Impulse-Approximation Dirac Optical Potential

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The impulse approximation to the Dirac optical potential for proton-nucleus elastic scattering is deduced from elementary considerations and found to be in remarkably good agreement with phenomenological parameters. The large difference between the real parts of scalar and vector optical potentials is explained by the impulse approximation.

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The phenomenological Dirac optical potential has been used extensively to fit proton-nucleus elastic scattering data.[1] The potentials so determined at low energy have recently been shown to be consistent with interactions used in relativistic models of infinite nuclear matter.[2-4] Shakin and collaborators[2] have shown that a Brueckner Hartree-Fock approach to infinite nuclear matter, starting from an NN potential based on NN phase shifts, yields a Dirac optical potential in agreement with the phenomenology at low energy. Other nuclear-matter approaches[3,4] based more generally on the idea of dominant scalar (attractive) and vector (repulsive) meson-exchange forces yield similar potential strengths. However, for energies greater than 300 MeV, the situation stands in contrast to standard multiple-scattering approaches to the optical potential[5,6] employing the Schrödinger equation which provide a direct and simple form for the optical potential based on the impulse approximation. The ingredients are free NN amplitudes and the nuclear density which are far from arbitrary, the former being determined by NN scattering experiments and the latter being rather tightly constrained by electron scattering data and Hartree-Fock mean-field theory.[7]

Multiple-scattering theory also provides a very clear basis for the physical origin of the imaginary (or absorptive) part of the p-nucleus optical potential.[8,9] In the impulse approximation the p-nucleus reaction cross section is due to quasifree NN scatterings and this is, in fact, known to be in accord with experiment at intermediate to high energy.

In the present paper, we employ the simplest notions of multiple-scattering theory to deduce the impulse-approximation Dirac optical potential. The key ingredients are the invariant NN amplitude in the space of Dirac spinors[10] and the nuclear density.

The elastic scattering of a proton by a spin-saturated nucleus is described by the fixed-energy Dirac wave equation

$$[\not{p} - m - \hat{U}_{00}(E)]\psi_{\vec{k},s}{}^{(+)} = 0, \quad (1)$$

where $p^0 = E$ is constant and $\hat{U}_{00}(E)$ is the elastic-scattering optical potential. The asymptotic momentum $\vec{k}$ is related to the energy by $E = (k^2 + m^2)^{1/2}$ (we employ units where $\hbar = c = 1$). The solution can be written as an integral equation:

$$\psi_{\vec{k},s}{}^{(+)}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u_s(\vec{k}) + \int d^3r' \langle\vec{r}|(\not{p}+m)/(p^2 - m^2 + i\eta)|\vec{r}'\rangle\langle\vec{r}'|\hat{U}_{00}|\psi_{\vec{k},s}{}^{(+)}\rangle. \quad (2)$$

Considering the asymptotic limit as $r \to \infty$, one finds

$$\psi_{\vec{k},s}{}^{+}(\vec{r}) \xrightarrow[r\to\infty]{} e^{i\vec{k}\cdot\vec{r}}u_s(\vec{k}) + (e^{ikr}/r)\{-[(E\gamma^0 - k\vec{\gamma}\cdot\hat{r} + m)/4\pi]\int d^3r' e^{-i\vec{k}'\cdot\vec{r}'}\langle\vec{r}'|\hat{U}_{00}|\psi_{\vec{k},s}{}^{(+)}\rangle\}, \quad (3)$$

where the term in curly brackets defines the scattering amplitude, and $\vec{k}' \equiv k\hat{r}$ is the outgoing wave momentum. Employing the identity[11]

$$E\gamma^0 - \vec{\gamma}\cdot\vec{k}' + m = 2m\textstyle\sum_{s'} u_{s'}(\vec{k}')\bar{u}_{s'}(\vec{k}'), \quad (4)$$

we can extract the scattering amplitude from Eq. (3) in the form

$$F_{s's}(\vec{k}', \vec{k}; E) = -(m/2\pi)\bar{u}_{s'}(\vec{k}')\int d^3r' e^{-i\vec{k}'\cdot\vec{r}'}\langle\vec{r}'|\hat{U}_{00}|\psi_{\vec{k},s}^{(+)}\rangle. \tag{5}$$

