Constraint on the Photino Mass from Cosmology

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A lower bound for the photino mass $m_{\tilde\gamma}$ as a function of the spin-0 fermion superpartner mass $m_{\tilde f}$ is derived as an extension of the calculation of Lee and Weinberg. The Majorana nature of the photino induces a p-wave threshold for annihilation $\tilde\gamma\tilde\gamma \to f\bar f$ into light fermions, and leads to a rather unexpected form for the bound: for 25 GeV $\lesssim m_{\tilde f} \lesssim$ 45 GeV, $(m_{\tilde\gamma})_{\rm min} \simeq m_\tau = 1.8$ GeV; for $m_{\tilde f} > 45$ GeV, $(m_{\tilde\gamma})_{\rm min}$ increases approximately ***linearly*** with $m_{\tilde f}$ to a value of 20 GeV when $m_{\tilde f} = 100$ GeV.

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The supersymmetric partner of the photon, the photino ($\tilde\gamma$), is very possibly the lightest supersymmetric particle. As such, it is stable, and the value of its mass, $m_{\tilde\gamma}$, assumes phenomenological importance. Since the photino is massless in a supersymmetric world, one may expect that $m_{\tilde\gamma}$ is related to the scale of supersymmetry breaking. For example, in some recently proposed locally supersymmetric grand unified models,[1] the photino is massless at tree level, and $m_{\tilde\gamma}$ is radiatively induced[2]:

$$m_{\tilde\gamma} \approx C(\alpha/2\pi) m_{\rm gravitino} \approx \tfrac{1}{100}\, m_{\rm gravitino} \qquad (1)$$

for $C \approx 10$, where C is an appropriate Casimir number. In another class of models, $m_{\tilde\gamma}$ may enter as a parameter in a nonperturbative modification of the gauge kinetic energy, in such a way that both $m_{\rm gravitino}$ and $m_{\tilde\gamma}$ are nonzero in the limit $m_{\rm Pl} \to \infty$.[3,4] One then obtains relations of the sort[3]

$$m_{\tilde\gamma}^2 \propto \sum m_{\rm boson}^2 - \sum m_{\rm fermion}^2 - N m_{\rm gravitino}^2 \qquad (2)$$

(N is a group factor), implying a possibly heavier photino mass. Hence, there is not yet a clear theoretical constraint on $m_{\tilde\gamma}$.

As noted by Weinberg,[5] the photino mass should be bounded from below by a cosmological consideration applied previously[6] to neutral lepton masses: namely, the contribution to the present mass density of such particles (those which survive annihilation) is no greater than that needed to close the universe.

It is the purpose of this note to quantify this cosmological result, taking into account the specific nature of photino couplings. The result of interest is displayed in Fig. 1, which shows the dependence of $(m_{\tilde\gamma})_{\rm min}$ on another phenomenologically significant parameter, the (almost) common mass $m_{\tilde f}$ of the spin-0 supersymmetric partners of the light fermions. In many models, $m_{\tilde f}$ and $m_{\tilde\gamma}$ are not independent, and the explication of this relation between them is of interest. The peculiar behavior of the bound can be traced to the presence of a p-wave barrier in the incident channel for the annihilation of cool photinos into the light fermions, while the s-wave annihilation into the heavy fermions (notably τ and c) will lead to sufficient depletion of $\tilde\gamma$'s only for $m_{\tilde f} < 45$ GeV. I will proceed to the derivation of the result, reserving further comment until later.

The low-energy effective interaction for the annihilations $\tilde\gamma\tilde\gamma \to f\bar f$ (f is a light fermion) is[7] (with λ the Majorana photino field)

$$\mathcal{L}_{\rm eff} = \sum_f (e^2 Q_f^{\,2}/2m_{\tilde f}^{\,2})(\bar\lambda\gamma_\mu\gamma_5\lambda)(\bar f\gamma^\mu\gamma_5 f), \qquad (3)$$

where $m_{\tilde f}$ is the (assumed) common mass of the scalar and pseudoscalar spin-0 $\tilde f$ particles and Q_f is the charge of f. The spread in masses among all spin-0 quarks and leptons is severely

