of the eliminated areas is taken to be a resistor with resistance R_w; similarly, what used to be a K bond is now taken to be a resistor R. The resistor-moving renormalization-group scheme yields the following equations:

$$R' = \frac{bl}{b(c-l-1)/R + 2b/R_w} + \frac{b(c-1)}{bc/R}, \quad R_w' = \frac{bl}{b(c-l-1)/2R + (b+1)/R_w - 1/2R} + \frac{b(c-l)}{(bc-1)/2R + 1/R_w}. \tag{5}$$

Equations (5) may be written as a single recursion relation in the variable $\alpha = R_w/R$. When b, $c \to \infty$, one has $\alpha \to 2$. Substituting in Eqs. (5), we find that

$$R' = [l/(c-l) + (c-l)/c]R = (bc)^{\bar{\zeta}}R, \tag{6}$$

where $\bar{\zeta} = 2 - D$. This result applies to all the cases considered in Eqs. (2). Again, this agrees with the result one expects for the abstract analytically continued, translationally invariant lattices.

Our several examples suggest that, within the Migdal approximation, our low-lacunarity fractals and the abstract "hypercubic" lattices have the same physical properties, for general noninteger D. It is clear, however, that the general statement near the beginning of this article requires further tests: One should compare general-D low- and high-temperature expansions, other renormalization-group schemes, exact calculations, etc. We hope that this paper will stimulate such further studies.

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Can Nuclear Interactions Be Long Ranged?

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A renormalizable relativistic quantum field theory of nuclear interactions is shown to possess not only Yukawa-type solutions, but also a topologically nontrivial one. It corresponds to a hadronic monopole, called a *hadroid*. Experimental evidence suggesting the existence of such a nuclear state is considered.

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The possibility that hadronic forces can be long ranged in the ground state was considered by Lee and Yang a number of years ago.[1] Their argument was based on the assumption that hadronic interactions are invariant under local non-Abelian gauge transformations. At that time it was believed that such invariance necessitated the existence of massless vector bosons, leading to a formal equivalence of the non-Abelian theory with electromagnetic gauge transformation. We now know that this is not the case. Thus their argument, as it stands, is not valid. It is known, though, that long-ranged interactions are possible in theories with massive vector fields. The vortex solution of Nielsen and Olesen to a U(1) gauge field theory and the 't Hooft–Polyakov monopole solution to a SU(2) gauge field theory are two such examples.[2]

Direct experimental evidence firmly indicates that in hadron-hadron collisions and in the normal nuclear ground state, or its low-energy excitations, nuclear forces are short ranged. Thus there is no other massless vector boson besides the photon. Such compelling evidence on the nature of the nuclear force in a highly excited state does not exist. In such a state nuclear forces could be quite different in their effective nature. That the effective force in the medium can be quite different from that in the vacuum is nothing new. A simple explanation of the Meissner effect is the assumption that the photon acquires an effective mass due to the induced screening current. A similar mechanism is in operation in the Nielsen-Olesen vortex solution. In this case the induced current cancels the mass. Thus it is not entirely novel to suggest the possibility that in an excited nuclear state hadronic forces can become long ranged as a result of the total screening to the vector field mass. The purpose of this work is to show how this can happen for nuclear interactions in a model that is both relevant for nuclear physics and theoretically consistent.

A number of relativistic quantum field models of nuclear interactions have been constructed that can account for a wealth of known nuclear structure data in terms of a few parameters.[3] These models predict the magnitude of the spin-orbit interaction in nuclei, single-particle energy levels, the shape and energy dependence of the optical potential, and the distribution of mass and charge in closed-shell nuclei. These results follow from the bold assumption, originally proposed by Duerr, Johnson, and Teller, that a nucleus is a relativistic system in which mesonic degrees of freedom play an important role.[4] Indeed, it is a minor miracle that a hadronic field theory is at all applicable to nuclear structure, let alone able to predict and correlate a large amount of diverse data. The existence of such models invites one to ask some deeper questions about the relativistic quantum field-theory approach to the study of nuclear structure and nuclear dynamics. Furthermore, phenomenological success can also serve as a springboard for theoretical speculation about new, undiscovered nuclear structures. These speculations would then rest on the hard rock of facts.