Introducing a Dirac T matrix appropriate to elastic p-nucleus scattering by the definition

$$\hat{U}_{00}|\psi_{\vec{k},s}^{(+)}\rangle = \hat{T}_{00}|\vec{k}\rangle u_s(\vec{k}), \tag{6}$$

where $|\vec{k}\rangle u_s(\vec{k})$ is the incoming plane-wave state, we have that the Dirac scattering amplitude involves plane-wave matrix elements of $\hat{T}_{00}$ as follows:

$$F_{s',s}(\vec{k}', \vec{k}; E) = -(m/2\pi)\bar{u}_{s'}(\vec{k}')\langle\vec{k}'|\hat{T}_{00}|\vec{k}\rangle u_s(\vec{k}). \tag{7}$$

The central point exploited here is that in any multiple-scattering theory, the T matrix is, in single-scattering approximation, the expectation value of a sum of two-body NN t matrices in the nuclear ground state $|0\rangle$:

$$\langle\vec{k}'|T_{00}^{(1)}|\vec{k}\rangle = \langle 0|[\sum_{i=1}^{A}\langle\vec{k}'|\hat{t}_i|\vec{k}\rangle]|0\rangle. \tag{8}$$

In this equation, $\hat{t}_i$ is the positive-energy Dirac spinor t matrix for the incident proton to scatter from nucleon i of the target nucleus and A is the mass number of the nucleus. In the impulse approximation, the NN t-matrix elements needed in Eq. (8) are equated to the on-shell NN amplitude for momentum transfer $\vec{q}=\vec{k}-\vec{k}'$. With our conventions, this relation is

$$-(m/2\pi)\langle\vec{k}'|\hat{t}_i|\vec{k}\rangle = 2ik\hat{\mathfrak{F}}(q^2, s)\exp(i\vec{q}\cdot\vec{r}_i), \tag{9}$$

where $\hat{\mathfrak{F}}$ is the Lorentz-invariant NN amplitude defined as follows[8,10]:

$$\hat{\mathfrak{F}} = F_S + F_V\gamma_1 + \gamma_2 + F_T\sigma_1^{\mu\nu}\sigma_{2\mu\nu} + F_P\gamma_1^5\gamma_2^5 + F_A\gamma_1^5\gamma_1^{\mu}\gamma_2^5\gamma_{2\mu}. \tag{10}$$

Subscripts 1 and 2 distinguish Dirac operators in the spinor space of the two scattering particles. The five complex amplitudes for scalar (S), vector (V), tensor (T), pseudoscalar (P), and axial vector (A) interactions depend on q^2 and s, the invariant energy parameter. They are determined directly from the NN phase shifts which parametrize the physical NN scattering data.[10]

The following relation for the single-scattering amplitude for proton-nucleus elastic scattering is deduced by combining Eqs. (7)–(9):

$$F_{s's}^{(1)}(\vec{k}', \vec{k}; E) = 2ik\bar{u}_{s'}(k')\textstyle\sum_i\langle 0|\hat{\mathfrak{F}}(q)\exp(i\vec{q}\cdot\vec{r}_i)|0\rangle u_s(\vec{k}). \tag{11}$$

Notice that for a spin-saturated nucleus, this matrix element involves a trace over struck-nucleon spins which eliminates all but the scalar (F_S) and time component of vector ($\gamma_1^0 F_V$) terms from the Dirac scattering amplitude of Eq. (10). Thus, Eq. (11) simplifies to

$$F_{s's}^{(1)}(\vec{k}', \vec{k}; E) = 2ik\bar{u}_{s'}(\vec{k}')[F_S(q)\rho_S(q) + \gamma_1^0 F_V(q)\rho_V(q)]u_s(\vec{k}), \tag{12}$$

where the scalar and vector form factors of the nucleus are defined by

$$\rho_S(q) = \langle 0|\textstyle\sum_i\exp(i\vec{q}\cdot\vec{r}_i)|0\rangle, \quad \rho_V(q) = \langle 0|\textstyle\sum_i\gamma_i^0\exp(i\vec{q}\cdot\vec{r}_i)|0\rangle. \tag{13}$$

Equation (12) defines the Dirac impulse approximation. It consists of a scalar and a vector term which are fully determined by the NN amplitudes and the nuclear density.

