Static and Dynamic Critical Magnetic Fields in Ising Spin-Glasses

W. Kinzel and K. Binder
Institut für Festkörperforschung der Kernforschungsanlage Jülich, D-5170 Jülich, Federal Republic of Germany
(Received 15 February 1983)

By Monte Carlo simulations it is shown that the magnetization $M(T,H)$ of the two-dimensional nearest-neighbor Edwards-Anderson model is proportional to H^{1-x}, essentially independent of temperature T, for fields $H < H_c^{eq}(T)$, with $H_c^{eq}(T) \propto T^{1/x}$, the exponent being $x \approx 0.28 \pm 0.06$. Irreversible behavior on a time scale t sets in for a field $H_c(t)$ where $H_c(t)/T_f(t)$ nearly follows the De Almeida–Thouless line, although no static nonzero freezing temperature T_f exists. Consequences for the interpretation of related experimental work are discussed.

PACS numbers: 75.40.Dy, 75.10.Hk, 75.30.Kz

In the infinite-range model of spin-glasses,[1,2] De Almeida and Thouless (AT)[3] have found a critical magnetic field $H_c(T)$, with $H_c(T) \propto (1 - T/T_f)^{3/2}$ for temperatures near T_f, for the onset of "replica-symmetry breaking." For $H < H_c(T)$ the model is nonergodic,[4,5] and truly irreversible behavior sets in. These model predictions very recently have inspired a large number of experiments where various critical magnetic fields for real spin-galss materials were identified (e.g., Refs. 6–10). The first experiment[6] locates a field $H_c^{eq}(T)$ from the temperature below which the field-cooled magnetization $M(T,H)$ is essentially temperature independent. Note that it is widely believed that by slow field cooling one finds the *equilibrium* magnetization. Other methods[7-10] locate various critical fields $H_c(t)$ from studying where slow magnetic relaxation disappears. The temperature variation of these fields follows rather nicely the form $H_c(t) \propto [1 - T/T_f(t)]^{3/2}$, although both $T_f(t)$ and $H_c(t)$ depend on the time scale t of observations.[7-10] These findings have been widely interpreted as evidence that real spin-glasses do have an Edwards-Anderson–type (EA) phase transition and an AT line. Of course, this conclusion is at variance with the theoretical prediction that the lower critical space dimension d^* for this transition is $d^* = 4$ (see, e.g., Refs. 11–14), and hence $T_f = 0$ for $d = 3$.

In this Letter we elucidate this problem by careful Monte Carlo studies of square Ising lattices with nearest-neighbor bonds distributed according to a Gaussian, $P(J) \propto \exp[-J^2/2(\Delta J)^2]$. For this model there is overwhelming evidence[13,15] that $T_f = 0$ in thermal equilibrium; Monte Carlo studies over "time scales" of 10^3 to 10^5 Monte Carlo steps per spin indicate onset of very slow relaxation of the spins near $kT_f(t)/\Delta J \lesssim 1$,[16] and many properties are (qualitatively) very similar[16-18] to experiment. Identifying critical fields in this model system (if they exist there also !) by methods analogous to the experimental ones hence should greatly clarify the significance of these critical fields.

Indeed the field-cooled "susceptibility" $M(T,H)/H$ is found to be independent of T for low temperatures, as shown by typical data in Fig. 1. These "plateau values" $M(T \to 0, H)/H$ become independent of cooling rate $|dT/dt|$ for small enough rates (Fig. 2), which is evidence that we observe the equilibrium magnetization.[19] For $H/\Delta J \gg 1$ M saturates and thus $M/H \propto H^{-1}$ trivially; but for