A desirable property that any nuclear field model should have is that it display known nuclear-interaction symmetries in a consistent way. This consistency can be expressed in the requirement that the theory be renormalizable. Only in this way is a systematic investigation of the neglected higher-order effects possible and can the physical validity of the model be assessed. The problem of renormalizability arises when isospin interactions that are mediated by the rho field are considered. Such interactions are necessary to understand the structure of ^{208}Pb nucleus. The only known way of constructing renormalizable non-Abelian field theories is through a local gauge principle.[5] Such an idea for hadronic interactions was advocated by Sakurai a long time ago.[6] The phenomenological consequences for nuclear matter were considered by Serot.[7] In this work I show that a SU(2) gauge field theory of nuclear interactions not only possesses the usual Yukawa-type solutions, but also a topologically nontrivial solution. It corresponds to a hadronic monopole. I call such a state a *hadroid*. That such a state should exist follows from the fact that the fourth homotopy group of SU(2) is nontrivial. This solution should be distinguished from the 't Hooft–Polyakov monopole, whose properties are determined by the second homotopy group of SU(2).

Isotopic-spin interactions in nuclei are mediated by the triplet of rho fields ρ^+,ρ^-,ρ^0 and the pion field. For present purposes the pion field can be consistently set equal to zero. A local SU(2) gauge field theory can be easily constructed. Since all components of the rho field have the same mass, the Higgs field must belong to the fundamental representation of SU(2). For the field tensor $\hat{F}_{\mu\nu}$, take

$$\hat{F}_{\mu\nu}=\frac{\partial}{\partial x_\mu}\hat{\rho}_\nu-\frac{\partial}{\partial x_\nu}\hat{\rho}_\mu+g\hat{\rho}_\mu\times\hat{\rho}_\nu. \quad (1)$$

The rho field develops a mass by the Higgs mechanism.[8] Take the Higgs doublet H to be

$$H=\begin{bmatrix} h^+ \\ h^0 \end{bmatrix}. \quad (2)$$

In the vacuum state we assume that H develops a nonvanishing constant expectation value given by

$$\langle H\rangle=\begin{bmatrix} 0 \\ V_0 \end{bmatrix}. \quad (3)$$

For this to happen the Higgs potential must be

$$V=(\lambda/4)(H^\dagger H-V_0^2)^2. \tag{4}$$

The Lagrangian for the rho field is then given by

$$\mathcal{L}=-\tfrac{1}{4}\hat{F}_{\mu\nu}\cdot\hat{F}_{\mu\nu}-\tfrac{1}{2}|(\partial/\partial x_\mu-\tfrac{1}{2}ig\hat{\tau}\cdot\hat{\rho}_\mu)H|^2-V. \tag{5}$$

By expanding the squared term in Eq. (5) we get the following:

$$\mathcal{L}=-\frac{1}{4}\hat{F}_{\mu\nu}\cdot\hat{F}_{\mu\nu}-\frac{1}{2}\left(\frac{\partial H}{\partial x_\mu}\right)^2+\frac{ig}{4}\left\{\left(\frac{\partial H}{\partial x_\mu}\right)^\dagger\hat{\tau}H-H^\dagger\hat{\tau}\frac{\partial H}{\partial x_\mu}\right\}\cdot\hat{\rho}_\mu-\frac{g^2}{8}H^\dagger H\hat{\rho}_\mu\cdot\hat{\rho}_\mu-V. \tag{6}$$

The rho field couples to a conserved isospin current given by

$$\hat{J}_\mu=\frac{ig}{4}\left\{\left(\frac{\partial H}{\partial x_\mu}\right)^\dagger\hat{\tau}H-H^\dagger\hat{\tau}\frac{\partial H}{\partial x_\mu}\right\}-\frac{g^2}{8}H^\dagger H\hat{\rho}_\mu, \tag{7a}$$

where

$$\partial\hat{J}/\partial x_\mu=0. \tag{7b}$$

From Eq. (6) it follows that the mass of all the three rho fields is the same and given by

$$m_\rho=(g/2)V_0. \tag{8}$$

In the vacuum state all isotopic-spin interactions that are mediated by the rho meson are short ranged. The range is given by the usual relation $R\sim m_\rho^{-1}$.

