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Violations of Bell's Inequality in Cooperative States

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Cooperative multiparticle quantum states in an Einstein-Podolsky-Rosen type of experiment are shown to violate a Bell inequality. Thus correlations can occur between wave packets each having N bosons which cannot be explained within local hidden-variable theories. This provides a way to test quantum measurement theory for multiparticle states. In these states, a local, positive probability distribution exists for all N. However, the representation of correlations with this probability distribution is different from that in standard (Bell-type) hidden-variable theories.

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Correlation measurements over spacelike distances are known to provide a stringent test of quantum theory.[1] Experiments of this type are often described as Einstein-Podolsky-Rosen[2] experiments after these authors' well-known discussion of the possible incompleteness of quantum mechanics. In recent experiments,[3] observed measurements were shown to violate a Bell inequality, which implies that they cannot be described within a large class of hidden-variable theories. The existence of experiments that allow a direct comparison between quantum theory and alternative theories has a great significance in view of the widespread acceptance of quantum theory in current physics. However, evidence to date is largely restricted to correlations of single cascade photons in atomic photoemission.[3]

In this Letter, a new test of the foundations of quantum theory is described. This test has the unusual property that it is a large-scale measurement in which each wave packet includes N particles. The motivation for this proposal is that previous Bell-inequality experiments have been restricted to low intensities, in fact to situations involving just one particle per detector. This restriction to low intensities also holds for other tests of measurable correlations that distinguish quantized from classical electrodynamics, as in the case of photon antibunching.[4] At a deeper level, one can inquire if all nonclassical effects in quantum measurement theory are limited to these relatively low particle numbers.

If possible, it is preferable to test quantum theory more rigorously than just at the single-particle level. The suggestion that nonlocal correlations and cooperative emission of particles be combined is therefore proposed. The central point of interest is whether any violation of a Bell inequality can occur in multiparticle states. In order to determine this, the correlations of the intensity of an electromagnetic field, as observed with polarized detectors, will be calculated for a specific cooperative state of N photon pairs. This state has similar properties to the superfluorescent[5] radiation of a group of N cascade photoemitters, in the approximation that all the atoms emit cooperatively on a short time scale.

The quantum-field state of interest is defined to be

$$|N\rangle = [a_1{}^\dagger b_1{}^\dagger + a_2{}^\dagger b_2{}^\dagger]^N |0\rangle / N!\,(N+1)^{1/2}. \qquad (1)$$

Here N is the number of quanta produced at energies E_a, E_b. The operators $a_1{}^\dagger, a_2{}^\dagger$ create orthogonally polarized quanta with energy E_a propagating in the $+x$ direction, which are polarized in the y,z directions, respectively. The operators $b_1^\dagger, b_2^\dagger$ behave identically, except that they create quanta with energy E_b which propagate in the $-x$ direction. In general, the four operators could correspond to any four orthogonal modes of a boson field. The state $|0\rangle$ is the ground state of the field.

It is clear that $|N\rangle$ is a well defined state of a quantum field.[6] I wish to calculate the possible results of measurements on this state according to quantum theory. The observable measurements will be defined here as intensity correlations of the structure

$$G^{IJ}(\gamma;N) = \langle N|A^I(1,\vec{a})A^J(\gamma,\vec{b})|N\rangle\,, \tag{2}$$

where

$$\begin{gathered}\gamma \equiv \cos^2\theta, \quad \vec{a} \equiv [a_1^\dagger, a_2^\dagger, a_1, a_2], \quad \vec{b} \equiv [b_1^\dagger, b_2^\dagger, b_1, b_2], \\ A^J(\gamma,\vec{a}) \equiv [\gamma^{1/2}a_1^\dagger + (1-\gamma)^{1/2}a_2^\dagger]^J[\gamma^{1/2}a_1 + (1-\gamma)^{1/2}a_2]^J, \quad A^J(\infty,\vec{a}) \equiv :[a_1^\dagger a_1 + a_2^\dagger a_2]^J:\,.\end{gathered} \tag{3}$$