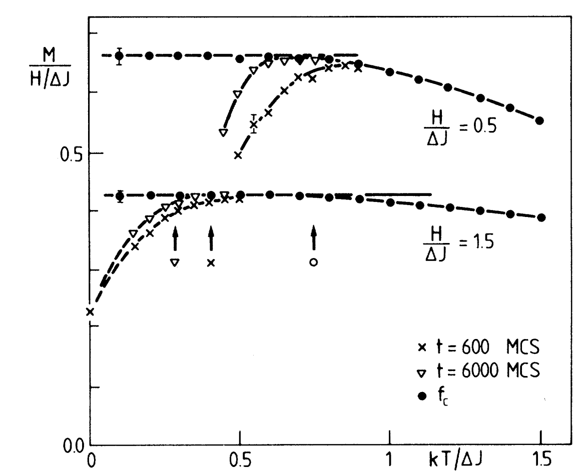

FIG. 1. Susceptibility $M/(H/\Delta J)$ plotted vs temperature for two values of the field, for 60×60 lattices with periodic boundary conditions. The field-cooled magnetization (full circles) is obtained from averages over 5–30 runs where T was cooled down from $kT/\Delta J = 1.6$ to $T = 0$ at a rate $dT/dt = 6.25 \times 10^{-5} J/\mathrm{kMCS}$ (MCS, Monte Carlo steps per spin). Other symbols show results where the system is cooled in zero field to the considered temperature and then the field is applied for a given time as indicated. Arrows illustrate the identification of critical fields (see text). Units of H are such that the zero-field susceptibility simply is $\Delta J/kT$.

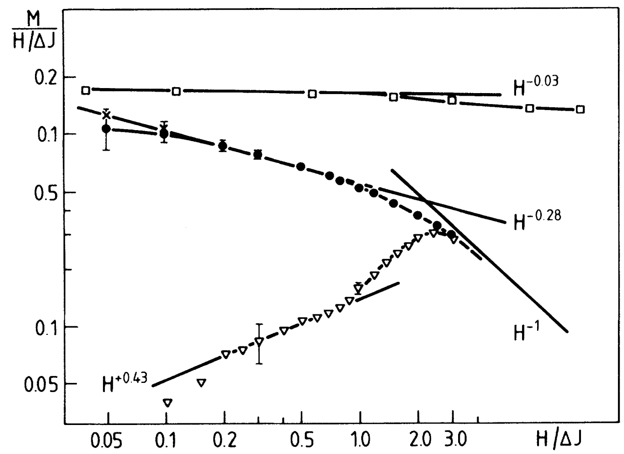

FIG. 2. Log-log plot of susceptibility $M(T\rightarrow 0)/(H/\Delta J)$ plotted vs field. Full circles denote field-cooled magnetization for $|dT/dt| = 2.5\times10^{-4}\Delta J$/kMCS, crosses for $|dT/dt| = (6.25$ and $1.5)\times10^{-5}\Delta J$/kMCS. Open squares denote experimental data of Ref. 6 for *Ag*-Mn(10.6%) (on arbitrary scales). Triangles are the magnetization obtained from systems cooled to $T=0$ without a field.

$0.1 \lesssim H/\Delta J \lesssim 1.0$ we observe a *nontrivial power law* $M/H \propto H^{-x}$, with $x \approx 0.28 \pm 0.06$. Thus there is no contradiction with the fact that the *zero-field* susceptibility for this model is divergent at $T=0$ [simple Curie law $\chi = \Delta J/kT$ (Ref. 13)]. We cannot, however, obtain this exponent with higher precision by studying smaller fields: (i) At smaller fields a still smaller cooling rate would be required, because at too high cooling rates the plateau value observed for M is too small (cf. full circle at $H/\Delta J = 0.05$, Fig. 2). (ii) While we estimate that the correlation length ξ_{EA} {measuring the decay of $[\langle S_0 S_R\rangle_T^2]_{av}$ with distance R} is much smaller than the lattice linear dimensions in the regime of T, H on which Fig. 2 is based,[13,15] for smaller H also much larger lattice sizes would be needed. Experimental data[6] behave like $M/H \propto H^{-0.09}$ for intermediate fields, while the effective exponent x crosses over to a still smaller value (≈ 0.03, Fig. 2) at smaller fields. The question arises whether this crossover reflects equilibrium properties or (as in the simulation) too large cooling rates at the *smallest* fields. In fact, after cooling *Au*Fe(8%) in small fields the magnetization still has not relaxed to its equilibrium value; see Lundgren, Svedlindh, and Beckmann.[20] Careful experiments on this question (and at still smaller fields) are needed, as the effective exponent x *must be strictly zero for* $H\rightarrow 0$ if the dimensionality d is at (or above) d^*, while a nonzero value of x is evidence for $d < d^*$.