The task here is to construct a new solution that does not correspond to the vacuum state, or small-energy excitations of it. For this purpose take

$$H=\hat{\tau}\cdot\vec{\mathbf{r}}\begin{pmatrix}0\\u(r)\end{pmatrix}, \tag{9a}$$

where

$$\hat{\tau}\cdot\vec{\mathbf{r}}=x\tau_x+y\tau_y+z\tau_z. \tag{9b}$$

Here the tau matrices are the usual Pauli 2×2 matrices acting on H. The function $u(r)$ must satisfy $u\to V_0/r$, $r\to\infty$. This is to keep the energy finite. For the rho field take the usual Wu–Yang–'t Hooft–Polyakov *Ansatz*,

$$\rho_c{}^a=\epsilon_{abc}x^bW(r). \tag{10}$$

Then if $W(r)$ and $u(r)$ are defined by

$$W(r)=(K-2)/gr^2, \tag{11a}$$

$$u(r)=h(r)/gr^2, \tag{11b}$$

the field equations for $K(r)$ and $h(r)$ are given by

$$r^2K''=K(K-1)(K-2)+\tfrac{1}{4}h^2K, \tag{12a}$$

$$r^2h''=\tfrac{1}{2}hK^2+(\lambda/g^2)(h^2-g^2V_0^2r^2)h. \tag{12b}$$

In deriving these equations the following relations are used:

$$\left\{\left(\frac{\partial H}{\partial x_c}\right)^\dagger\hat{\tau}H-H^\dagger\hat{\tau}\frac{\partial H}{\partial x_c}\right\}^a=2i\epsilon_{abc}x^bu^2, \tag{13a}$$

$$\hat{\tau}\cdot\hat{\rho}_\mu\,\partial H/\partial x_\mu=-2iW(r)H. \tag{13b}$$

The mechanism that is in action here can be seen from Eqs. (10) and (13a). The induced isospin current of Eq. (13a) is proportional to the rho field itself. A total screening of the rho mass is possible. This is achieved by giving the function $W(r)$ the asymptotic behavior $W(r)\to-2/gr^2$. This is to be contrasted with the 't Hooft–Polyakov solution, where $W(r)\to-1/gr^2$ asymptotically. The difference stems from the fact that in the present case the Higgs field is in the doublet representation. This is necessary here, since all three components of the rho field must be massive in vacuum.

The functions $K(r)$ and $h(r)$ must satisfy boundary conditions given by $K(r)\to2+br^2$, $h\to cr^2$, $r\to0$; $K\to0$, $h\to gV_0r$, $r\to\infty$. The coefficients b and c are to be determined by fulfilling the asymptotic condition for $K(r)$ and $h(r)$. The other parameters are the coupling constant g and the strength of the Higgs potential λ. The value of V_0 is fixed by the mass of the rho meson, which is taken to be 780 MeV. The coupling constant g is just the rho-pi coupling $g_{\rho\pi\pi}$ and is taken to be $g^2/4\pi=2.0$. The Higgs strength λ must be determined phenomenologically by fitting nuclear structure data. A convenient choice of λ for use here is the Prasad-Sommerfield limit, $\lambda=0$. The 't Hooft–Polyakov monopole has an exact analytic solution in this case. I did not succeed in finding one, and so the equations are solved numerically. A search is made for the values of b and c so that at a distance of 4.5 fm from the origin the asymptotic conditions are satisfied to at least 0.1%. Solutions are numerically stable when higher precision is sought. In Fig. 1 the functions $K(r)$ and $h(r)$ are shown.

