$E2$ Transition Densities and Proton Shell Structure in ^{88}Sr, ^{89}Y, and ^{90}Zr

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Electron scattering data have been used to obtain transition charge densities for the low-lying $E2$ transitions in ^{88}Sr, ^{89}Y, and ^{90}Zr. These charge densities show a characteristic shape indicative of their microscopic constituency, modified by core-polarization effects. A good description of transitions in the even-even nuclei is obtained by using the measured single-particle transitions in ^{89}Y as effective densities.

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We have investigated the transition charge densities of the low-energy quadrupole excitations in the neutron closed-shell nuclei ^{88}Sr, ^{89}Y, and ^{90}Zr. The measured transition densities in ^{89}Y for the transitions from the $\frac{1}{2}^-$ ground state to the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ excited states are used to study configuration mixing in the neighboring even nuclei.

The transitions investigated in this experiment are shown in Fig. 1. With the exception of the second 2^+ state in ^{88}Sr, all transitions have been probed already by electron scattering. These measurements[1-3] were taken at momentum transfers less than 0.7 fm^{-1} except for the first 2^+ states in ^{88}Sr and ^{90}Zr, which have been measured[4,5] up to a momentum transfer of 2.5 and 2.2 fm^{-1}, respectively. In order to get reliable transition densities, it is important to map out the electron scattering form factor up to two times the Fermi momentum[6] at forward and backward angles. None of the previous experiments determined the 2^+ transition densities of these nuclei adequately. Thus, for example, the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states in ^{89}Y have been interpreted by Fivozinsky *et al.*[2] as a weak-coupling doublet obtained by the coupling of the $2p_{1/2}$ proton to the collective 2^+ state of the ^{88}Sr core. This model predicts identical shapes for the two transition densities. Our experiment demonstrates that this interpretation is incorrect. Instead, we are in agreement with the results of transfer-reaction experiments[7,8] where substantial spectroscopic factors were found for the $2p_{3/2}{}^{-1}$ and the $1f_{5/2}{}^{-1}$ configurations.

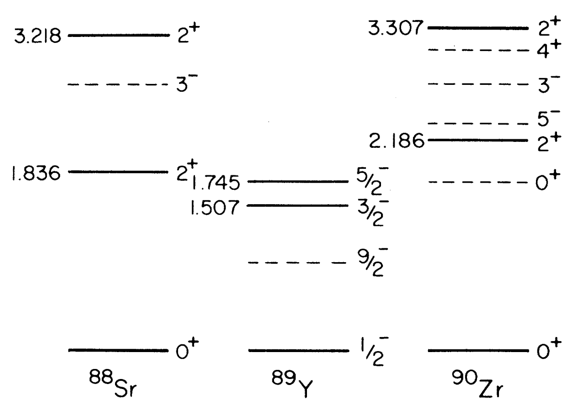

FIG. 1. Low-energy quadrupole excitations in the nuclei under study.

The experiment was performed at the Massachusetts Institute of Technology–Bates electron scattering facility with the high-resolution spectrometer.[9] Forward scattering data were taken at 45° and 90° and backward scattering data at 160° covering a momentum transfer range of $0.4 \leq q \leq 3.0\ \mathrm{fm}^{-1}$ and $0.7 \leq q \leq 2.8\ \mathrm{fm}^{-1}$, respectively. We have also included all existing cross sections[2-5,10] in our data analysis. Such a complete set of electron scattering data allows a separation of the transverse and longitudinal form factors.

The transition charge and current densities for the levels under consideration were extracted with use of the following iterative procedure: The forward scattering data were first fitted by using the Fourier-Bessel expansion analysis (FBA) for the transition charge density $\rho_{\mathrm{tr}}(r)$.[6] Then the backward (160°) data were fitted by keeping $\rho_{\mathrm{tr}}(r)$ fixed and allowing the transition current $J_{L,L+1}(r)$ to vary. In the next step, $J_{L,L+1}(r)$ was fixed allowing $\rho_{\mathrm{tr}}(r)$ to vary so as to further improve the fit to the forward data. The last two steps were repeated until no further improvement in χ^2 could be observed. The high-momentum-transfer uncertainty was treated with an exponential upper limit[6] while the large-radius behavior ($r > 6.4$ fm) was biased towards a shape of the form $re^{-\alpha r}$, where α was fitted to the data.[6]

Our data for the 2.186- and the 3.307-MeV levels of ^{90}Zr including the data of Phan Xuan-Ho *et al.*[5] and the $B(E2)$ values of Metzger[11] were found to be consistent with $J_{L,L+1}(r) = 0$. This is not the case for the 2^+ states of ^{88}Sr at 1.836 and 3.218 MeV. In this case the nonvanishing $J_{L,L+1}(r)$ transition current was assumed to be solely due to $\pi(2p_{1/2}, 1f_{5/2}^{-1})$ and $\pi(2p_{1/2}, 2p_{3/2}^{-1})$ configurations whose amplitudes were allowed to vary.