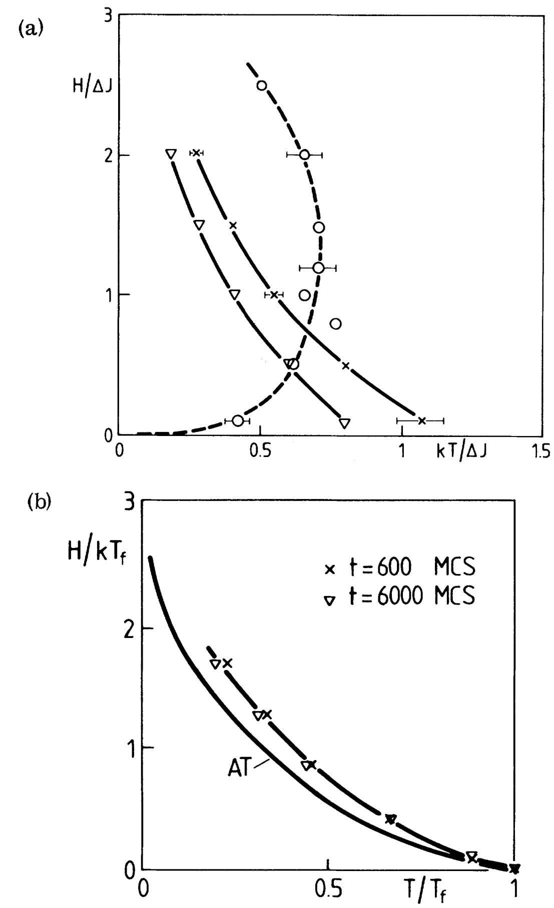

FIG. 3. (a) Static critical field $H_c^{eq}(T)$, open circles, and dynamic critical fields $H_c(t)$ for $t = 600$ MCS (crosses) and $t = 6000$ MCS (triangles) plotted vs temperature. (b) Normalized dynamic critical field plotted vs normalized temperature [note that $T_f(t)$ is observation-time dependent; see, e.g., Ref. 21].

As $T\rightarrow 0$ also the magnetization $M_0(t)$ found from putting systems cooled without field into a field becomes time independent, because the slow relaxation (involving barrier hopping) is completely frozen out (Fig. 1). The resulting susceptibility also follows a power law, $M/H \propto H^y$, with $y \approx 0.43$ in the regime of fields where our data are most reliable.

From our data we obtain the field $H_c^{eq}(T)$ where the plateau begins (Fig. 1, arrow with open circle) and the fields $H_c(t)$ [defined from the points where $M_0(t)$ and M differ by 3%, other arrows in Fig. 1]. These critical fields are displayed in Fig. 3. The static field $H_c^{eq}(T)$ is consistent at small T with a variation $H_c^{eq}(T) \propto T^{1/x}$, which one predicts from a scaling assumption

$\chi(T,H) \propto T^{-1}\tilde{\chi}((kT/\Delta J)^{1/x}/H)$ which yields T-independent plateau values if $\tilde{\chi}(z) \propto z^x$ for $z \ll 1$. The nonlinear susceptibility

$$\chi_{nl} \equiv \partial^2\chi/\partial H^2|_{H=0} = (J/kT)^3 \sum_R [\langle S_0 S_R \rangle_T{}^2]_{av}$$

then varies as $\chi_{nl} \propto T^{-1-2/x} \cong T^{-8\pm1}$, which is consistent with the observed behavior $\chi_{nl} \propto T^{-7}$.[15,18] This interpretation suggests that H_c^{eq} is not sharply defined, but only reflects a crossover at $z \approx 1$. For larger T the curve describing $H_c^{eq}(T)$ in the H-T plane then strongly increases and bends backwards again (Fig. 3, broken curve). Such a behavior is reminiscent of the variation of the position of the static susceptibility maximum of CuMn and GdAl spin-glasses in very small fields.[22]