Generally the proton-nucleus scattering involves substantial multiple-scattering effects and Eq. (12) does not provide an adequate approximation to the scattering amplitude. Most of the multiple-scattering effect can be simply taken into account by use of the optical potential $\hat{U}_{00}$ which is iterated to all orders to define the elastic T matrix as in Eq. (6). With expansion to leading order, Eq. (6) reveals that the impulse-approximation optical potential (the part which is first order in the NN t matrix) must be equal to the impulse-approximation T matrix of Eq. (8). This procedure neglects nuclear-medium modifications of the NN interaction, off-shell effects, and intrinsic corrections of order A^{-1} to the optical potential. Implicitly we assume the operator form of the NN amplitudes in Dirac representation to obtain

$$\langle\vec{k}'|U_{00}^{(1)}|\vec{k}\rangle \simeq \langle\vec{k}'|T_{00}^{(1)}|\vec{k}\rangle = -(4i\pi k/m)[F_S(q)\rho_S(q) + \gamma_1^0 F_V(q)\rho_V(q)]. \tag{14}$$

The coordinate-space optical potential is found by Fourier transformation. If we approximate the NN amplitudes by forward ($q=0$) values $F_{S0} \equiv F_S(q^2=0)$ and $F_{V0} \equiv F_V(q^2=0)$, the coordinate-space optical potential takes the simple form

$$U_{00}^{(1)}(\vec{r}) = -(4i\pi k/m)[F_{S0}\rho_S(r) + \gamma_1^0 F_{V0}\rho_V(r)], \tag{15}$$

which is the Dirac equivalent of the "$t\rho$" approximation with $\rho_S(r)$ and $\rho_V(r)$ being the scalar and vector densities.

A matrix transformation has been derived which relates the Dirac amplitudes of Eq. (10) to the usual c.m.-frame *NN* amplitudes in Pauli spin representation:

$$(2ik_c)^{-1}f_c = A + B\vec{\sigma}_1\cdot\vec{\sigma}_2 + iqC(\sigma_{1n}+\sigma_{2n}) + D\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q} + E\sigma_{1z}\sigma_{2z}. \tag{16}$$

For forward scattering, this matrix may be inverted to provide an analytical form for the Dirac amplitudes needed in Eq. (15), i.e.,

$$F_{S0} = [2\gamma^2(1+\gamma)]^{-1}\left[\gamma(2\gamma+1)A_0 + (2\gamma^2-1)B_0 + 2m\gamma(2\gamma^2-1)\left(\frac{\gamma+1}{\gamma-1}\right)^{1/2} C_0 - \frac{2\gamma^2-1}{2(\gamma-1)}E_0\right], \tag{17}$$

$$F_{V0} = [2\gamma^2(1+\gamma)]^{-1}\left[\gamma A_0 - B_0 - 2m\gamma\left(\frac{\gamma+1}{\gamma-1}\right)^{1/2} C_0 + \frac{1}{2(\gamma-1)}E_0\right], \tag{18}$$

where $\gamma = s^{1/2}/2m$ is the ratio of *NN* center-of-mass energy to nucleon mass and $A_0 = A(q^2=0)$, etc. Furthermore, the forward amplitudes A_0, B_0, and E_0 can be expressed directly in terms of observables:

$$A_0 = \frac{\sigma(1-i\rho)}{8\pi}, \quad B_0 = \frac{-\Delta\sigma_T(1-i\rho_T)}{16\pi}, \quad E_0 + B_0 = \frac{-\Delta\sigma_L(1-i\rho_L)}{16\pi}, \tag{19}$$

where σ is the total *NN* cross section and $\Delta\sigma_T$ and $\Delta\sigma_L$ are cross-section differences in pure transverse and longitudinal spin states, respectively. The forward spin-flip amplitude C_0 is fixed by *NN* polarization observables and it plays a dominant role in causing F_{S0} and F_{V0} to be large in magnitude and opposite in sign. Equations (15) and (17)–(19) provide direct links between *NN* observables and the Dirac optical potential. The presence of double-spin-flip amplitudes B_0 and E_0 in the Dirac optical potential is interesting as these terms are not present in standard treatments; however, the effects are not large.