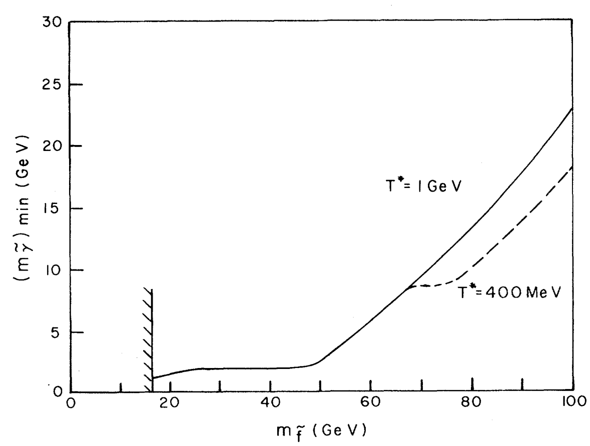

FIG. 1. The behavior of the lower bound on photino mass as a function of $m_{\tilde f}$ (spin-0 fermion mass). The region to the left of the crosshatching is disallowed by experiment. The label T^* on the curves denotes the temperature at which the entropy of the quark-gluon plasma attains its massless ideal-gas form.

constrained by $\Delta Q=0$ flavor-changing phenomenology,[8] and, in addition, $|m_{\tilde{q}u}-m_{\tilde{l}}|/m_{\tilde{q}u}$ is predicted to be small in supergravity grand unification theories.[1] Hence it is sufficiently accurate to take a common mass $m_{\tilde{f}}$ *independent of* f in Eq. (3).

The annihilation rate at low $\tilde{\gamma}\tilde{\gamma}$ momenta is calculated to be (for Majorana $\tilde{\gamma}$'s)

$$(|\vec{v}_{\rm rel}|\sigma)_{\tilde{\gamma}\tilde{\gamma}}=(e^4/4\pi)m_{\tilde{f}}^{-4}\sum_f Q_f^{\,4}[\tfrac{4}{3}(m_{\tilde{\gamma}}^{\,2}-m_f^{\,2})|\tfrac{1}{2}\vec{v}_{\rm rel}|^2+m_f^{\,2}], \tag{4}$$

where $|\vec{v}_{\rm rel}|$ is the relative velocity of the photinos. The momentum dependence in Eq. (4) can be understood as follows: For massless fermions, only the $J=1$ part of the axial coupling [in Eq. (3)] contributes. The Fermi statistics of the photinos requires that at low energies they be in a spin-0 s wave or spin-1 p wave. For $J=1$, only the latter is possible, giving rise to a p-wave barrier at $\tilde{\gamma}\tilde{\gamma}$ threshold. The factor of $\frac{4}{3}$ comes from averaging over a p-wave $(1+\cos^2\theta)$ factor. The term in $m_f^{\,2}$ is the remnant of the $\tilde{\gamma}\tilde{\gamma}$ s-wave annihilation.

I now replace $v_{\rm rel}^{\,2}$ in Eq. (4) by its thermal average $6k_BT/m_{\tilde{\gamma}}$ $[\frac{1}{2}(\frac{1}{2}m_{\tilde{\gamma}})v_{\rm rel}^{\,2}=\frac{3}{2}k_BT]$. At the same time, e^4 is rewritten as $2(4\sin^2\theta_W m_W^{\,2})G_F^{\,2}$ in order to bring $v\sigma$ into the standard weak interaction form used in Ref. 6:

$$(v\sigma)_{\tilde{\gamma}\tilde{\gamma}}=(G_F^{\,2}/2\pi)(4\sin^2\theta_W)^2(m_W/m_{\tilde{f}})^4\sum_f Q_f^{\,4}[2(m_{\tilde{\gamma}}^{\,2}-m_f^{\,2})x+m_f^{\,2}], \tag{5}$$

where $x=k_BT/m_{\tilde{\gamma}}$.

Because of the differing x and $m_{\tilde{\gamma}}$ dependence of the cross section from that used in Ref. 6, the results of the exact integration of the rate equation are not directly applicable. However, the analytic approximation obtained in that reference can be used (modified to take into account the $\tilde{\gamma}\tilde{\gamma}$ p-wave factor x), with a result for the relic abundance that is approximately 25% high. As an approximation, I will carry out the analytic approximation and downgrade the resulting abundance by a factor of 0.8.

