\rho_p(r)] + c_1[\rho_n(r)-\rho_p(r)]\} - (1+\mu/2M)^{-1} C_0 4\rho_n(r)\rho_p(r), \tag{4}$$

where μ and M are the pion and nucleon masses, respectively. The parameters b_0, b_1, B_0 and c_0, c_1, C_0, were fitted by Friedman, Gal, and Mandelzweig[2] to the very precise data on pionic atoms[6] (including those with an excess of neutrons), and they obtained the values

$$\begin{aligned} &b_0 = -0.017\mu^{-1}, \quad b_1 = -0.13\mu^{-1}, \quad \mathrm{Im}B_0 = 0.0475\mu^{-4}, \quad \mathrm{Re}B_0/\mathrm{Im}B_0 = -1, \\ &c_0 = 0.21\mu^{-3}, \quad c_1 = 0.17\mu^{-3}, \quad \mathrm{Im}C_0 = 0.0425\mu^{-6}, \quad \mathrm{Re}C_0/\mathrm{Im}C_0 = -0.6. \end{aligned} \tag{5}$$

If we take Eqs. (3)–(5) in the limit when $\rho_p \to 0$, we get the optical potential of a negative pion with a piece of neutron matter:

$$(2\mu/4\pi)V(\vec{r}) = \beta_0\rho_n(r) - \beta_1 \nabla\cdot\rho_n(r)\nabla, \tag{6}$$

with

$$\begin{aligned} &\beta_0 = 0.147(1+\mu/M)\mu^{-1}, \\ &\beta_1 = -0.38(1+\mu/M)^{-1}\mu^{-3}. \end{aligned} \tag{7}$$

For the purpose of comparison, it is instructive to construct the pion–neutron-matter optical potential theoretically, which we write in momentum space as

$$\begin{aligned} V(\vec{k},\vec{k}') &= t_{\pi n}(\vec{k},\vec{k}')\rho_n(\vec{k}-\vec{k}') \\ &= (4\pi/2\mu)(\beta_0 + \beta_1\vec{k}\cdot\vec{k}')\rho_n(\vec{k}-\vec{k}'), \end{aligned} \tag{8}$$

with the strengths β_0 and β_1 given in terms of the experimental scattering lengths[7] $a_L^{2I,2J}$ by

$$\begin{aligned} \beta_0 &= -a_0^{31}(1+\mu/M) = 0.098(1+\mu/M)\mu^{-1}, \\ \beta_1 &= -(a_1^{31}+2a_1^{33})(1+\mu/M)^{-1} \\ &= -0.379(1+\mu/M)^{-1}\mu^{-3}. \end{aligned} \tag{9}$$

Thus, we see that the "theoretical" strengths (9) are quite close to those given by Eq. (7), particularly the very important p-wave part.

The pion–neutron-matter optical potential (6) implies a pion-nucleon amplitude with zero range, which can lead to unphysical behavior.[8] In order to avoid these problems, we will take into account the finite range of the pion-nucleon interaction by making the replacement

$$V(\vec{k},\vec{k}') \to V(\vec{k},\vec{k}')\Lambda^4/(k^2+\Lambda^2)(k'^2+\Lambda^2),$$

where we will use for the range Λ the value $\Lambda = 1$ GeV/c which has been obtained by several theoretical studies.[9-13]

Since we do not know the density of the piece of neutron matter, we will assume that it is similar to that of ordinary nuclei, that is, we will use the Fermi distribution[14]

$$\begin{aligned} &\rho_n(r) = \rho_0[1+e^{(r-R)/d}]^{-1}, \\ &d = 0.569\ \mathrm{fm}, \quad R = r_0 N^{1/3}, \end{aligned} \tag{10}$$

where the parameter r_0 will be varied so as to study the dependence of the binding energy on the radius of the distribution.

I obtained the ground-state solution of a pion in a piece of neutron matter (which I found to correspond to the state with angular momentum $L=0$) by transforming Eq. (1) into momentum space and solving the resulting integral equation with a 40-point Gauss mesh, which gives the binding energy with an accuracy of about 0.1 MeV. I show in Fig. 1 the results for the binding energy of the pion, for several values of the parameter r_0 in Eq. (10). As we see, the pion can be bound more easily if r_0 is small, which is understandable if we consider that the density $\rho_n(r)$ in the central region increases when r_0 decreases, and therefore the optical potential becomes stronger.