Figure 1 shows the scalar ($\overline{S}$) and vector ($\overline{V}$) terms in the optical potentials of Eq. (15) based on central nuclear density $\rho_S = \rho_V = 0.16$ fm^{-3}. The *NN* amplitudes used are an isospin average of *pp* and *pn* amplitudes calculated from a recent phase-shift solution.[12] There is qualitative agreement between the impulse approximation and the phenomenological potentials—in particular the very large scalar-vector difference is explained. The ratio $-\mathrm{Re}\overline{V}/\mathrm{Re}\overline{S}$ shown in the figure is within 5% of the empirical values. From Eqs. (17) and (18), the spin-flip term (C_0) dominates the

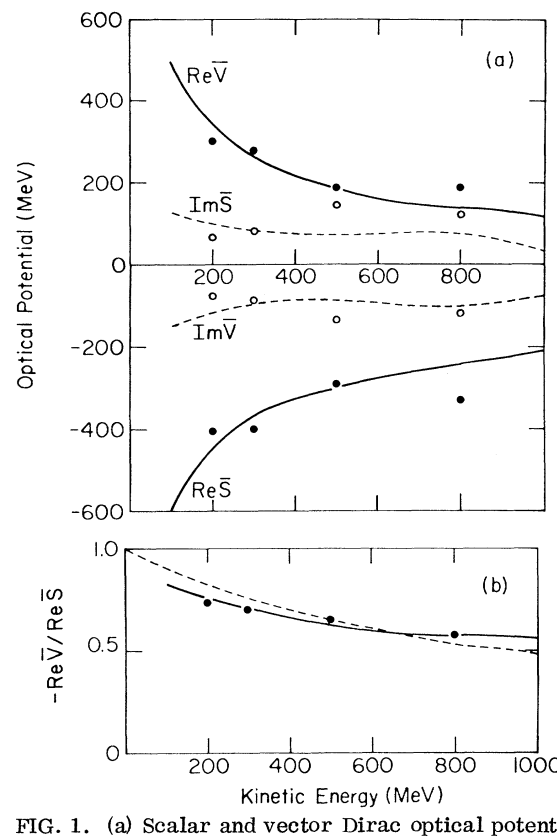

FIG. 1. (a) Scalar and vector Dirac optical potentials, $\overline{S}$ and $\overline{V}$, respectively, for $\rho_S = \rho_V = 0.16/\mathrm{fm}^3$ nuclear density vs proton laboratory kinetic energy. The solid lines show the real parts and the dashed lines show the imaginary parts based on impulse approximation. The filled and unfilled circles show corresponding results found by phenomenological analysis of proton-nucleus scattering data. (b) Ratio of real vector to real scalar strengths based on impulse approximation (solid line). Points are based on phenomenological fits to data. Dashed line shows the ratio m/E_{lab}.

scalar-vector difference and it explains the approximate energy dependence m/E_{lab} seen in Fig. 1(b). The imaginary parts of the optical potentials are in accord with "$t\rho$" and thus the basic reaction is quasifree *NN* scattering. Empirical strength parameters[13] shown in the figure are uncertain by about 10% to 15% (eight additional parameters were varied).[13] Indeed the impulse approximation based on Eq. (14) has been found to give an excellent fit to 500-MeV p-^{40}Ca data (cross section, analyzing power, and "Q") *without any adjustable parameters.*[14] Thus the agreement of Dirac impulse approximation with data goes well beyond the qualitative agreement seen in the figure. The impulse-approximation results are evidently too large at low energy (~100 MeV) and in this region nuclear-medium effects are expected to be important.[2]

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