The reduced abundance $f=n/T^3$ at $T=0$ is found by integrating the rate equation

$$df/dx=\gamma f^{\,2} \tag{6}$$

from the freezeout value of x, $x=x_{\rm fr}$, to $x=0$. Here

$$\gamma(x)=\tilde{C}m_{\tilde{\gamma}}\mu^2(x),\quad \tilde{C}=N_F^{-1/2}G_F^{\,2}m_{\rm Pl}(45/32\pi^5)^{1/2}(4\sin^2\theta_W)^2(m_W/m_{\tilde{f}})^4,$$
$$\mu^2(x)=\mu_0^{\,2}+2\mu_1^{\,2}x,\quad \mu_0^{\,2}=\sum_f Q_f^{\,4}m_f^{\,2}\theta(m_{\tilde{\gamma}}-m_f),\quad \mu_1^{\,2}=\sum_f Q_f^{\,4}(m_{\tilde{\gamma}}^{\,2}-m_f^{\,2})\theta(m_{\tilde{\gamma}}-m_f), \tag{7}$$

and N_F is the effective number of degrees of freedom at freezout.[6] The freezeout temperature is found by setting

$$df_0/dx=\tilde{C}m_{\tilde{\gamma}}\mu^2(x)f_0^{\,2} \tag{8}$$

with f_0 given in Ref. 6. The only difference in procedure is that now the coefficient γ in Eq. (6) is x dependent.

Integrating the rate equation (6) using the linear form (7) for γ, I find

$$f(0)=(\tilde{C}m_{\tilde{\gamma}})^{-1}(\mu_0^{\,2}x_{\rm fr}+\mu_1^{\,2}x_{\rm fr}^{\,2})^{-1}. \tag{9}$$

With the aforementioned reduction of this approximation by 20%, the present matter density due to $\tilde{\gamma}$'s is then (with $m_W=81$ GeV)

$$\rho_{\tilde{\gamma}}=0.8\,\tfrac{4}{11}\,T_\gamma^{\,3}m_{\tilde{\gamma}}f(0)\kappa \tag{10a}$$
$$\cong(0.72\times10^{-28}\ {\rm g\ cm^{-3}})\kappa^{1/2}(\mu_0^{\,2}x_{\rm fr}+\mu_1^{\,2}x_{\rm fr}^{\,2})^{-1}[m_{\tilde{f}}/(100\ {\rm GeV})]^4 \tag{10b}$$

at $T_\gamma=2.8$ K. (All masses are in gigaelectronvolts.) The factor κ in (10a) adjusts for the inequality $T_{\tilde{\gamma}}\neq T_\gamma$ due to reheating of the photon gas by annihilation of particles when $T<T_{\rm fr}$.[9] $\kappa=1$ if $m_e\lesssim T_{\rm fr}\lesssim m_{\mu,\pi}$; $\kappa=\frac{43}{69}\cong0.62$[10] if $m_{\mu,\pi}\lesssim T_{\rm fr}\lesssim T^*$, where T^* is the temperature at which the entropy of the quark-gluon plasma is approximately equal to its massless, ideal-gas value. If $T_{\rm fr}\gtrsim T^*$, then $\kappa=\frac{43}{205}=0.21$.[10] The value of T^* will be discussed in the conclusion: Some estimates based on finite-temperature QCD will give $T^*\sim1$ GeV; other heuristic arguments suggest $T^*\sim400$ MeV. The $\kappa^{1/2}$ in Eq. (10b) is the result of a partial cancellation of κ by $N_F^{\,1/2}$ $(\propto\kappa^{-1/2})$ as $m_{\tilde{\gamma}}$ crosses thresholds.

The freezeout condition (8) gives $x_{\rm fr}\simeq\frac{1}{17}$ to $\frac{1}{22}$ over a range of $m_{\tilde{\gamma}}=1.5$ to 25 GeV and $m_{\tilde{f}}=20$

to 100 GeV (see below for details). Thus to a good approximation

$$\mu_0^2 + \mu_1^2 x_{fr} \simeq \sum_f Q_f^4(m_f^2 + x_{fr} m_{\tilde\gamma}^2)\theta(m_{\tilde\gamma} - m_f) . \quad (11)$$

The requirement that $\rho_{\tilde\gamma} \lesssim 10^{-29}$ g cm^{-3} for $T_\gamma = 2.8$ K leads to a condition on the masses:

$$\sum_f Q_f^4(m_f^2 + x_{fr} m_{\tilde\gamma}^2) \gtrsim 7.2 x_{fr}^{-1}(m_{\tilde f}/100)^4 \kappa^{1/2} . \quad (12)$$