In the absence of nucleons, the range of nuclear

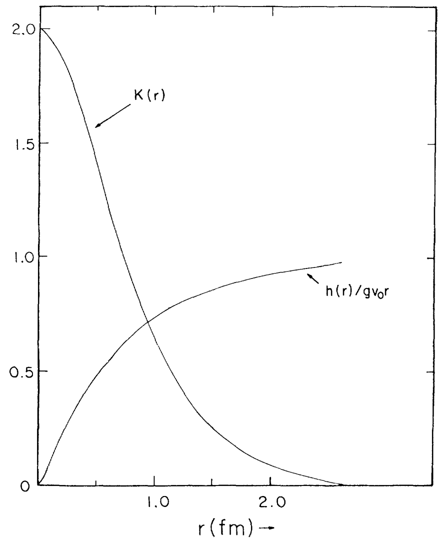

FIG. 1. The functions $K(r)$ and $h(r)$ as a function of radius in fermis.

interactions is determined by the inverse of the exchanged quanta. For the rho meson this corresponds to 0.25 fm. From Fig. 1 we see that the rho-meson mass does not set the scale in the hadroid phase. The extension of the hadroid is enormous compared with the Compton wavelength of the rho. The empty glueball can also be populated by nucleons. In this case the Dirac equation must be considered and the problem solved self-consistently, just as in the case of normal nuclear structure. Some qualitative remarks can be made. If the charge and mass number of the hadroid are small, the soliton solution we obtained will not be greatly modified. The geometric properties of such a state should be independent of the charge and mass number. Its reaction cross section must be enormous. This is not because the nucleons stick out so far, but because the rho field has undergone a metamorphosis. The nucleons in a hadroid are almost incidental appendages to the mesonic solution.

The incidental nature of nucleons in the hadroid does not mean that there cannot be a rich particle structure in the hadroid. The mesonic potential can be viewed as a shell potential in which various nuclear excitations can be supported. In exact analogy with normal nuclear structure predicted by the relativistic quantum field theory, I expect to find single-particle excitations. Various transitions between excited states of the hadroid can occur. The alignment of nuclear spin can also lead to rotational states. These are properties of the hadroid that can be searched for experimentally. All of them are calculable in the present model.

The basis for the above considerations is a relativistic quantum field theory of nuclear interactions. That such an approach is relevant and indeed successful in describing known nuclear structure has been investigated and reported.[3] Thus where are the hadroids? To answer this question I summarize the pertinent experimental facts about the anomalously short mean free path effect in projectile fragments resulting from heavy-ion collisions.[9] These states have been called anomalons. First, the anomalons are copiously produced. This means that a hadronic interaction is involved in their production. Second, the anomalous state persists for at least 10^{-10} sec. This means that the hadronic interaction, although producing the anomalous state, is inhibited from destroying it. Third, the reaction cross section of the anomalous state is huge. If a 6% concentration of anomalons in the projectile fragments is assumed, then the best fit to the data is obtained by assuming that the reaction cross section of the anomalous state is 10 times as large as that for a normal nucleus. Anomalous ^{4}He would "look" as big as ^{208}Pb. Fourth, no charge decay modes of the anomalon have been observed. Evidence suggests that in the collisions experienced by the anomalous state, the anomalous behavior is retained by the heavier fragment. Any of these properties is very difficult to understand in terms of conventional nuclear theory.

The promotion of isospin invariance of nuclear interactions to a local gauge symmetry, as required by the renormalizability of the model, directly leads to a topologically nontrivial nuclear excitation. It is a soliton state possessing unusual properties. I have shown that in this state, the rho mass does not set the length scale. The mass is screened by an induced isospin current. The hadroid will have an enormous reaction cross section, once it is produced. Its copious production is possible by an associated production mechanism. The topological quantum number of the hadroid will inhibit the strong interactions from destroying the state. Since the meson field is the essence of the hadroid, its destruction must involve the disintegration of the meson field. The tunneling of the hadroid into a normal nuclear state must also be severely inhibited by the fact that the mesonic field configurations in the hadroid and in a normal nucleus are radically different. This difference is not local, but global. A detailed analysis of the lifetime is still to be made. For this, the electromagnetic and weak interactions must be included into the theory in a consistent way. This will perhaps involve the extension of the gauge group.

From the above discussion we see that the hadroid is an economical way of understanding the anomalously short mean free path effect of projectile fragments. It is thus tempting to identify this effect as evidence for the hadroid.

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