In ^{89}Y, the corresponding states at 1.507 MeV ($J = \frac{3}{2}^-$) and 1.745 MeV ($J = \frac{5}{2}^-$) can be excited not only via a quadrupole transition, but also through $M1$ and $M3$ transitions, respectively, because of the nonvanishing spin of the ground state. Single-particle predictions show that except near the diffraction minimum, the $M1$ contribution to the scattering cross sections of the $\frac{3}{2}^-$ level is small, in most cases even smaller than the statistical uncertainties. On the other hand, the $M3$ contribution in the $\frac{5}{2}^-$ level is significant and dominates the backward scattering cross sections at high momentum transfer. The shape of the $M1$ and $M3$ form factors was determined by fitting $M1$ and $M3$ excitations at 3.487 and 3.635 MeV in our ^{88}Sr data, which correspond to the same particle-hole transitions. Their strength was determined by fitting the cross sections in the diffraction minima where the $E2$ contributions are small. After correction for these contributions, the $E2$ cross sections for the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states in ^{89}Y were treated in the same way as those for the 2^+ states in ^{88}Sr. The uncertainties from this subtraction procedure are negligible in the $\frac{3}{2}^-$ state. In the $\frac{5}{2}^-$ state, they cause small uncertainties in the forward scattering data, but fairly large ones in the backward scattering data. Therefore, for the $\frac{5}{2}^-$ level, only the transition charge density can be extracted with reasonable precision.

The resulting transition charge densities for all six transitions are shown in Fig. 2. They show quite different structures, indicating that the weak coupling does not describe these states in ^{89}Y at all. Instead, the levels in ^{89}Y should be interpreted as proton-hole transitions to the $2p_{3/2}$

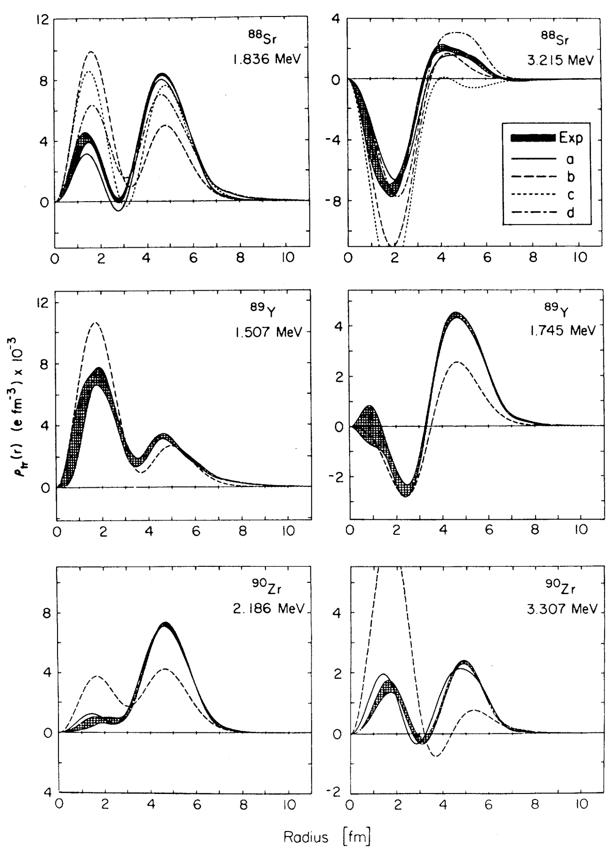

FIG. 2. Transition charge densities for the $E2$ excitations discussed in this paper. The theoretical results are curve a, two-component fit; curve b, BP; curve c, RPA; and curve d, BP with effective charge.

and the $1f_{5/2}$ orbits, respectively. This interpretation agrees qualitatively with the shape of the transition charge densities, where the $\frac{3}{2}^-$ transition charge corresponds (except for some core polarization) to the $(2p)^2$ shape, while the transition charge of the $\frac{5}{2}^-$ level corresponds to the product of the $2p_{1/2}$ and the $1f_{5/2}$ radial distributions.

We have interpreted the absence of the weak-coupling doublet in ^{89}Y as an indication that the lowest 2^+ state in ^{88}Sr is not a collective state. Instead, the two lowest 2^+ states represent mainly two orthogonal mixtures of the two allowed proton configurations: $\pi(2p_{1/2}, 1f_{5/2}^{-1})$ and $\pi(2p_{1/2}, 2p_{3/2}^{-1})$. In Table I, we list spectroscopic amplitudes for these configurations obtained from transfer reactions[12] as well as the random-phase approximation (RPA) and broken-pair (BP) calculations.