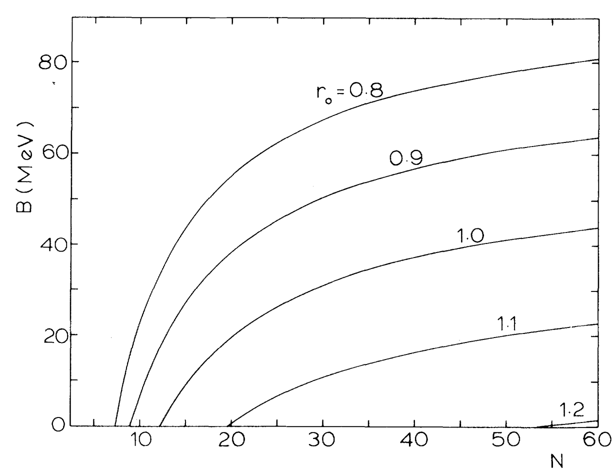

FIG. 1. Binding energy of a pion in the piece of neutron matter described by Eq. (10), as a function of the number of neutrons N.

In order to investigate whether Z negative pions and the neutron distribution (10) can form a stable bound state, we need to know, in addition to the binding energies of Fig. 1, the self-energy of the N neutrons and the Coulomb energy of the pions. Therefore, I describe next the procedure used for calculating these two quantities.

The self-energy of infinite neutron matter has been calculated recently by Jackson *et al.*,[3] using the correlated basis functions (CBF) scheme,[15] in which they have taken into account the perturbative CBF correction to second order. From their results for $e(p_F)$, the self-energy per neutron obtained from the Bethe-Johnson potential[4] (see Table 4 and Fig. 10 of Ref. 3), we calculate the self-energy of a finite distribution of neutrons as

$$E_n = \int e(p_F)\rho_n(r)\,d^3r, \qquad (11)$$

where the Fermi momentum p_F is related to the density of the neutron distribution (10) by

$$p_F = [3\pi^2\rho_n(r)]^{1/3}. \qquad (12)$$

The energy associated with the Coulomb repulsion between the pions will be calculated by assuming that the total charge Ze is distributed uniformly in a sphere of radius $R=r_0N^{1/3}$, which gives simply

$$E_C = \frac{3}{5}\frac{\hbar c\alpha}{r_0}\frac{Z^2}{N^{1/3}}. \qquad (13)$$

We can now obtain the self-energy of a system of N neutrons and Z pions by adding the self-energy of the neutrons given by Eq. (11) to the Coulomb energy (13) and subtracting the binding energy of the Z pions, that is, for a given choice of the parameter r_0,

$$E_Z^{\,N}(r_0) = E_n(r_0, N) - ZB(r_0, N) + \frac{3}{5}\frac{\hbar c\alpha}{r_0}\frac{Z^2}{N^{1/3}}. \qquad (14)$$

If the self-energy of Eq. (14) is negative, then it is possible to form a bound state characterized by the two integers (N, Z).

As an example, let us consider the case $N=20$, $r_0=1.0$ fm, which corresponds to having all the neutrons in closed shells and a density of neutrons similar to that of an ordinary nucleus.[14] We get, using Fig. 1 and Eqs. (10)–(13), that (in MeV)

$$E_Z^{\,20}(1.0) = 252.5 - 20.0Z + 0.318Z^2. \qquad (15)$$

The self-energy (15) becomes negative if $18 \leq Z \leq 45$, so that for these values of Z and N it is possible to form bound states at a neutron matter density similar to that of ordinary nuclei. The binding energies obtained from Eq. (15) for the case of 20, 30, and 40 pions are 20.3, 61.3, and 38.7 MeV, respectively.

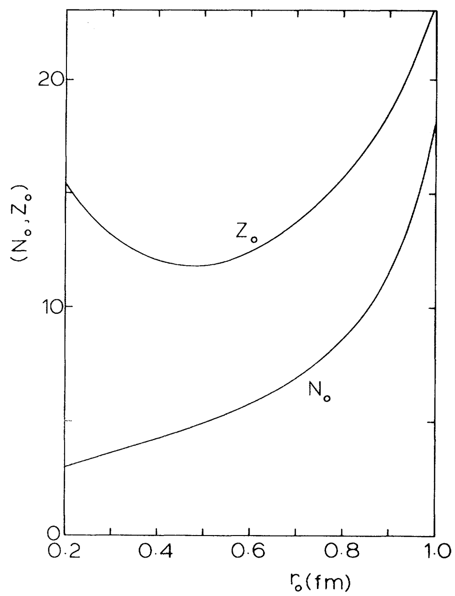

FIG. 2. The set of numbers (N_0, Z_0), as a function of the parameter r_0.

We can use Eq. (14) to find the set of numbers (N_0, Z_0) such that N_0 is the minimum number of neutrons that can be bound with the mechanism of Eq. (14) and Z_0 is the corresponding number of pions. I show the set (N_0, Z_0) in Fig. 2 as a function of the parameter r_0. We see that the bound states with the smaller number of neutrons and pions will occur with a value of r_0 around 0.5 fm.

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