The correlation $G^{IJ}(\gamma;N)$ is proportional to the probability[7] of observation of I quanta at position X, of type "a," with polarizer angle θ_a; and of J quanta at position $-X$, of type "b," with polarizer angle θ_b. The coefficient γ depends on the relative polarizer angle $\theta \equiv \theta_a - \theta_b$. The measurement $G^{IJ}(\infty;N)$ is identical to $G^{IJ}(\gamma;N)$ except that there is no polarizer in the detection of the J type-"b" quanta. In the case of symmetric observations, relative probabilities are defined as

$$g_N^J(\theta) \equiv G^{JJ}(\gamma;N)/G^{JJ}(\infty;N). \tag{4}$$

The result of counting J photons at an idealized detector is a binary experiment, in the sense that there are either J photocounts observed or not. It is known that it is just the binary character of the measurement, together with the polarizer adjustment, that allows one to obtain Bell's inequalities.[8] Hence Bell's inequalities are directly obtained for this type of correlation measurement.

The no-enhancement axiom[3] then reduces Bell's inequalities to a form applicable to relative probabilities. In terms of a hidden-variable theory of this type, one particularly useful Bell-type inequality[3] is

$$3g(\theta) - g(3\theta) - 2 \equiv \Delta(\theta) \le 0. \tag{5}$$

One can now ask a simple question, on combining Eqs. (1)–(4). Are there any combinations of numbers of particles and angles for which Eq. (5) is violated in quantum theory? If this were the case, then the predictions of quantum measurement theory would be testable in a region where one might expect classical ideas of measurement to be valid, that is, in the region of large numbers of particles per quantum state.[9]

To answer this, the quantum intensity correlation functions can be calculated with standard techniques, giving the result

$$\begin{gathered}G^{IJ}(\gamma;N) = \left(\frac{1}{N+1}\right)\sum_{n=0}^{N-J}\frac{n![(N-n)!]^2}{(N-I)!(N-J-n)!}\sum_{i=0}^{I}\binom{I}{i}^2\binom{N-J}{n-i}(1-\gamma)^i\gamma^{I-i}\,, \\ G^{IJ}(\infty;N) = \left(\frac{N!}{(N+1)(N-I)!}\right)\sum_{n=0}^{N-J}\frac{(N-n)!}{(N-J-n)!}\,.\end{gathered} \tag{6}$$

Limits of these expressions that are of particular simplicity occur either for $I=J=N$ (counting all the photons) or for $I=J=1$ (counting just one photon). These combine to give relative probabilties

$$\begin{gathered}g_N^1(\theta) = [N-1+\gamma(N+2)]/3N, \\ g_J^J(\theta) = \gamma^J\end{gathered} \tag{7}$$

for the cases $I=J=1$ and $I=J=N$, respectively.

For the case of $N=1$, these results are identical, and reproduce the standard Bell-inequality violation.[3] For $J=1$, the relative correlation function for large N does not violate the inequality. In fact the results have a close relationship with a random, classically correlated polarization model.

For $J=N$, however, there is a strikingly different behavior. The correlation is sharply peaked near $\theta=0$, with a stronger correlation than in any classical model. For all $J=N$, the inequality [Eq. (5)] is violated at finite θ. In fact for large $J=N$, $g_J^J(\theta)$ tends to a limiting behavior as a function of $\theta\sqrt{J}$:

$$\lim_{J\to\infty} g_J^J(\theta) = \exp(-J\theta^2/2). \tag{8}$$

In the limit of $J=N\to\infty$, the largest violation of the inequality is given analytically by $(\frac{8}{3})^{1.125}-2 = 0.332\,45\ldots$, which occurs at $\theta = \frac{1}{2}[(\ln 3)/J]^{1/2}$.

This limiting behavior is shown graphically in Fig. 1. It is worth emphasizing that unlike the case of photon antibunching or other low-order correlations, there is a nonzero limit to the violation of the classical inequality at large particle number.