For ^{89}Y the BP calculations[13,14] essentially verify our interpretation for these states with the difference that they yield smaller spectroscopic factors. In Fig. 2, we display the BP predictions for all transitions measured. It would seem that these calculations reproduce the essential structure, but are missing the core-polarization aspect.

It has been customary to account for core polarization by rescaling the transition densities with an effective charge. We can see from Fig. 2 that such a procedure could not succeed in our case. Instead, we use the ^{89}Y results as measurements of effective transition densities which include core polarization for the renormalized $1f_{5/2} \rightarrow 2p_{1/2}$ and the $2p_{3/2} \rightarrow 2p_{1/2}$ transitions.

These effective transition densities can be used immediately to predict the BP transition densities for the 2^+ states in ^{88}Sr. Use of the effective transition densities for this case gives a prediction that includes core-polarization effects. These predictions are shown in Fig. 2 for the ^{88}Sr levels. Considering the substantial uncertainties in the calculation of the spectroscopic factors, the agreement is reasonable. This supports the idea of the separation between valence particles and core polarization, where the core polarization is essentially the coupling of the particle-hole transitions within the valence space to core particle-hole transitions. This coupling varies only slightly from nucleus to nucleus since the core particle-hole excitations are not strongly influenced by the changes in the valence configuration. A similar conclusion was obtained in our comparison[15,16] of $E5$ transitions in ^{89}Y and ^{90}Zr.

We can also use the effective densities to fit the two amplitudes in ^{88}Sr. The results are given in Table I. Even though the experimental shape is not exactly reproduced we estimate that the uncertainty in the fitted amplitudes is less than 10%. However, since the fitted amplitudes are based on the theoretical amplitudes in ^{89}Y, it is not clear whether the observed discrepancy is a failure of the theory in ^{89}Y or in ^{88}Sr.

The structure of the lowest 2^+ transitions in ^{90}Zr is quite different. The first 2^+ state arises essentially from the recoupling of a proton pair in the $1g_{9/2}$ orbit. This is confirmed by the surface-peaked nature of the transition (see Fig. 2). On the other hand, the similarity in shape between the transition charge density of the second 2^+ state in ^{90}Zr and that of the first 2^+ state in ^{88}Sr suggests that this 3.307-MeV state is largely a configuration in which the pair in the $1g_{9/2}$ orbit is coupled to 0^+, but the remaining ^{88}Sr core is in its lowest 2^+ state. The strength of this core excitation is reduced with respect to ^{88}Sr, since part of the ground state of ^{90}Zr consists of a $2p_{1/2}^2$ configuration, which blocks transitions to the $2p_{1/2}$ orbit of the ^{88}Sr core. Further, some mixing with the $1g_{9/2}^2$ configuration of the first 2^+ state will occur. This can be seen in the transition density of the 2^+ state and leads to a reduction of the transition density at the nuclear surface without affecting the nuclear interior. Thus we see that the pairing aspect is essential to an understanding of the character of the low-lying states in ^{90}Zr.

TABLE I. Spectroscopic amplitudes in ^{88}Sr.

	2^+(1.836 MeV)		2^+(3.218 MeV)	
	$2p_{1/2}, 2p_{3/2}^{-1}$	$2p_{1/2}, 1f_{5/2}^{-1}$	$2p_{1/2}, 2p_{3/2}^{-1}$	$2p_{1/2}, 1f_{5/2}^{-1}$
RPA	0.85	0.85	0.88	−0.63
BP	0.87	0.54	0.65	−0.81
Fit	0.62	0.87	0.65	0.58

We fitted both densities in ^{90}Zr as linear combinations of a $1g_{9/2}{}^2$ configuration and the ^{88}Sr $2_1{}^+$ transition density. Summing the squares of the amplitudes for the ^{88}Sr $2_1{}^+$ configuration gives a total strength of 0.30. This must be interpreted as the probability of finding the $2p_{1/2}$ orbit empty in the ground state of ^{90}Zr—a result that is in good agreement with transfer reactions.[7]

In conclusion, we can say that by using the measured transition densities for the single-particle transitions in ^{89}Y as effective densities that include the effect of core polarization, we have a sensitive and powerful probe for the nuclear structure of the low-lying 2^+ states of ^{88}Sr and ^{90}Zr. The states in ^{88}Sr turn out to be orthogonal combinations of the $\pi(2p_{1/2}, 2p_{3/2}{}^{-1})$ and $\pi(2p_{1/2}, 1f_{5/2}{}^{-1})$ configurations, which result in very different transition densities for these two states, while the second 2^+ state in ^{90}Zr can be rather well described as an excitation of the ^{88}Sr core, the two last protons acting as (partly blocking) spectators.

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