It is convenient to parametrize $x_{fr} = (20\xi)^{-1}$, $0.85 \lesssim \xi \lesssim 1.1$. Then Eq. (12) becomes

$$(\sum_f Q_f^4 m_f^2) + \tfrac{1}{20}\xi^{-1} m_{\tilde\gamma}^2 (\sum_f Q_f^4) \gtrsim 144\xi[m_{\tilde f}/(100 \text{ GeV})]^4 \kappa^{1/2} \text{ GeV}^2 . \quad (13)$$

The sum in Eq. (13) is over open channels. As a reasonable approximation, and for clarity of exposition, I take e, μ, u, d, and s fermions as massless, and $m_c \simeq m_\tau = 1.8$ GeV, $m_b \simeq 4.5$ GeV. The assumed degeneracy of c and τ will play a completely negligible role in the results, and the thresholds can be separated if so desired. I then consider the following cases:

(i) $m_{\tilde\gamma} < m_\tau$.—The first term in Eq. (13) is then absent, and we obtain the bound (with $\kappa = 1$)

$$m_{\tilde\gamma} \gtrsim (33 \text{ GeV})\xi[m_{\tilde f}/(100 \text{ GeV})]^2 , \quad (14)$$

which is inconsistent with $m_{\tilde\gamma} < m_\tau$ unless $m_{\tilde f} < 23\xi^{-1/2}$ GeV. Since the PETRA lower bound on $m_{\tilde f}$ is about 17 GeV,[11] we find from (14) that $m_{\tilde\gamma} \gtrsim 1.0\xi$ GeV. In this mass range ξ turns out to be about 0.86 (see below), so that there is a narrow experimentally allowed range (17 GeV $\lesssim m_{\tilde f} < 25$ GeV) in which $(m_{\tilde\gamma})_{\min}$ lies between 0.9 and 1.8 GeV (see Fig. 1).

(ii) $m_{\tilde\gamma} > m_\tau$.—For $m_{\tilde\gamma} < m_b$, Eq. (13) reads

$$\tfrac{43}{27}(1.8)^2 + \tfrac{115}{27}\tfrac{1}{20}\xi^{-1} m_{\tilde\gamma}^2 > 144\xi(m_{\tilde f}/100)^4 \kappa^{1/2} , \quad (15a)$$

and for $m_{\tilde\gamma} > m_b$

$$\tfrac{43}{27}(1.8)^2 + \tfrac{1}{27}(4.5)^2 + \tfrac{116}{27}\tfrac{1}{20}\xi^{-1} m_{\tilde\gamma}^2 > 144\xi(m_{\tilde f}/100)^4 \kappa^{1/2} . \quad (15b)$$

It is easy to see from Eq. (15a) that *if* $m_{\tilde f} \lesssim 44\xi^{-1/4}$ *GeV, the lower bound on* $m_{\tilde\gamma}$ *is simply the spectral one* ($m_{\tilde\gamma} \gtrsim m_\tau$). The physical origin of this result is apparent: If $m_{\tilde f} < (44 \text{ GeV})\xi^{-1/4}$, the s-wave $J^P = 0^-$ $\tilde\gamma\tilde\gamma$ annihilation into $\tau\bar\tau + c\bar c$ is large enough to deplete the photino abundance to give a mass density below ρ_{crit}. The dependence on m_f^2 (rather than $m_{\tilde\gamma}^2$) of this part of $\langle v\sigma\rangle$ removes any restriction on $m_{\tilde\gamma}$ beyond the spectral one. It is also important to note the insensitivity of this result to ξ, which turns out to be $\simeq 0.92$ in this region of $m_{\tilde f}$.

However, if $m_{\tilde f} \gtrsim 44\xi^{-1/4}$ GeV, Eqs. (15a) and (15b) do impose a lower bound on $m_{\tilde\gamma}$ (other than spectral), and this bound grows approximately linearly with $m_{\tilde f}$ until $m_{\tilde f} \simeq 100$ GeV. To obtain the bound on $m_{\tilde\gamma}$ from (15a) and (15b), a value of ξ $[=(20x_{fr})^{-1}]$ as a function of $m_{\tilde f}$ is needed. This is arrived at self-consistently: An initial value of $\xi = 1$ is used in Eq. (15) to generate a set $\{m_{\tilde f}, (m_{\tilde\gamma})_{\min}\}$. These are then fed into Eq. (8) to obtain new values of x_{fr} (or ξ) as a function of $m_{\tilde f}$ and $m_{\tilde\gamma}$. The procedure converges very rapidly, and the result is that x_{fr} may be parametrized roughly as

$$x_{fr}^{-1} = 16 + 5.3 m_{\tilde f}/(100 \text{ GeV}) ,$$

or

$$\xi = (20x_{fr})^{-1} = 0.80 + 0.27 m_{\tilde f}/(100 \text{ GeV}) . \quad (16)$$