The dynamic critical fields $H_c(t)$ increase monotonically as T decreases. This is expected, since the curves $H_c(t)$ basically represent contours of constant relaxation time in the H-T plane, and since we expect that with increasing fields both ξ_{EA} and the free-energy barrier dominating the relaxation time[15,21] will decrease. But the fact that $H_c(t)$ in scaled form nearly follows the AT line [Fig. 3(b)] is a surprise: Recall that there is no equilibrium freezing transition and hence also no replica-symmetry breaking in our model. Thus we feel that Fig. 3(b) rather reflects dynamic scaling associated with the transition at $T=0$.

In conclusion, we have shown that a variety of characteristic fields exist for spin-glass models even without a static freezing transition. A physical explanation for the temperature dependence of these fields remains to be given. Comparing our results with real materials encounters the difficulty, of course, that an Ising spin might correspond to a whole cluster of strongly correlated magnetic moments.[23] Without performing such a coarse graining explicitly, it is not possible to convert quantitatively the scales for t, H, etc., from the computer experiment to real systems. But the qualitative similarity of the results is again encouraging, and thus the "folklore" mapping of mean-field results (valid for the infinite-range model) to the real world has to be considered with great care.

[1]S. F. Edwards and P. W. Anderson, J. Phys. F 5, 965 (1975).

[2]D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. 35, 1792 (1975).

[3]J. R. L. De Almeida and D. J. Thouless, J. Phys. A 11, 983 (1978).

[4]H. Sompolinsky, Phys. Rev. Lett. 47, 935 (1981); H. Sompolinsky and A. Zippelius, Phys. Rev. B 25, 6860 (1982).

[5]N. D. Mackenzie and A. P. Young, Phys. Rev. Lett. 49, 301 (1982).

[6]P. Monod and H. Bouchiat, J. Phys. (Paris), Lett. 43, L45 (1982).

[7]R. V. Chamberlin, M. Hardiman, L. A. Turkevich, and R. Orbach, Phys. Rev. B 25, 6720 (1982).

[8]Y. Yeshurun and H. Sompolinsky, Phys. Rev. B 26, 1487 (1982).

[9]N. Bontemps, J. Rajchenbach, and R. Orbach, J. Phys. (Paris), Lett. 44, L47 (1983).

[10]M. B. Salamon and J. L. Tholence, J. Appl. Phys. 53, 7684 (1982), and to be published.

[11]R Fisch and A. B. Harris, Phys. Rev. Lett. 38, 785 (1977).

[12]A. J. Bray and M. A. Moore, Phys. Rev. Lett. 41, 1068 (1978), and J. Phys. C 12, L441 (1979).

[13]I. Morgenstern and K. Binder, Phys. Rev. B 22, 288 (1980), and Z. Phys. B 39, 227 (1980).

[14]J. R. Banavar and M. Cieplak, Phys. Rev. Lett. 82, 832 (1982).

[15]A. P. Young, Phys. Rev. Lett. 50, 917 (1983).

[16]K. Binder and K. Schröder, Phys. Rev. B 14, 2142 (1976).

[17]W. Kinzel, Phys. Rev. B 19, 4594 (1979).

[18]K. Binder, Z. Phys. B 48, 319 (1982).

[19]Further evidence that slow field cooling yields equilibrium magnetization is the agreement with magnetizations obtained from transfer-matrix methods; see I. Morgenstern, Phys. Rev. B 25, 6067 (1982).

[20]L. Lundgren, P. Svedlindh, and O. Beckmann, Phys. Rev. B 26, 3990 (1982).

[21]W. Kinzle, Z. Phys. B 46, 59 (1982), and Phys. Rev. B 26, 6303 (1982).

[22]B. Barbara, A. P. Malomzemoff, and Y. Imry, Physica (Utrecht) 108B+C, 1289 (1981).

[23]K. Binder, Z. Phys. B 26, 339 (1977).