In view of this nonclassical behavior, it seems possible that no local, positive probabilistic representation[10] of the state $|N\rangle$ exists. Nevertheless, a suitable positive representation for $|N\rangle$ can in fact be found for any N, including $N=1$. This is the generalized positive P representation[11,12] in which the wave function and its correlations are represented by use of a positive distribution $P(\vec{\alpha},\vec{\beta})$ with $\vec{\alpha}=(\alpha_1^\dagger,\alpha_2^\dagger,\alpha_1,\alpha_2)$, $\vec{\beta}=(\beta_1^\dagger,\beta_2^\dagger,\beta_1,\beta_2)$. In the case of $|N\rangle$, the result is

$$G^{IJ}(\gamma;N)=\int P(\vec{\alpha},\vec{\beta})A(1,\vec{\alpha})A(\gamma,\vec{\beta})\,d^8\alpha\,d^8\beta,$$

where

$$P(\vec{\alpha},\vec{\beta})=\left\{\frac{|(\alpha_1^\dagger+\alpha_1^*)(\beta_1^\dagger+\beta_1^*)+(\alpha_2^\dagger+\alpha_2^*)(\beta_2^\dagger+\beta_2^*)|^{2N}}{(2\pi)^8(N+1)(N!)^2 2^{4N}}\right\}\exp[(-|\vec{\alpha}|^2-|\vec{\beta}|^2)/2]. \tag{9}$$

Correlations like Eqs. (2) and (3) are simply calculated on averaging over all complex values of $\vec{\alpha}$ and $\vec{\beta}$. One may reasonably infer, since a calculation with $P(\vec{\alpha},\vec{\beta})$ reproduces the quantum predictions, that Bell-inequality violations do not rule out all local, positive distribution representations. In fact, the structure of Eq. (9) is identical to that proposed by Bell for hidden-variable theories, except that the function A has values that are complex.

It is clearly useful to describe a simple way to prepare these states, by adapting the original cascade experiment.[3] This would require an input of atoms in the upper state of an $F=0, F=1, F=0$ cascade level structure, under circumstances similar to those used to observe superfluorescence.[5] In order to define the field mode, an optically pumped beam with a well defined number (N) of atoms should be incident on the mode waist of an interferometer. This should be resonant at each transition frequency, with a larger bandwidth than the atomic natural linewidth, to allow cooperative emission to take place. The correlations of emitted photons would then be measured with use of polarized photodetectors at the interferometer outputs. It is straightforward to show, in the case $I=J=N$, that relative correlations are given within quantum theory by Eq. (7), and hence should violate the Bell inequality given by Eq. (5).

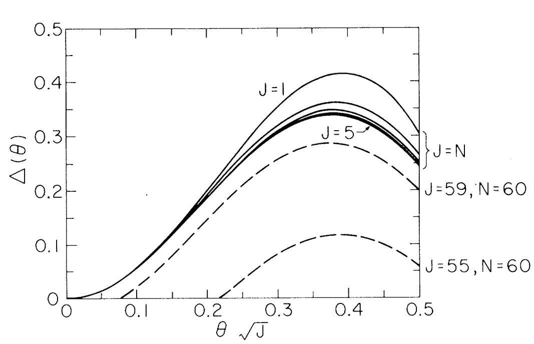

FIG. 1. The violation of the Bell inequality as a function of the reduced angle $\theta\sqrt{J}$, for $J=N=1,2,3,4,5$ (solid lines) and for $J=55, 59$ ($N=60$) (dashed lines).

Irrespective of the techniques used to prepare them, states of the structure implied by Eq. (1) can provide new tests of quantum measurement theory. These tests are certainly not restricted to quantum optics, since only the general boson commutation relations are needed to violate Eq. (5). It is clear from this analysis that the degree of quantum or classical correlation depends on the order of the measured correlation. While classical behavior may occur relative to single-particle detection, distinctly quantum properties are predicted to occur in the higher-order correlations that correspond to multiparticle measurements. Thus the usual classical paradigm that a large number of particles in one state have classical measurement properties is not necessarily true.

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