The resulting lower bound $(m_{\tilde\gamma})_{\min}$ is plotted against $m_{\tilde f}$ in Fig. 1. The behavior outlined in the text is clearly seen. The threshold factor $(1 - m_f^2/m_{\tilde\gamma}^2)^{1/2}$, and the inequality $m_c \neq m_\tau$, both ignored in obtaining Fig. 1, introduce negligible changes in the result. The effect of changing κ from 1 to 0.62 when $x_{fr} m_{\tilde\gamma} > m_\pi$ has been included in both curves. The solid curve assumes that the quark-gluon entropy does not attain its ideal massless value until $T^* = 1$ GeV, whereas the dashed curve assumes that this occurs for $T^* = 400$ MeV.

Several comments are in order:

(1) Clearly the most favorable situation for models in which the photino derives its mass from radiative corrections is that $m_{\tilde f} \lesssim 45$ GeV, in which case $m_{\tilde\gamma}$ can theoretically be as small as ~2 GeV. This situation is also (coincidentally) the most propitious for the detection of double photino production at PETRA.[12]

(2) If $m_{\tilde f} \sim 70$ GeV or larger, then the minimum photino mass is large ($\gtrsim 10$ GeV), and the radiative origin becomes unlikely. Models such as those in Ref. 3 (with all their attendant ambiguity) can accomodate large photino masses.

(3) When $m_{\tilde\gamma} \gtrsim 4$ GeV, the freezeout temperature $T_{fr} \cong \frac{1}{20} m_{\tilde\gamma}$ lies above $T_c \cong 200$ MeV, the quark-hadron transition point. It is then appropriate to insert a factor $\kappa = [g(m_e < T < m_{\mu,\pi})/g(T = T_{fr})]^{1/2}$ in Eq. (15b). [Here $g = S/(RT)^3$ is the degeneracy factor for the entropy in a comoving volume R^3 at temperature T.] The question of significance

becomes the following: At what temperature $T^* > T_c$ does g assume its (large) value corresponding to an ideal gas of massless quarks and gluons? An estimate based on the $q\bar{q}$ interaction energy[13] relative to the kinetic energy implies that $T^* \cong 440$ MeV and the dashed curve reflects this number. However, it is crucial to note that there are nonperturbative effects in the quark-gluon plasma which are *much stronger* than the $q\bar{q}$ interaction. Self-mass effects for gluons and quarks are especially important. For example, a massless quark will grow[14] a chirally invariant mass $m_{qu}(T)$ as a result of polarization effects. To lowest order in g, $m_{qu}(T) \cong 1.3gT$.[14] Iteration of this effect leads to the calculation of the entropy of a noninteracting gas of quarks of mass $m_{qu}(T)$; this attains 50% of the massless form only when $m_{qu}(T)/T \lesssim 2$, or $g^2/4\pi \lesssim 0.20$. With $g^2/4\pi = (2\pi/9)\ln(4T/\Lambda_{\rm QCD})$ ($4T \cong$ mean c.m. energy of quark collisions) and $\Lambda_{\rm QCD} \cong 0.2$ GeV, this does not occur until $T \cong 5$ GeV! The thermodynamics is similarly strongly affected by effective mass generation for the gluons. In this context, it has been noted[14,15] that the $O(g^3)$ contribution to the free energy (the Debye-Hückel effect) is comparable to the ideal-gas value for $g^2/4\pi \gtrsim 0.25$, or $T < 0.8$ GeV. The effects of possible magnetic mass growth[14,16] are more speculative. Recent Monte Carlo calculations[17] for SU(2) suggest that the transverse modes of the gluon propagate with an effective mass $m_{\rm mag}(T) \cong 0.25g^2T$. With a modest extrapolation to $m_{\rm mag}(T) = \frac{3}{2}(0.25g^2T)$ for SU(3), the same analysis as above indicates a confluence with massless ideal-gas entropy for $T > 550$ MeV. In view of this discussion one may favor T^* to lie between 0.4 and 1.0 GeV. The two branches in the figure reflect these boundaries.

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